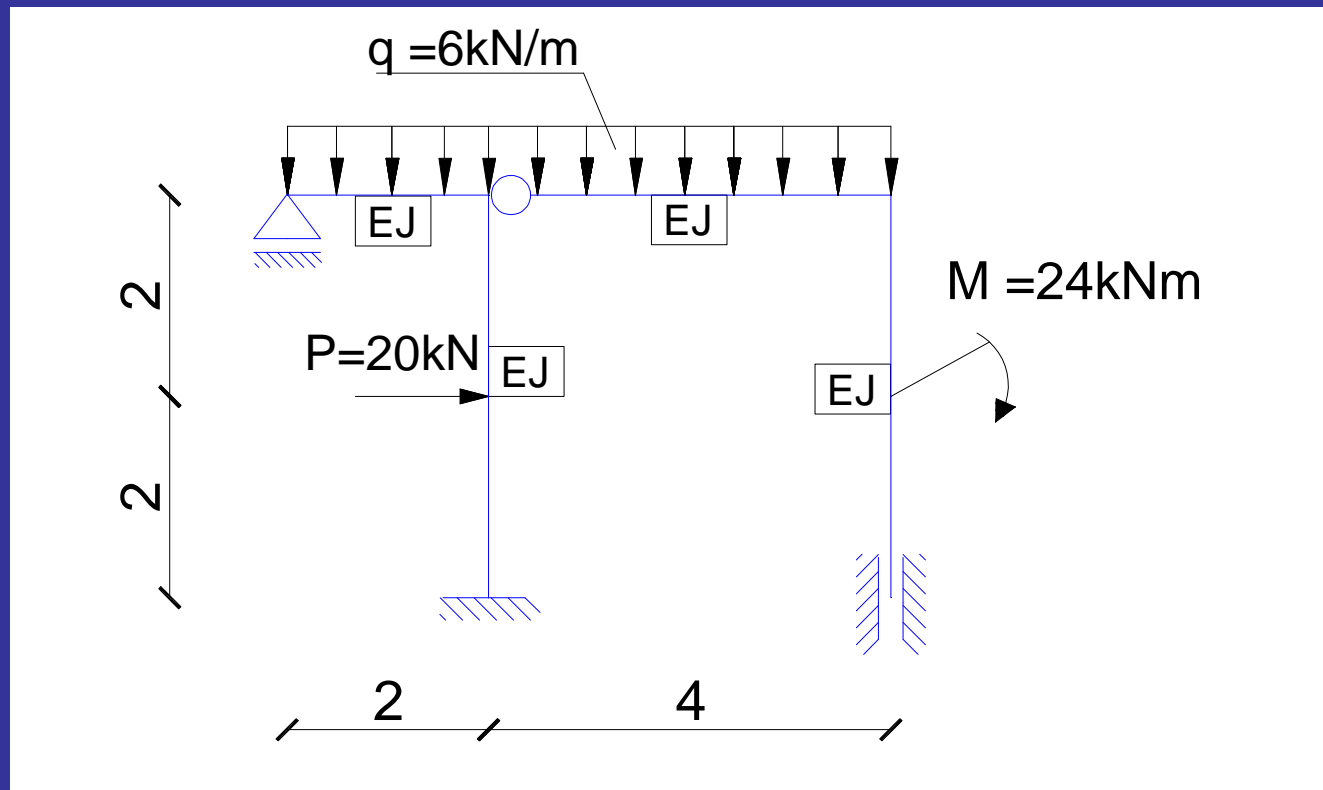


# Metoda Sił

Rysowanie wykresów sił wewnętrznych  
w ramach statycznie niewyznaczalnych

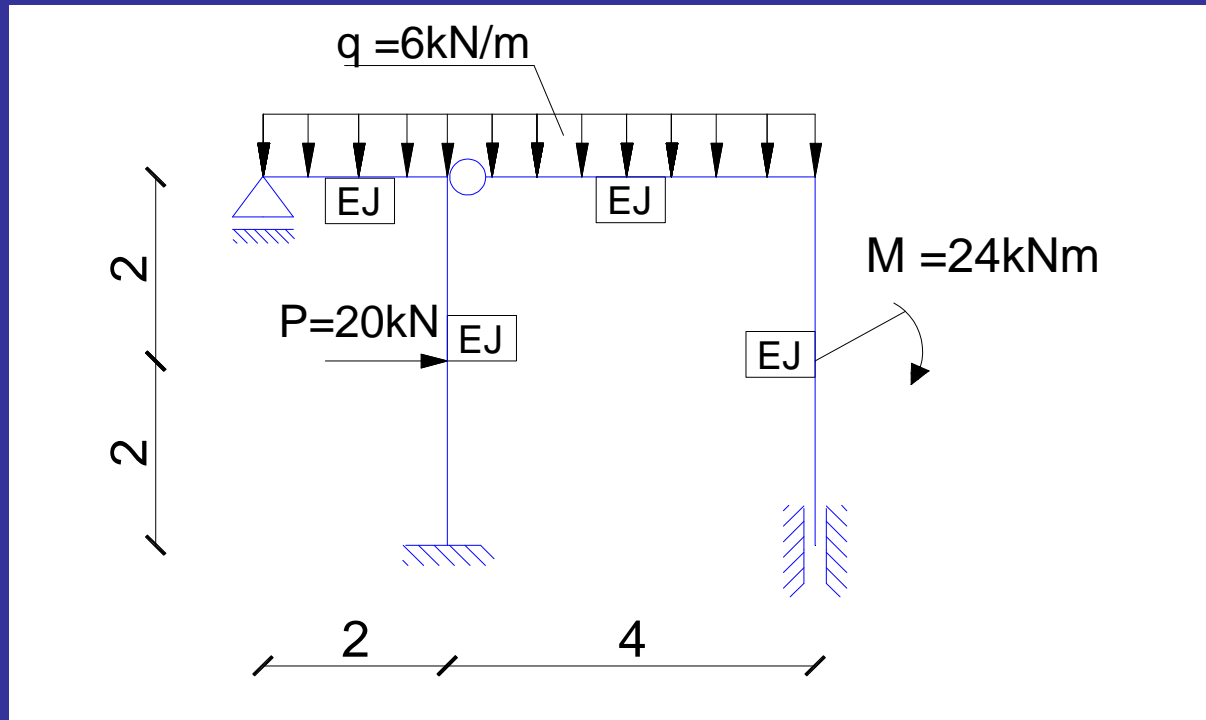
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Zadanie: Narysuj wykresy sił N, T, M.  
Zadanie rozwiąż metodą sił.



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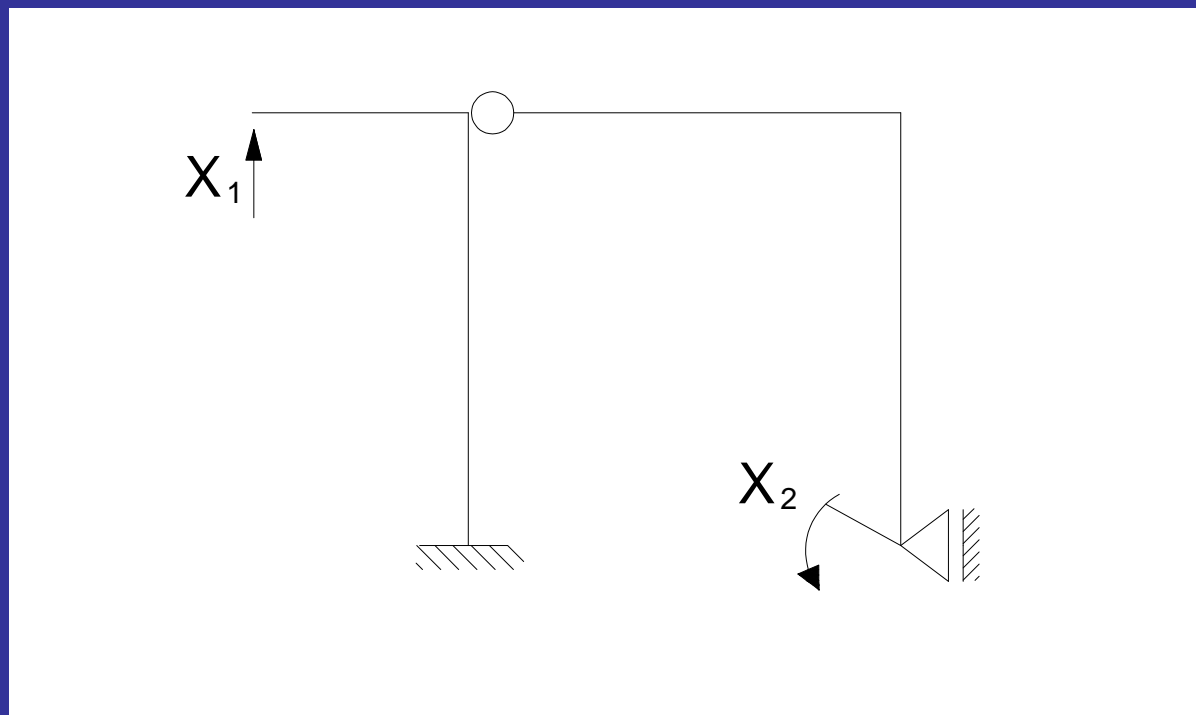
## Określenie stopnia statycznej niewyznaczalności :



$$n_s = l_r - l_p - 3 = 6 - 1 - 3 = 2$$

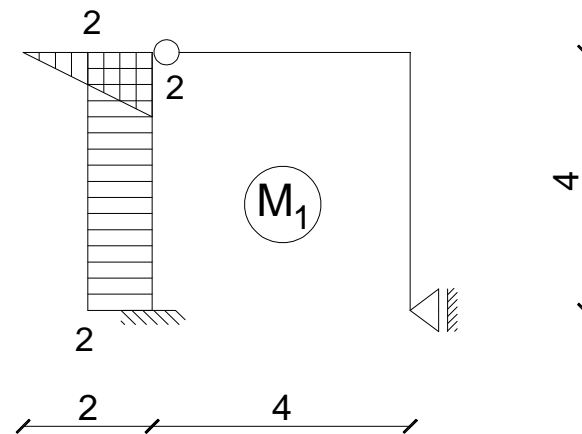
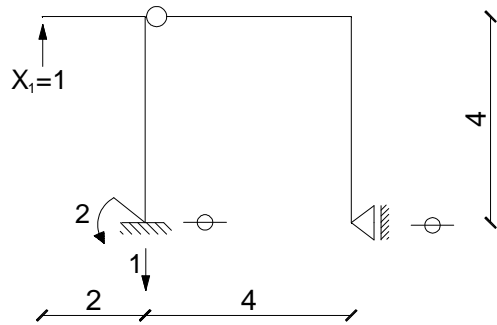
Układ dwukrotnie statycznie niewyznaczalny

## Schemat podstawowy statycznie wyznaczalny:

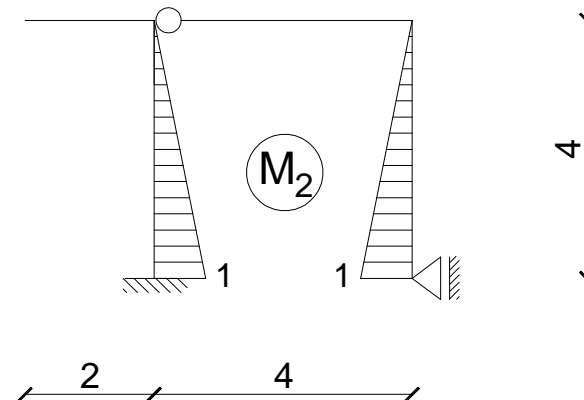
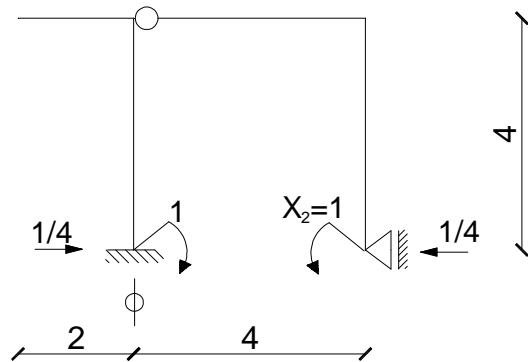


*dr inż. Hanna Weber*

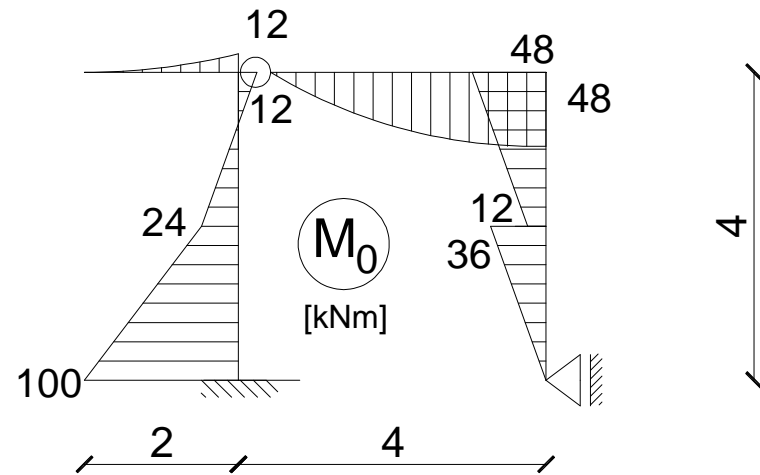
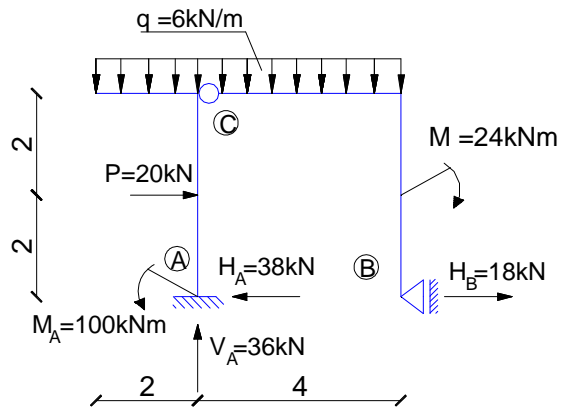
# Wykres $X_1=1$



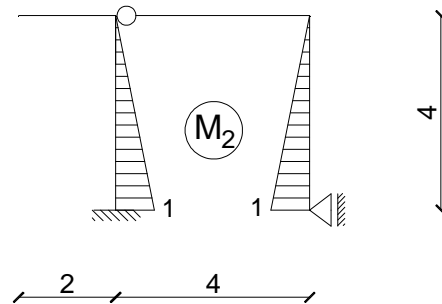
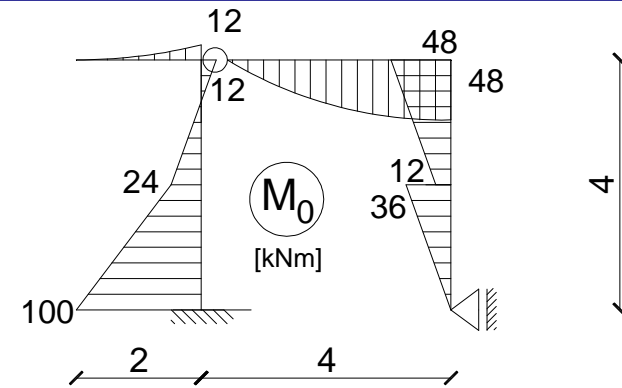
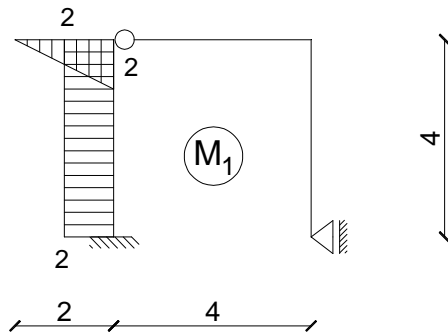
# Wykres $X_2=1$



# Wykres od obciążenia zewnętrznego:



## Całkowanie wykresów:



$$\delta_{11} = \frac{1}{EI} \left( \frac{1}{2} \cdot 2 \cdot 2 \cdot \frac{2}{3} \cdot 2 + 2 \cdot 4 \cdot 2 \right) = \frac{56}{3EI}$$

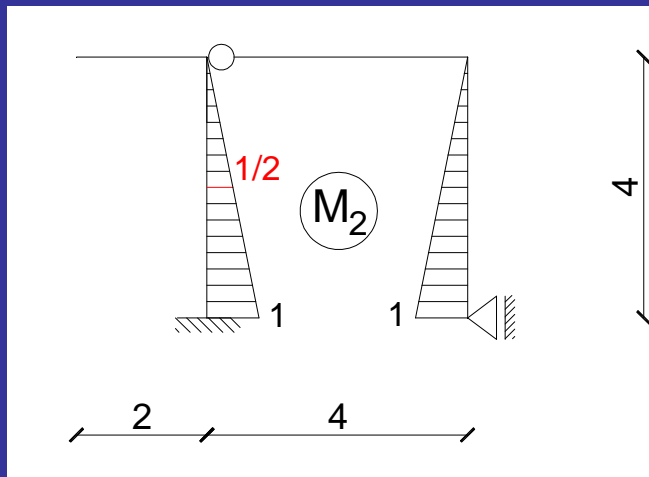
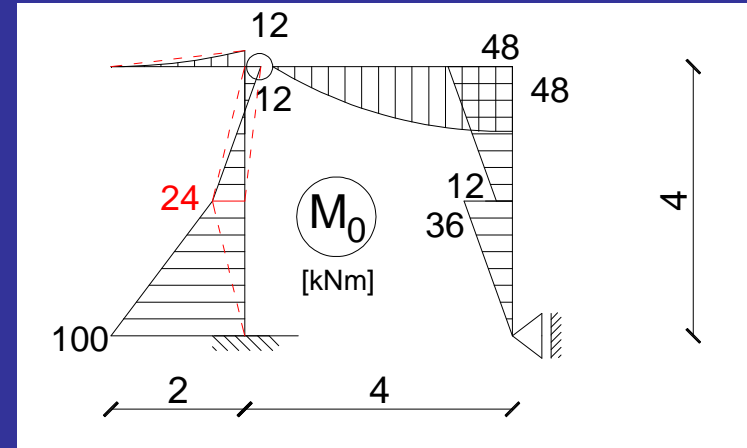
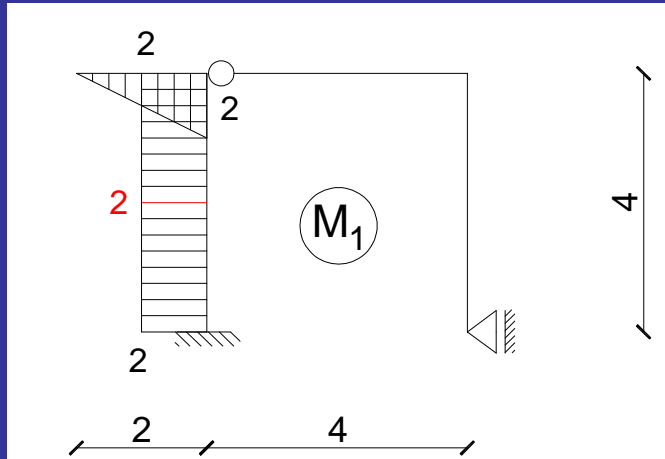
$$\delta_{12} = -\frac{1}{EI} \left( 2 \cdot 4 \cdot \frac{1}{2} \cdot 1 \right) = -\frac{4}{EI} = \delta_{21}$$

$$\delta_{22} = \frac{1}{EI} \left( \frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 \right) = \frac{8}{3EI}$$

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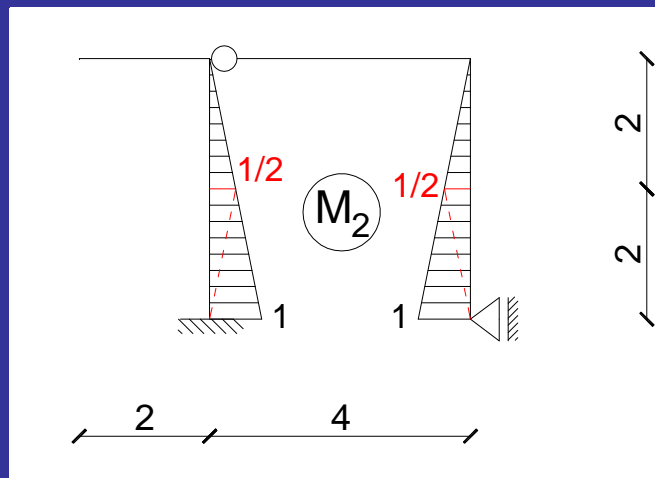
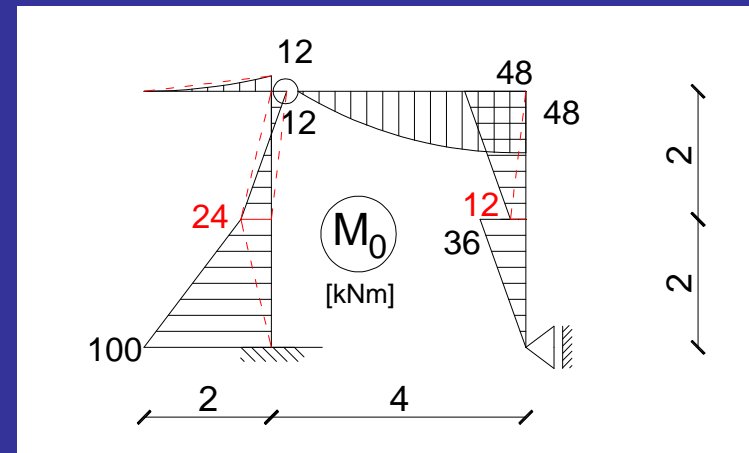
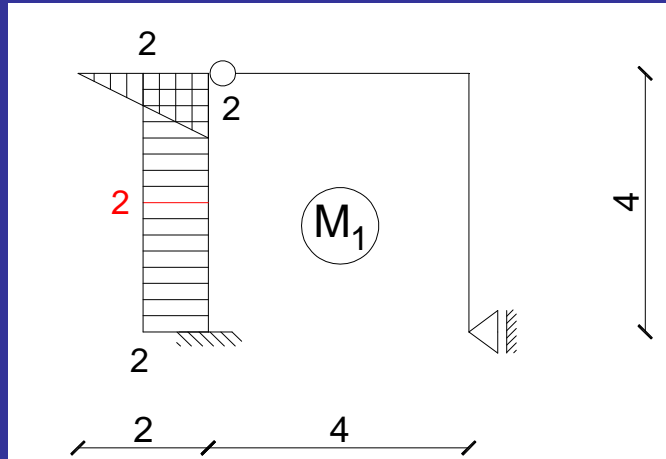
## Całkowanie wykresów:



$$\delta_{10} = \frac{1}{EI} \left( -\frac{1}{2} \cdot 12 \cdot 2 \cdot \frac{2}{3} \cdot 2 + \frac{2}{3} \cdot \frac{6 \cdot 2^2}{8} \cdot 2 \cdot \frac{1}{2} \cdot 2 \right) + \frac{1}{EI} 2 \cdot 2 \cdot \left( \frac{24}{2} - \frac{12}{2} \right) + \frac{1}{EI} 2 \cdot 2 \cdot \left( \frac{100}{2} + \frac{24}{2} \right) = \frac{260}{EI}$$

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## Całkowanie wykresów:



$$\begin{aligned} \delta_{20} = & \frac{1}{EI} \left( \frac{1}{2} \cdot \frac{1}{2} \cdot 2 \cdot \left( \frac{1}{3} \cdot 12 - \frac{2}{3} \cdot 24 \right) - \frac{1}{2} \cdot \frac{1}{2} \cdot 2 \cdot \left( \frac{2}{3} \cdot 24 + \frac{1}{3} \cdot 100 \right) \right) \\ & - \frac{1}{EI} \frac{1}{2} \cdot 1 \cdot 2 \cdot \left( \frac{2}{3} \cdot 100 + \frac{1}{3} \cdot 24 \right) + \frac{1}{EI} \frac{1}{2} \cdot \frac{1}{2} \cdot 2 \cdot \left( \frac{2}{3} \cdot 12 + \frac{1}{3} \cdot 48 \right) \\ & + \frac{1}{EI} \frac{1}{2} \cdot 36 \cdot 2 \cdot \left( \frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1 \right) = -\frac{208}{3EI} \end{aligned}$$

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Układ równań kanonicznych metody sił  
dla schematu dwukrotnie statycznie  
niewyznaczalnego:

$$\begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{10} = 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{20} = 0 \end{cases}$$

$$\begin{cases} \frac{56}{3EI} \cdot X_1 - \frac{4}{EI} \cdot X_2 + \frac{260}{EI} = 0 \\ -\frac{4}{EI} \cdot X_1 + \frac{8}{3EI} \cdot X_2 - \frac{208}{3EI} = 0 \end{cases}$$

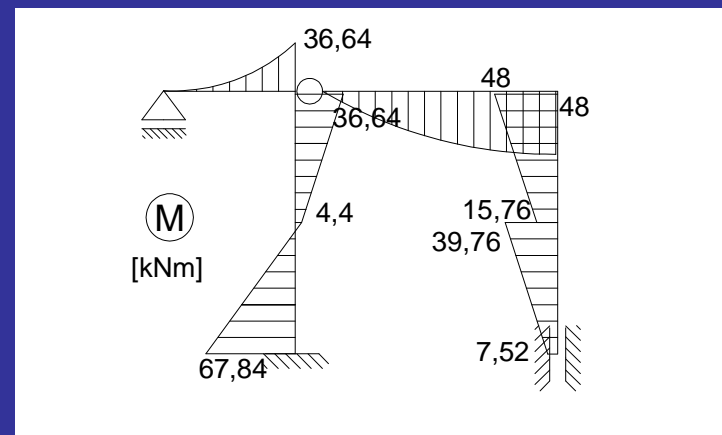
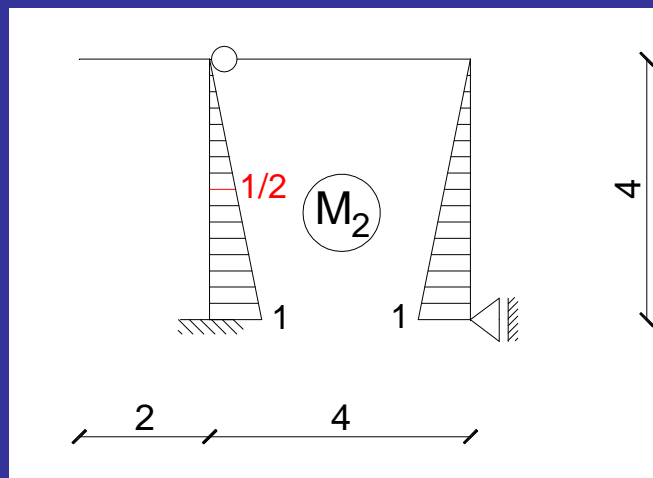
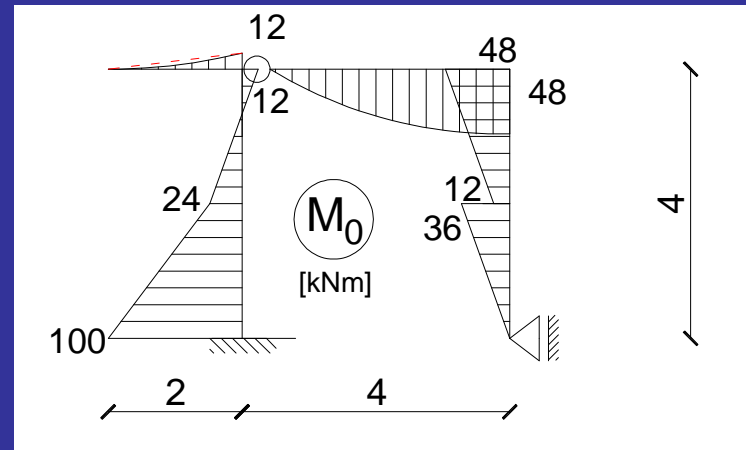
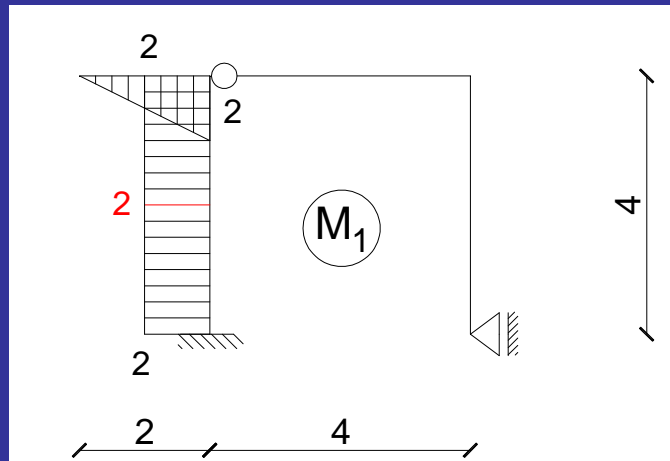
↓

$$X_1 = -12,32kN$$

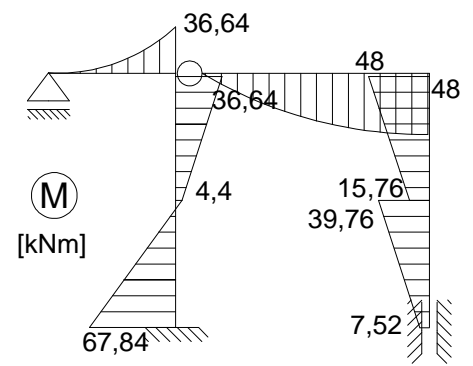
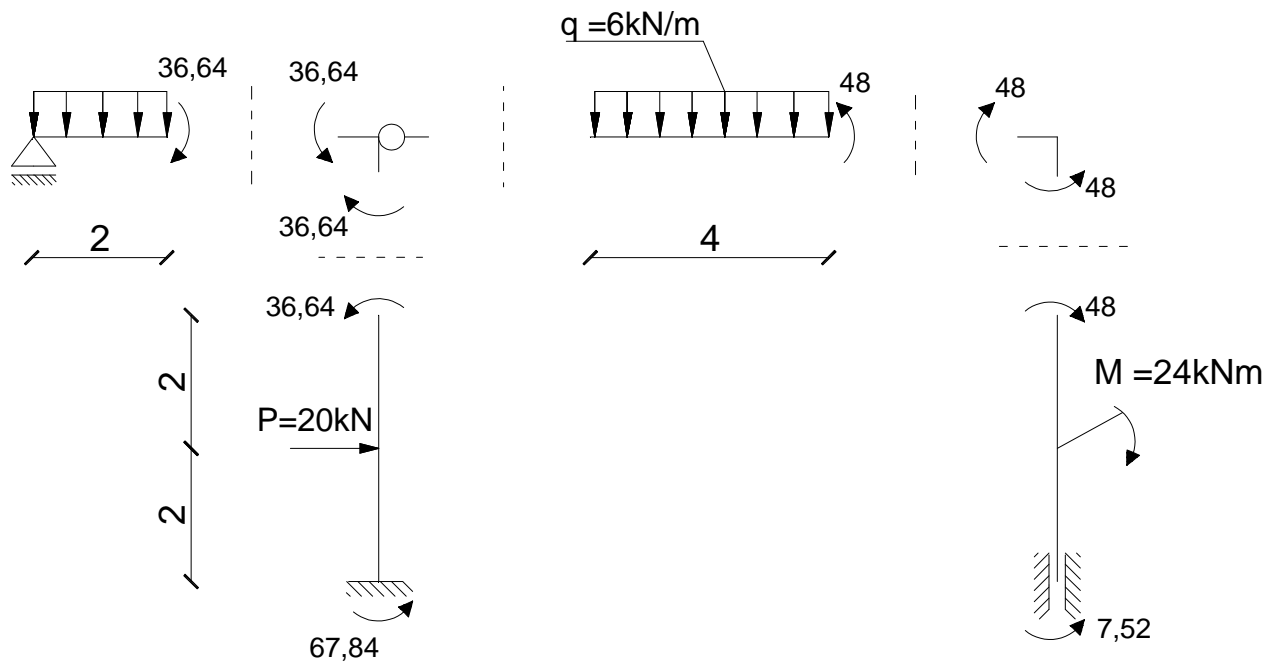
$$X_2 = 7,52kNm$$

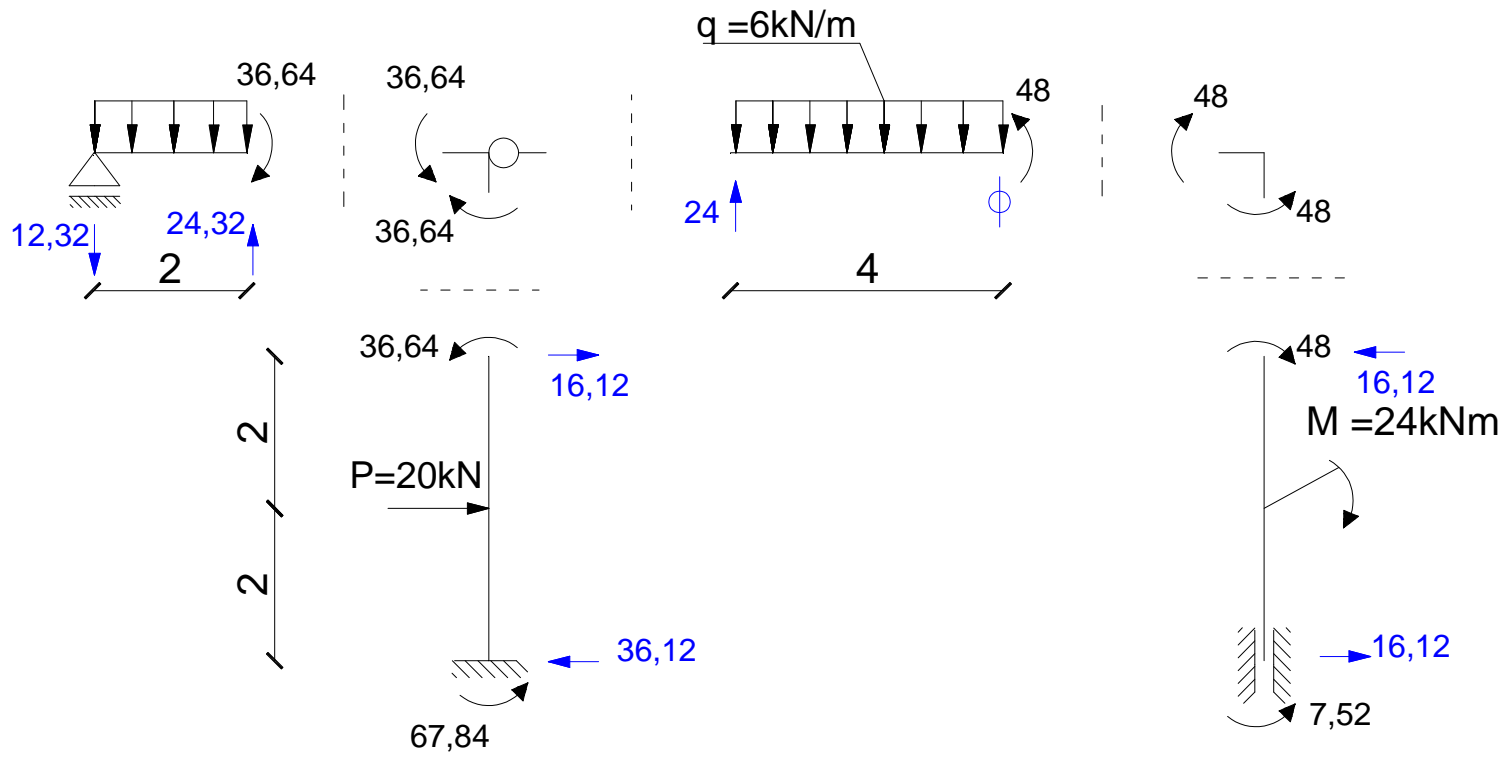
*dr inż. Hanna Weber*

# Tworzenie ostatecznego wykresu momentów:

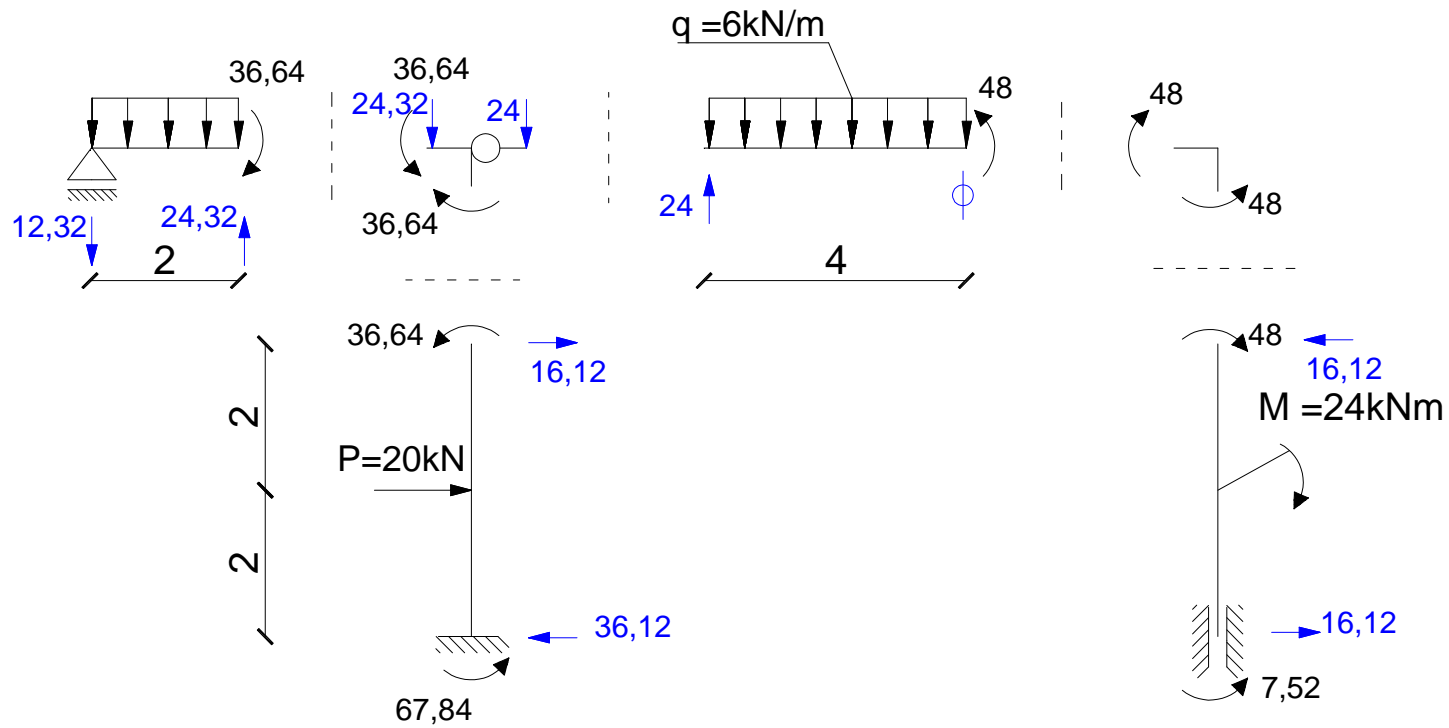


$$M_i = M_{i1} \cdot X_1 + M_{i2} \cdot X_2 + M_{i0}$$

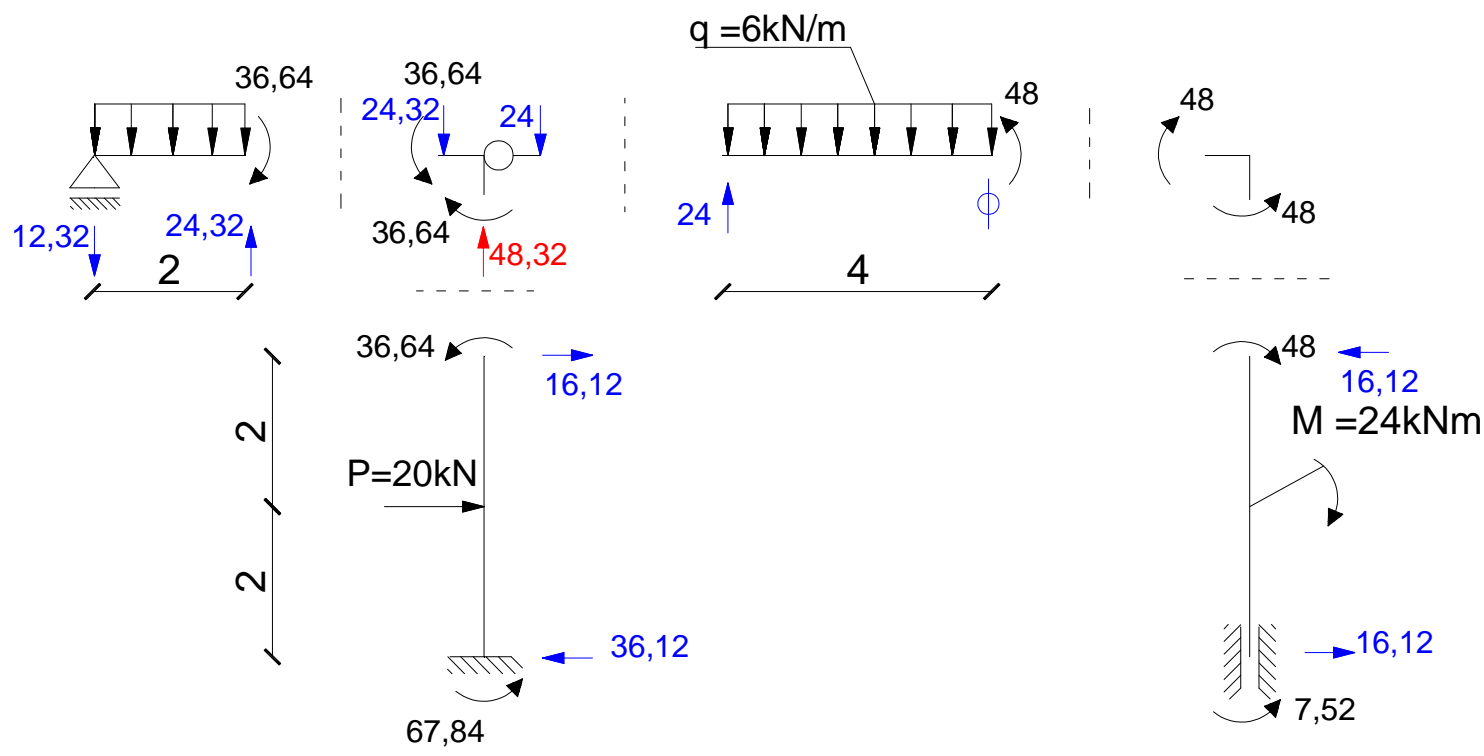




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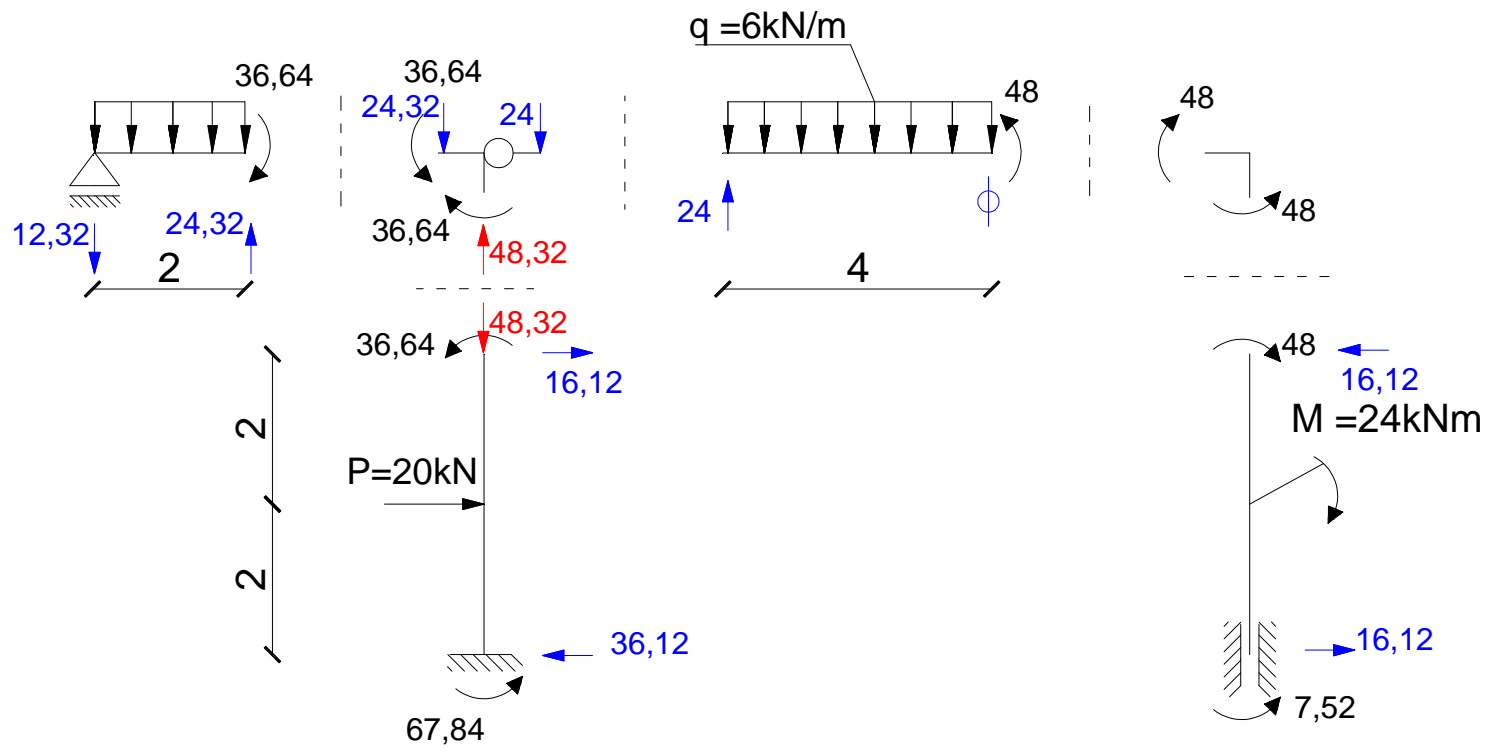


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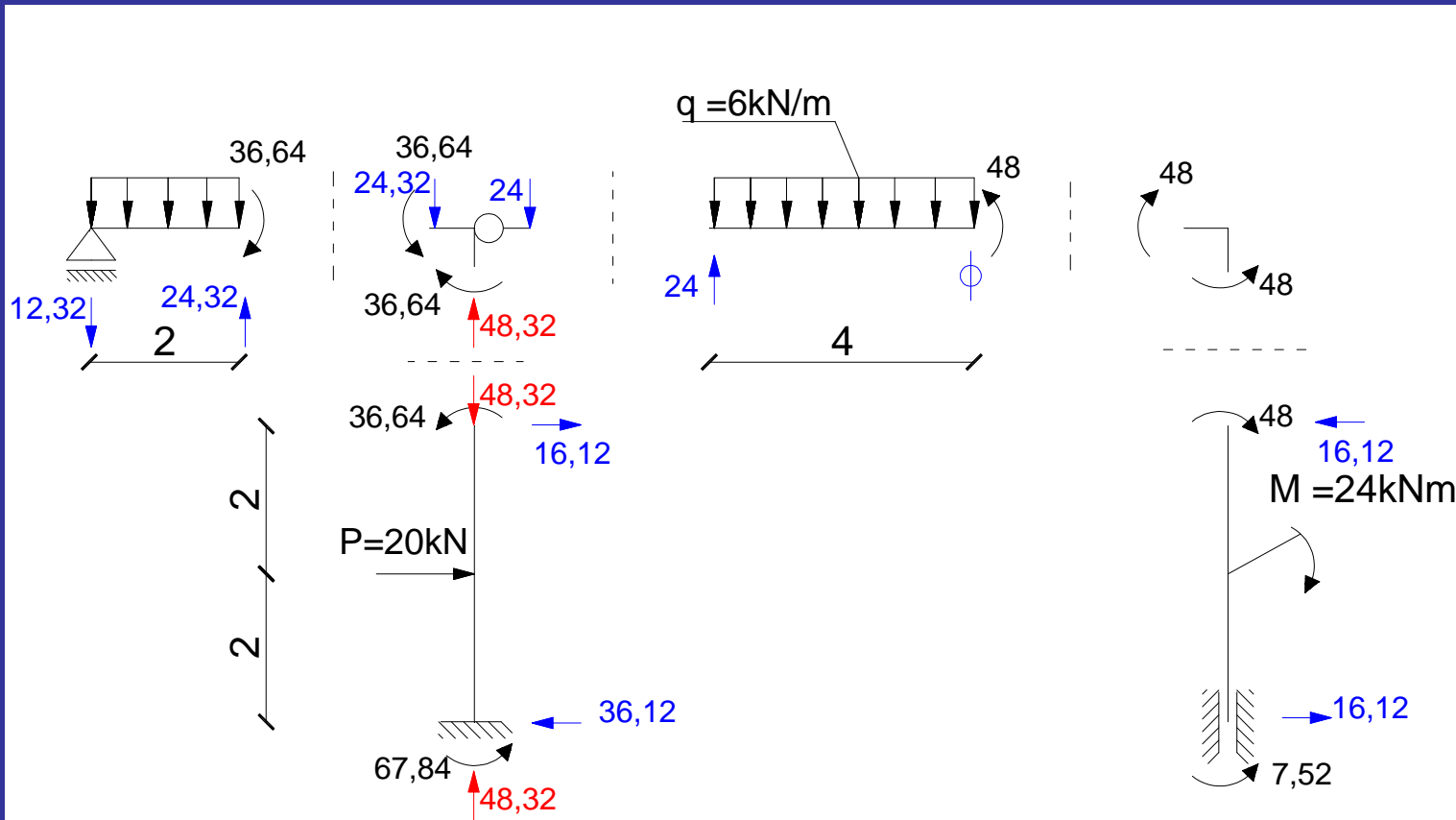


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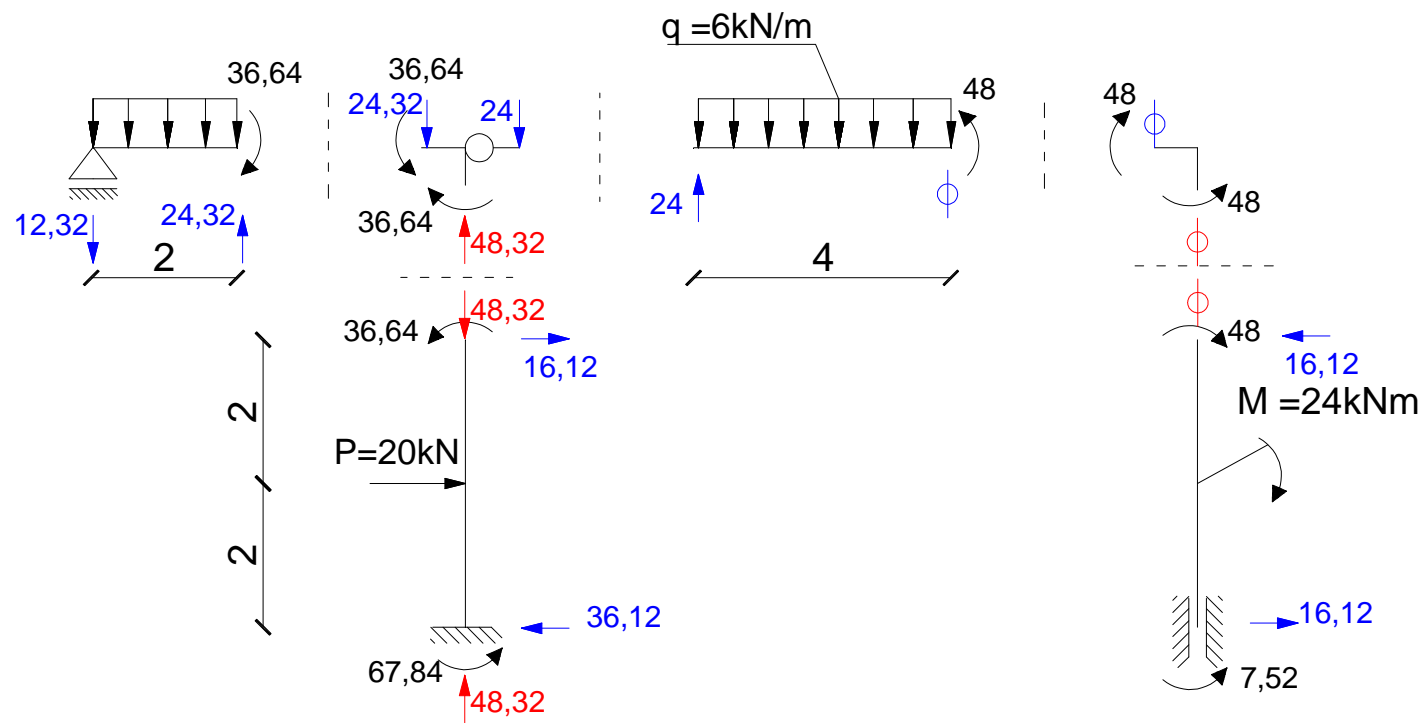




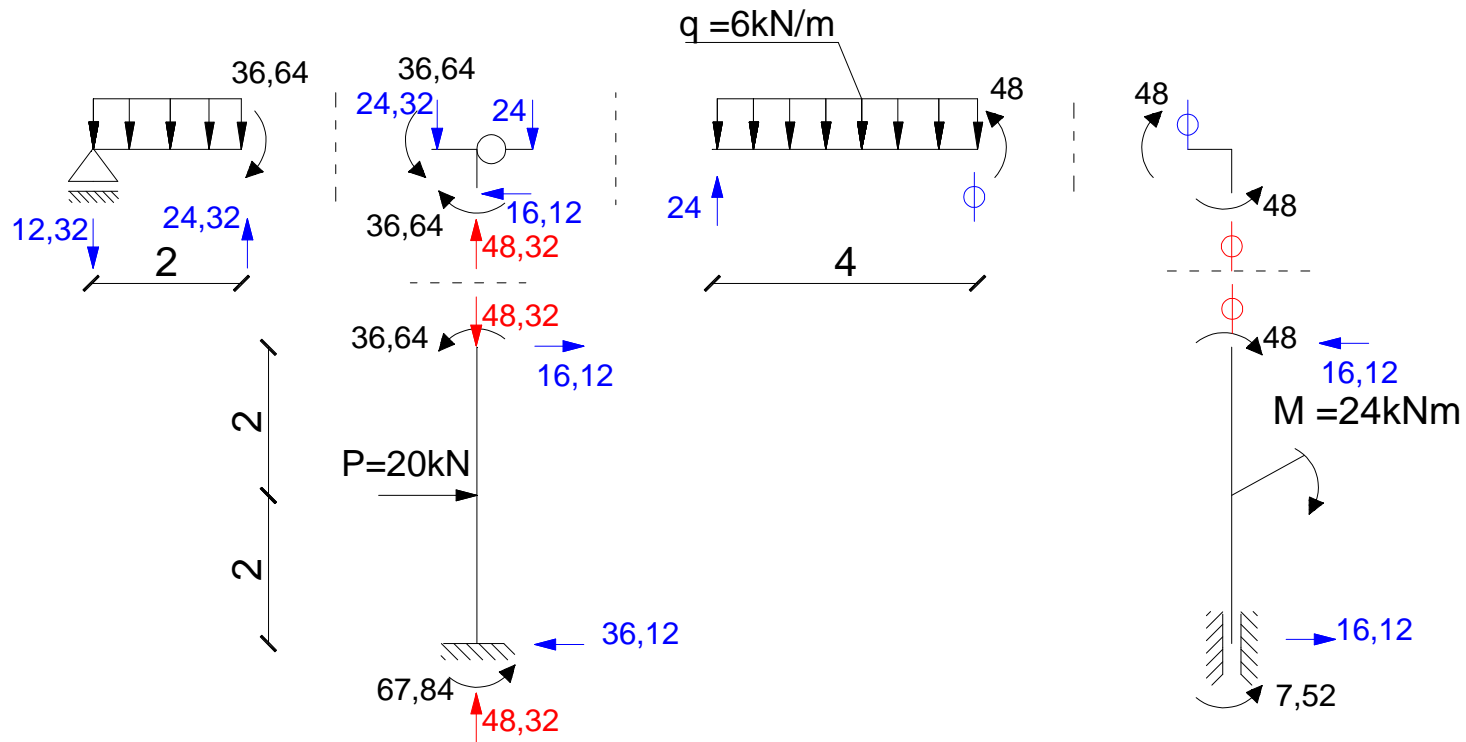
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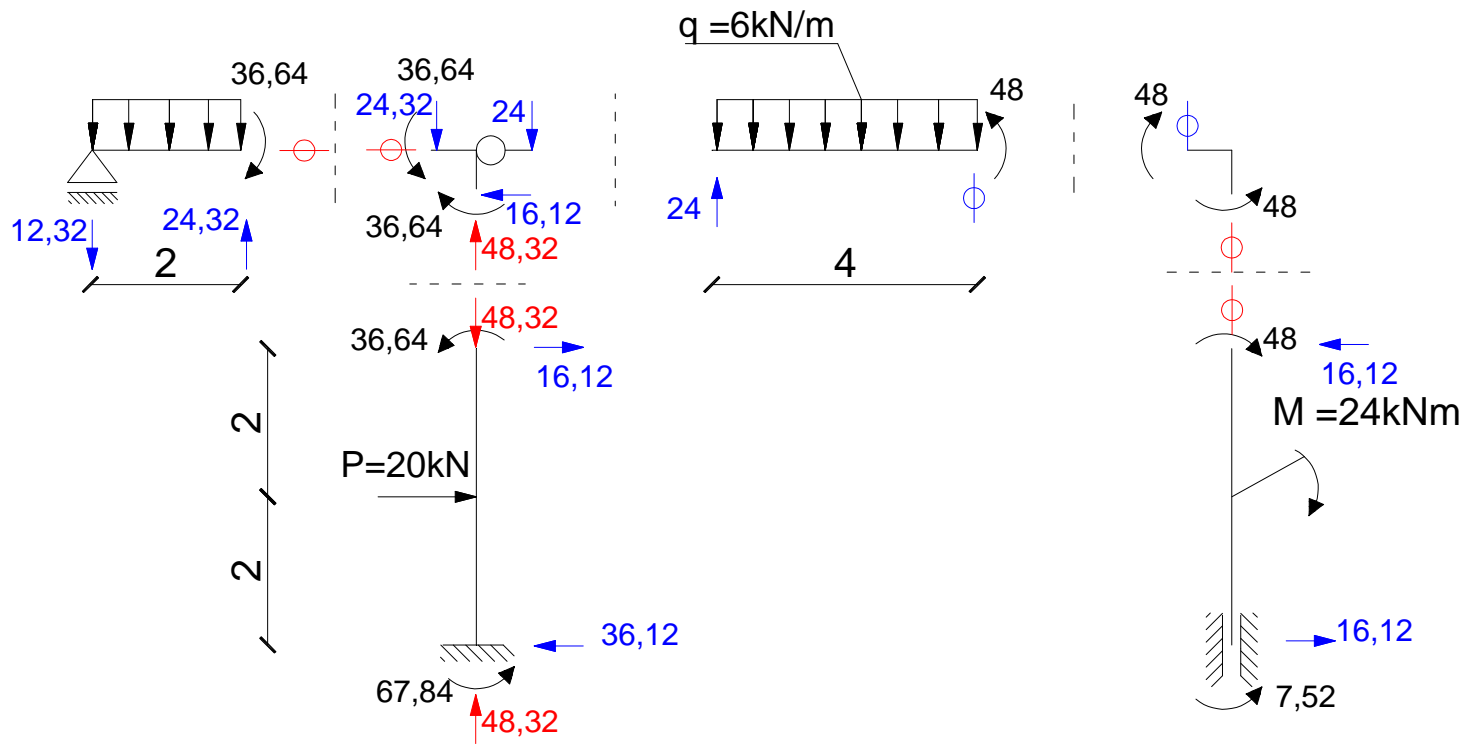
dr inż. Hanna Weber



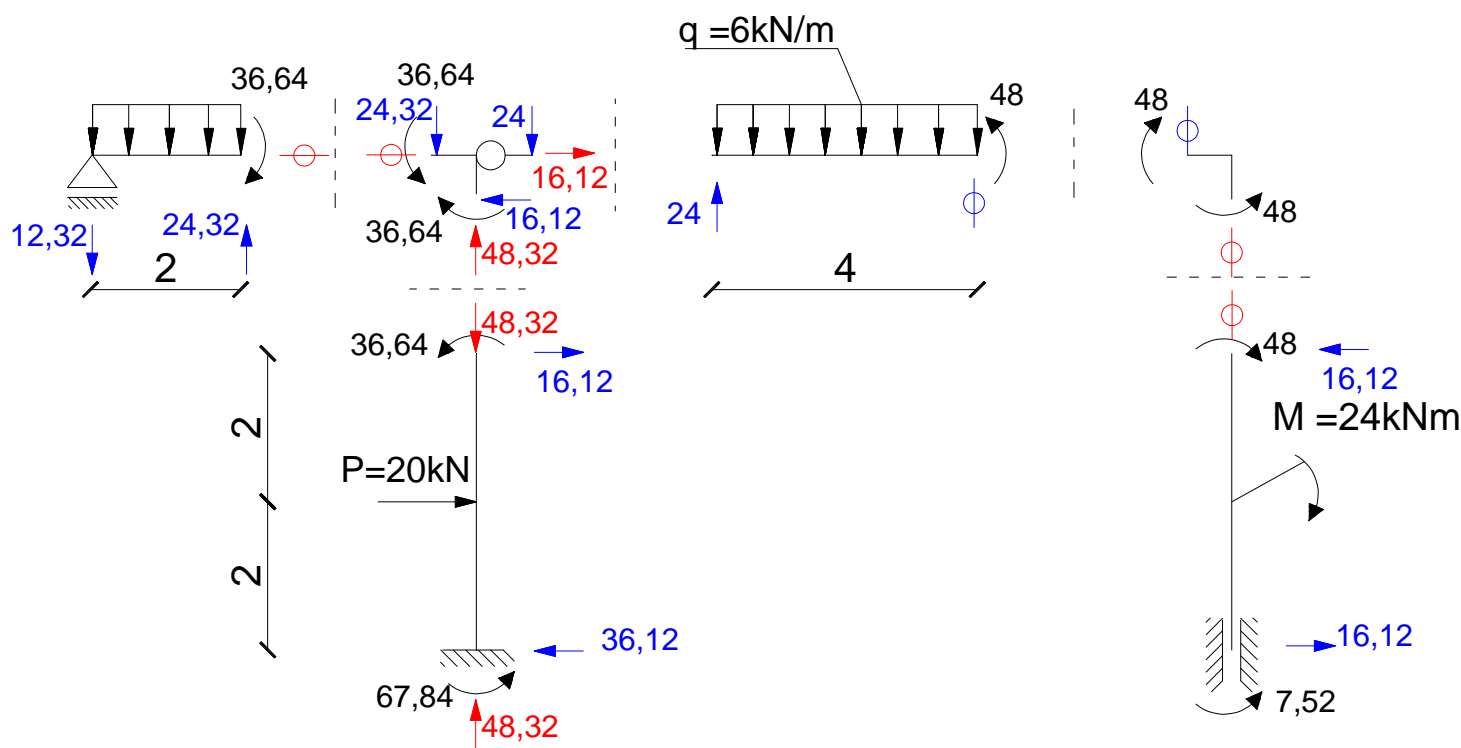
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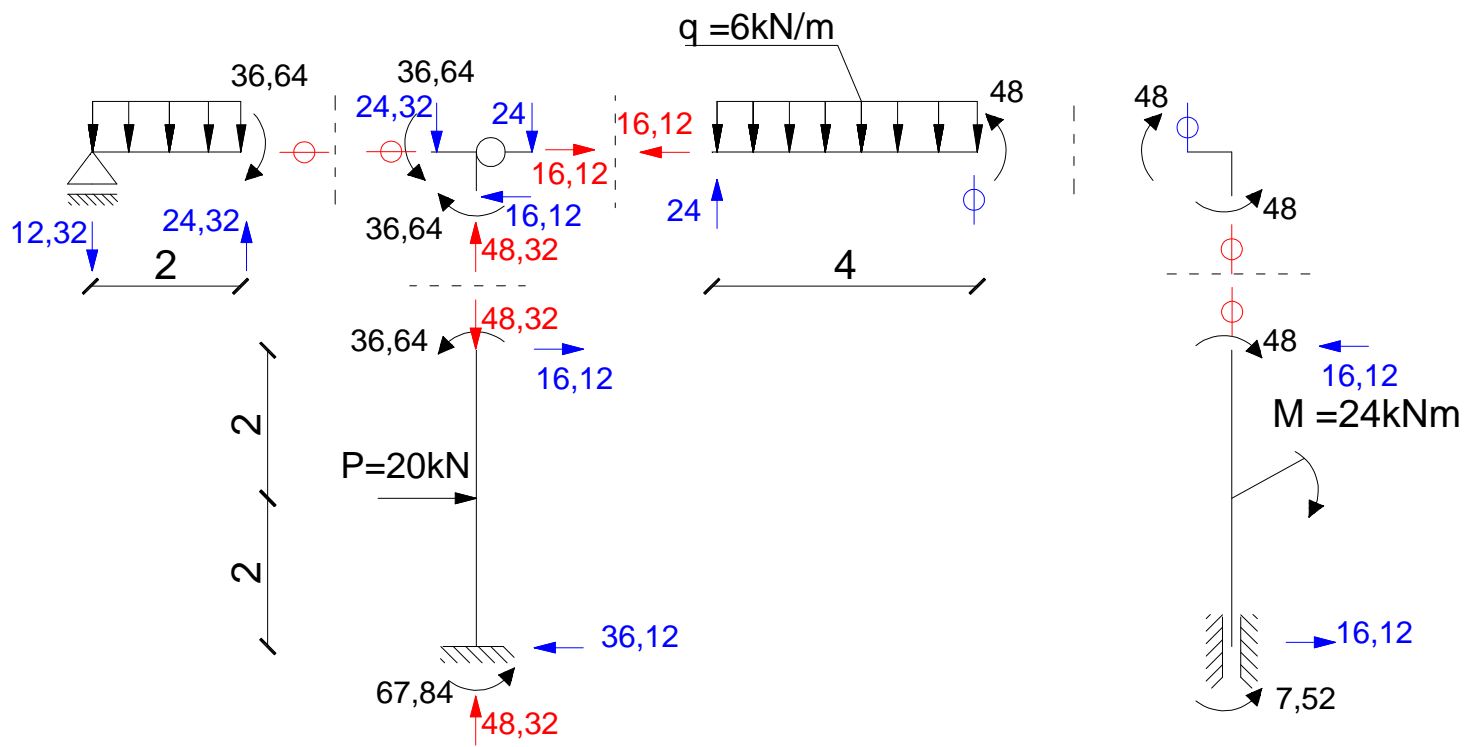
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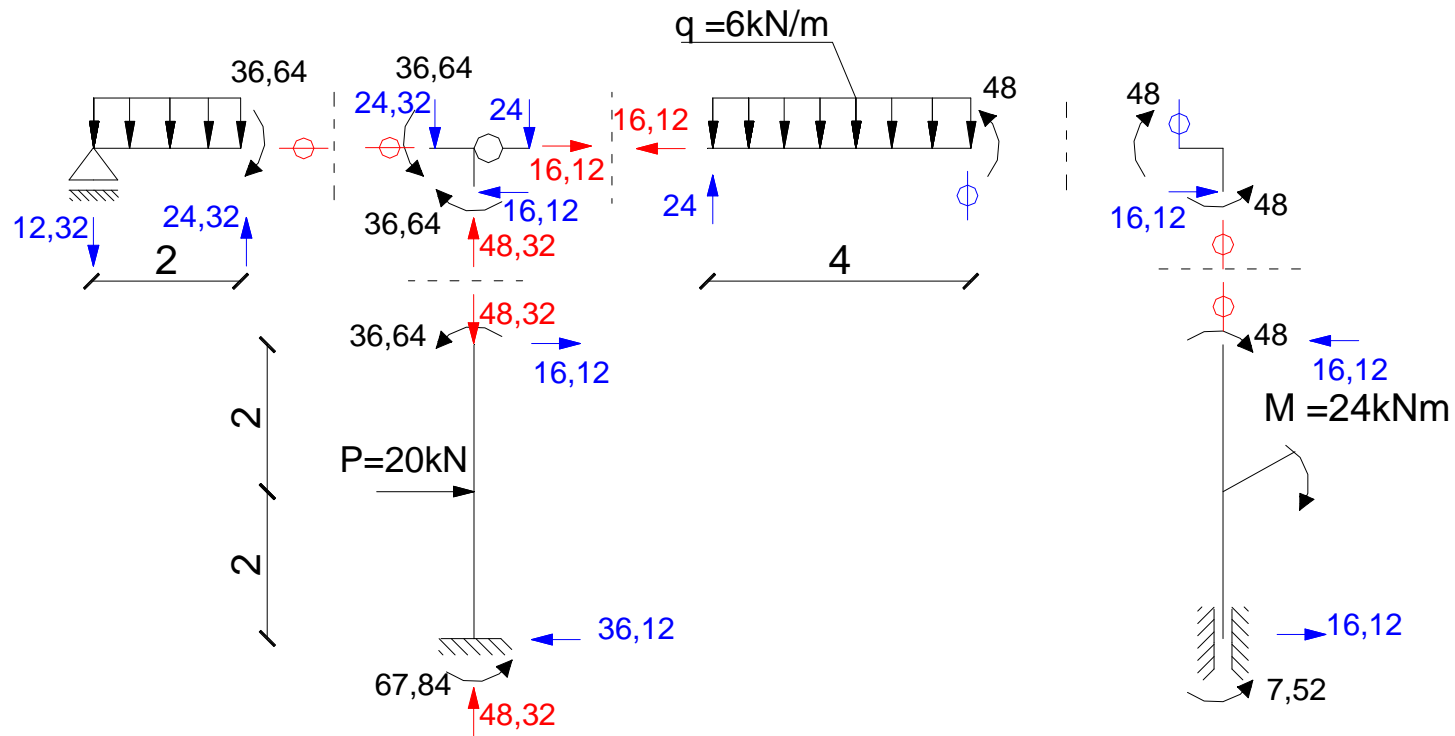
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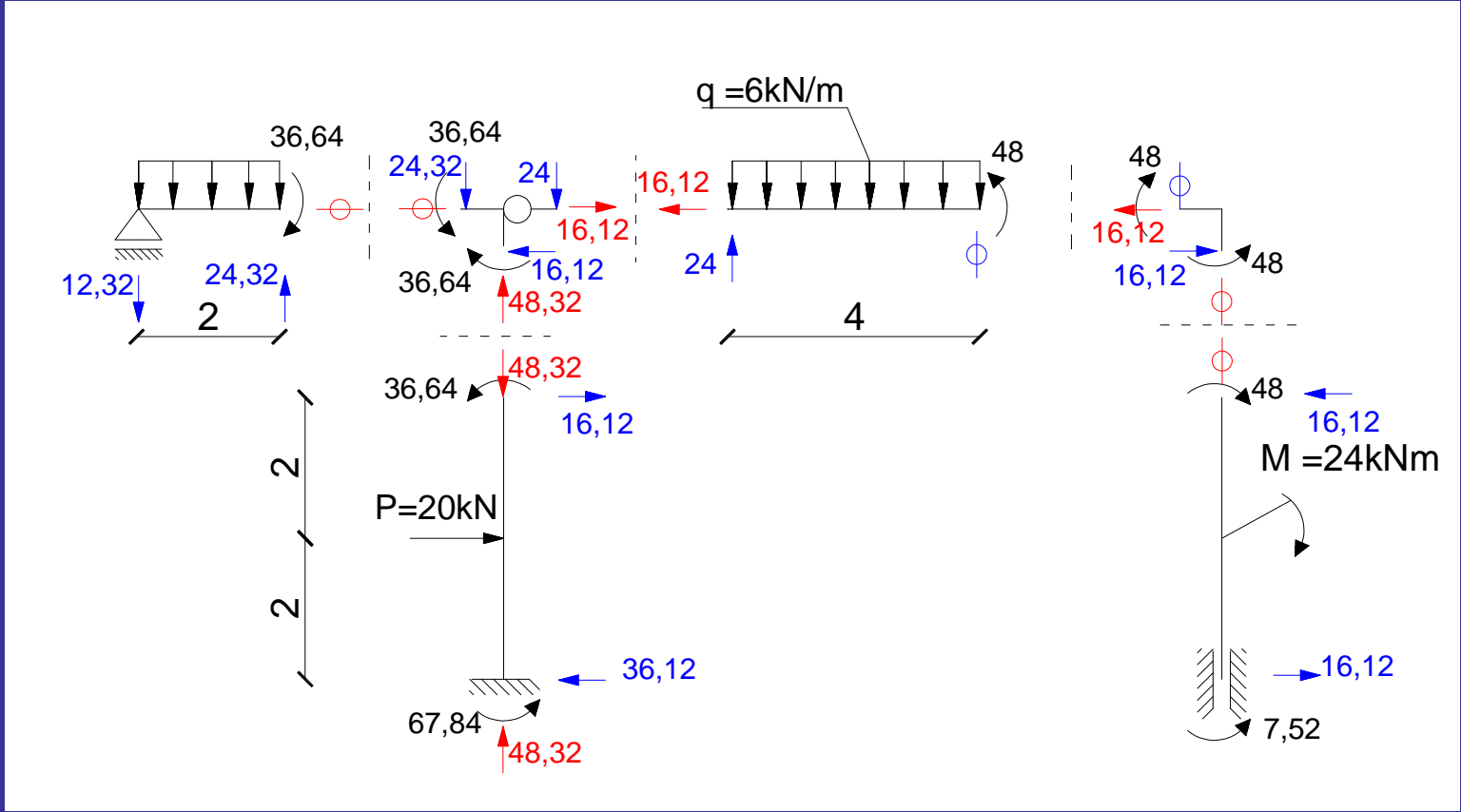


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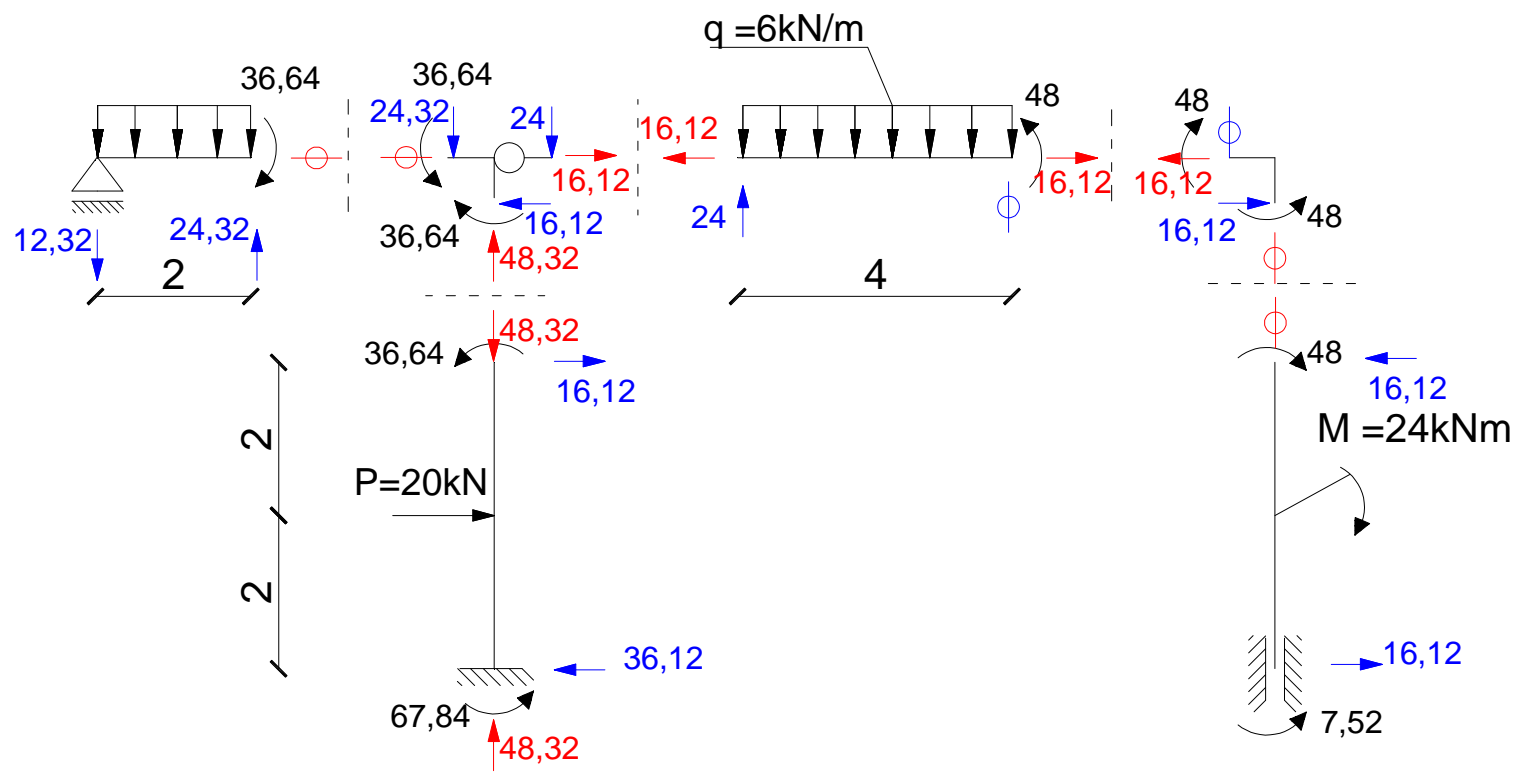


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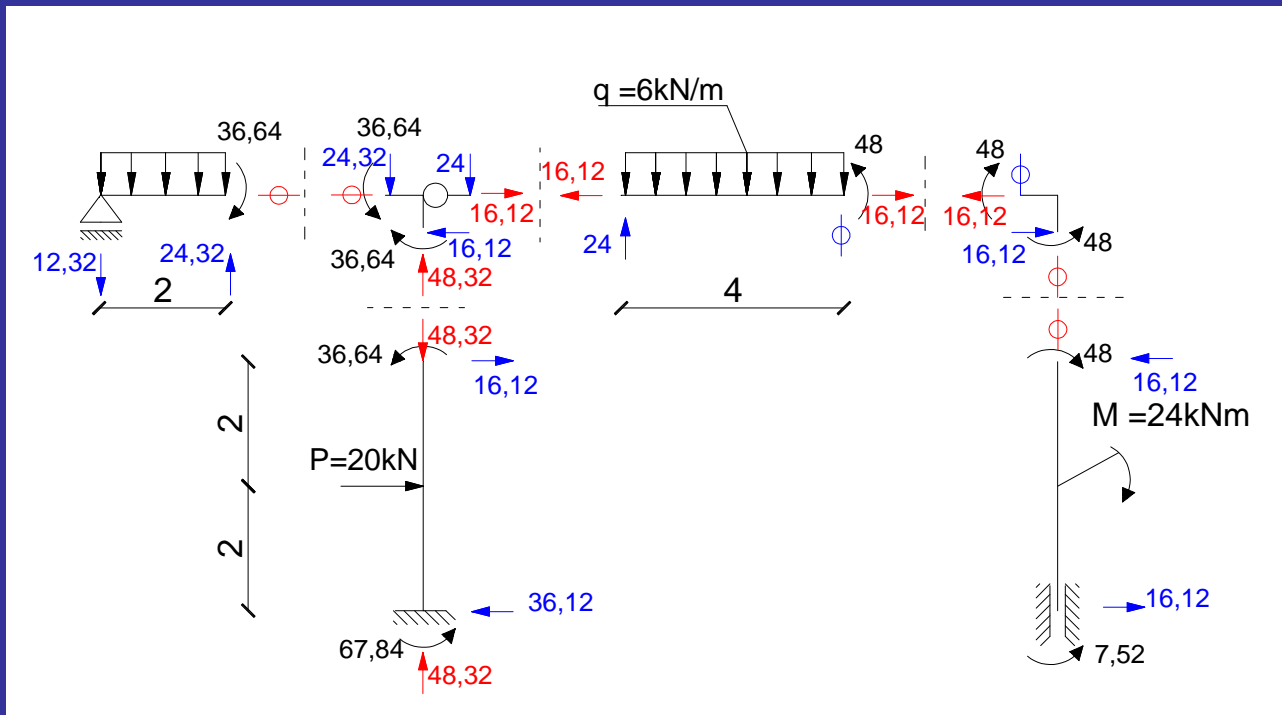




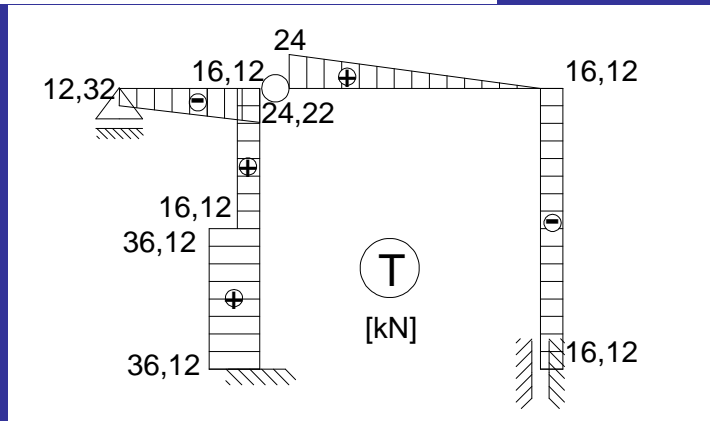
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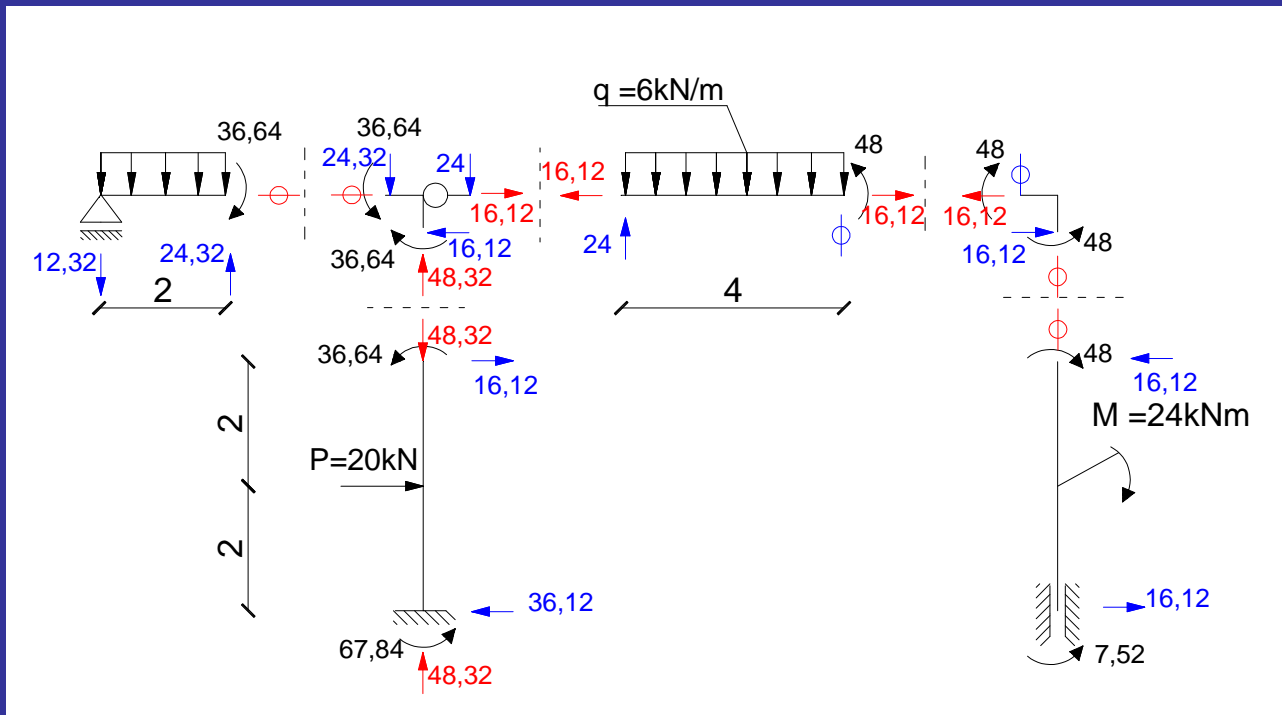


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Wykres sił tnących:





Wykres sił normalnych:

