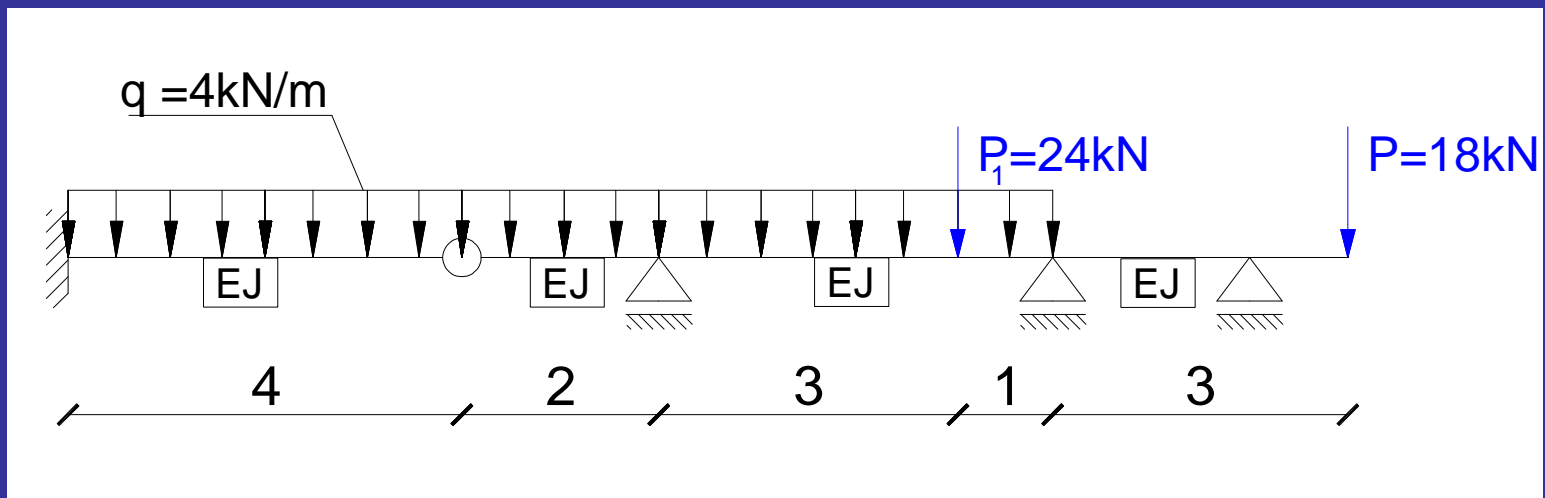
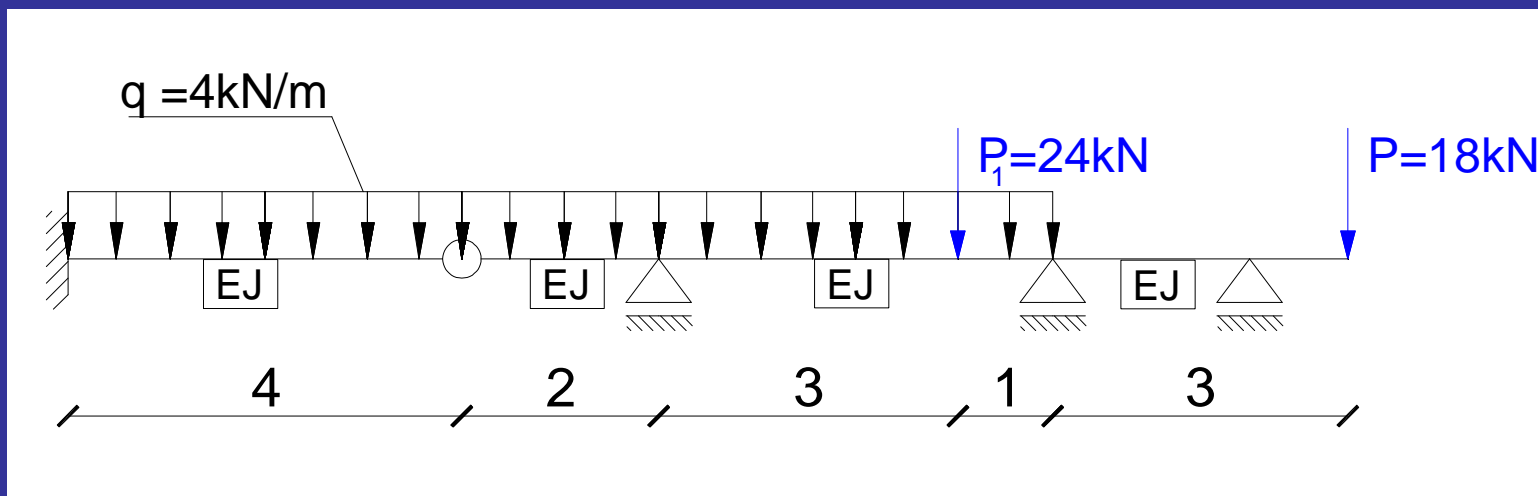


Zadanie: Narysuj wykresy sił N, T, M.
Zadanie rozwiąż metodą sił.



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Określenie stopnia statycznej niewyznaczalności :

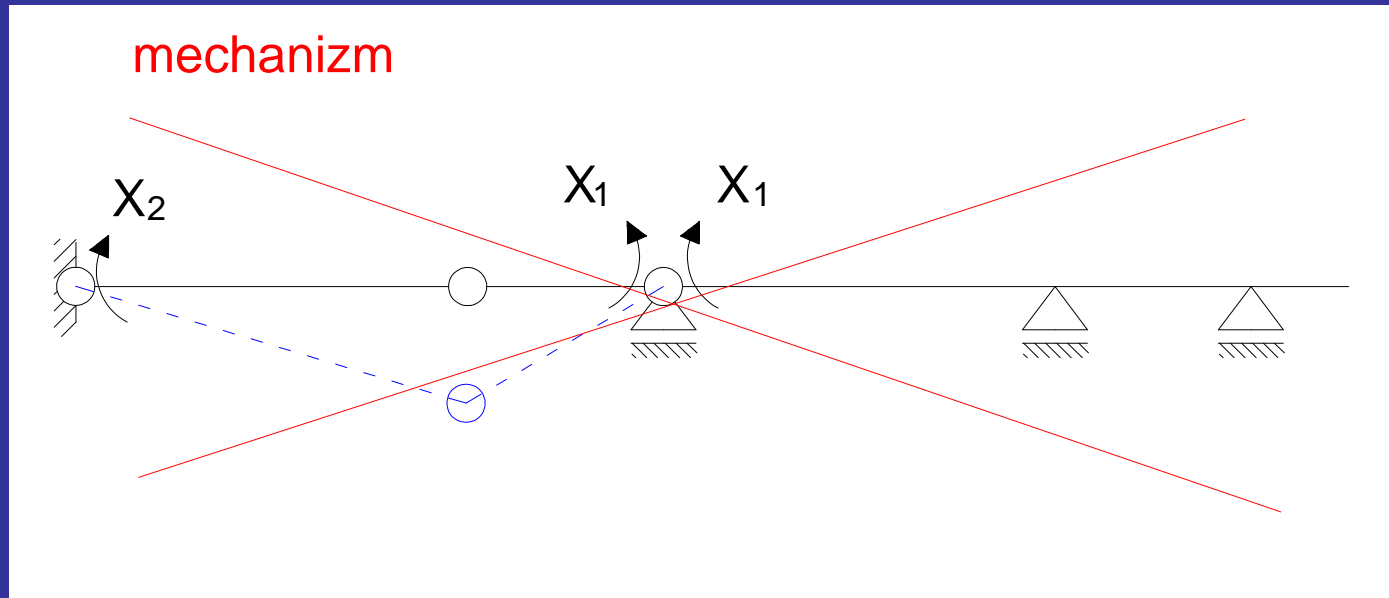


$$n_s = l_r - l_p - 3 = 6 - 1 - 3 = 2$$

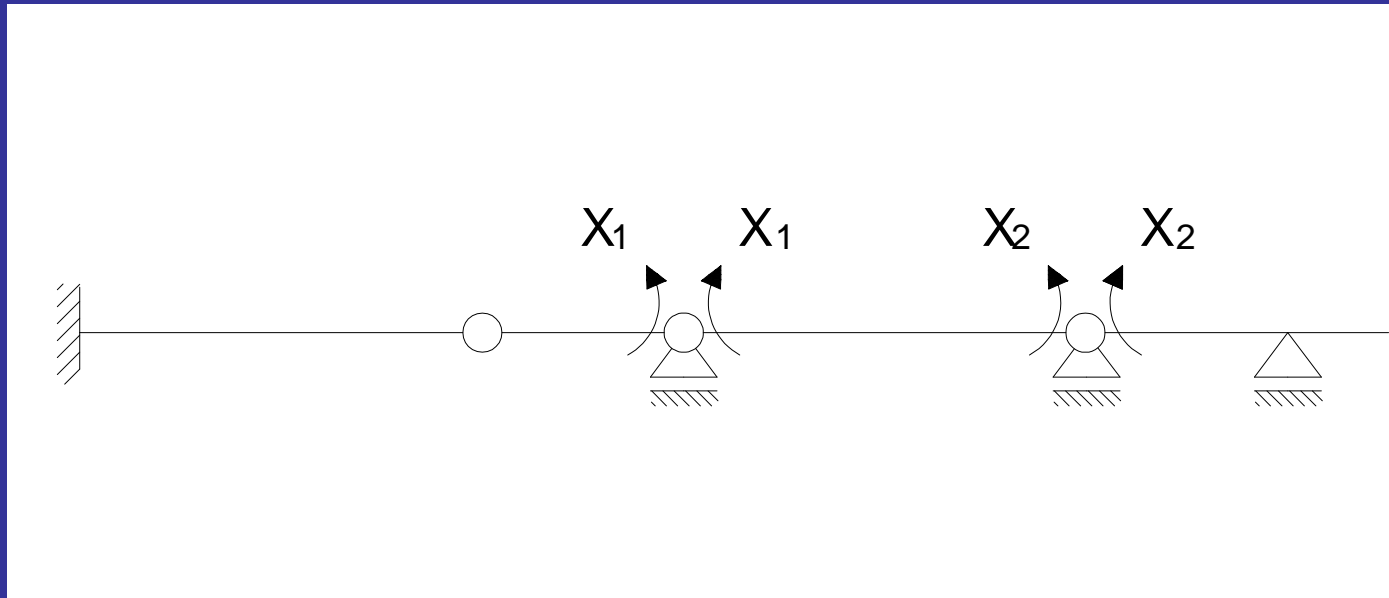
Układ dwukrotnie
statycznie niewyznaczalny

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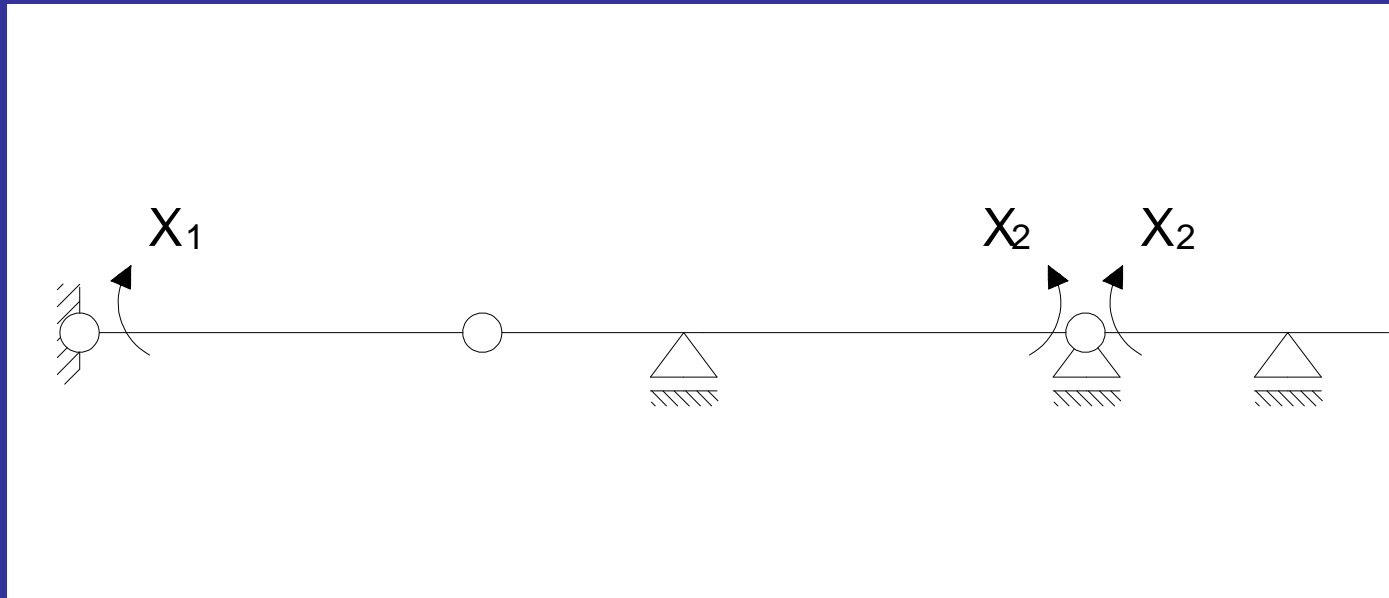
Dobór schematu podstawowego:



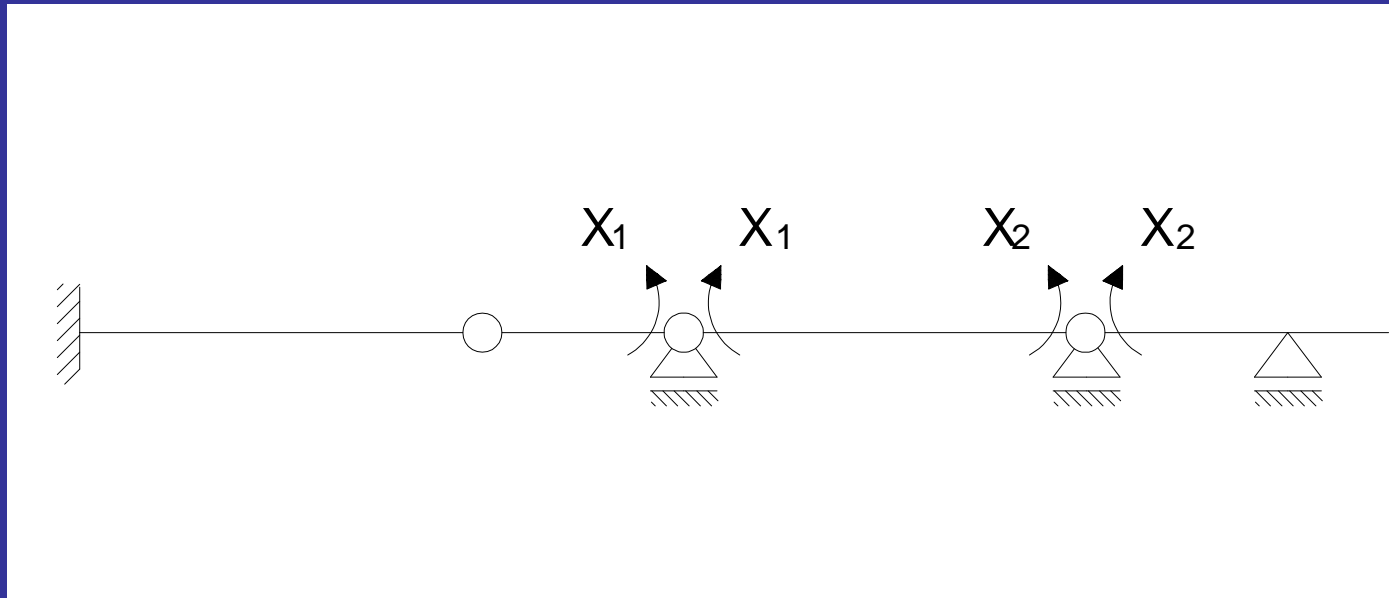
Dobór schematu podstawowego:



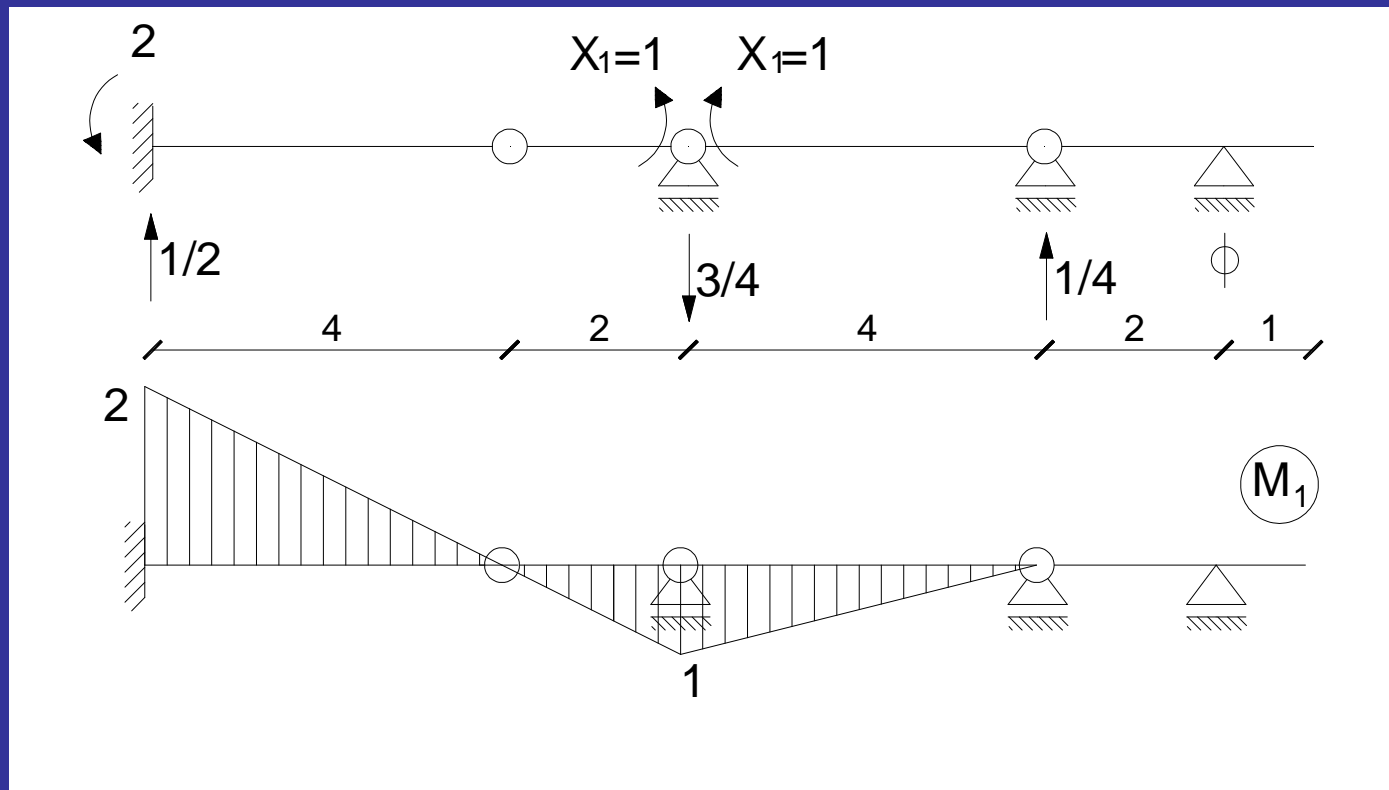
Schemat alternatywny:



Schemat podstawowy:

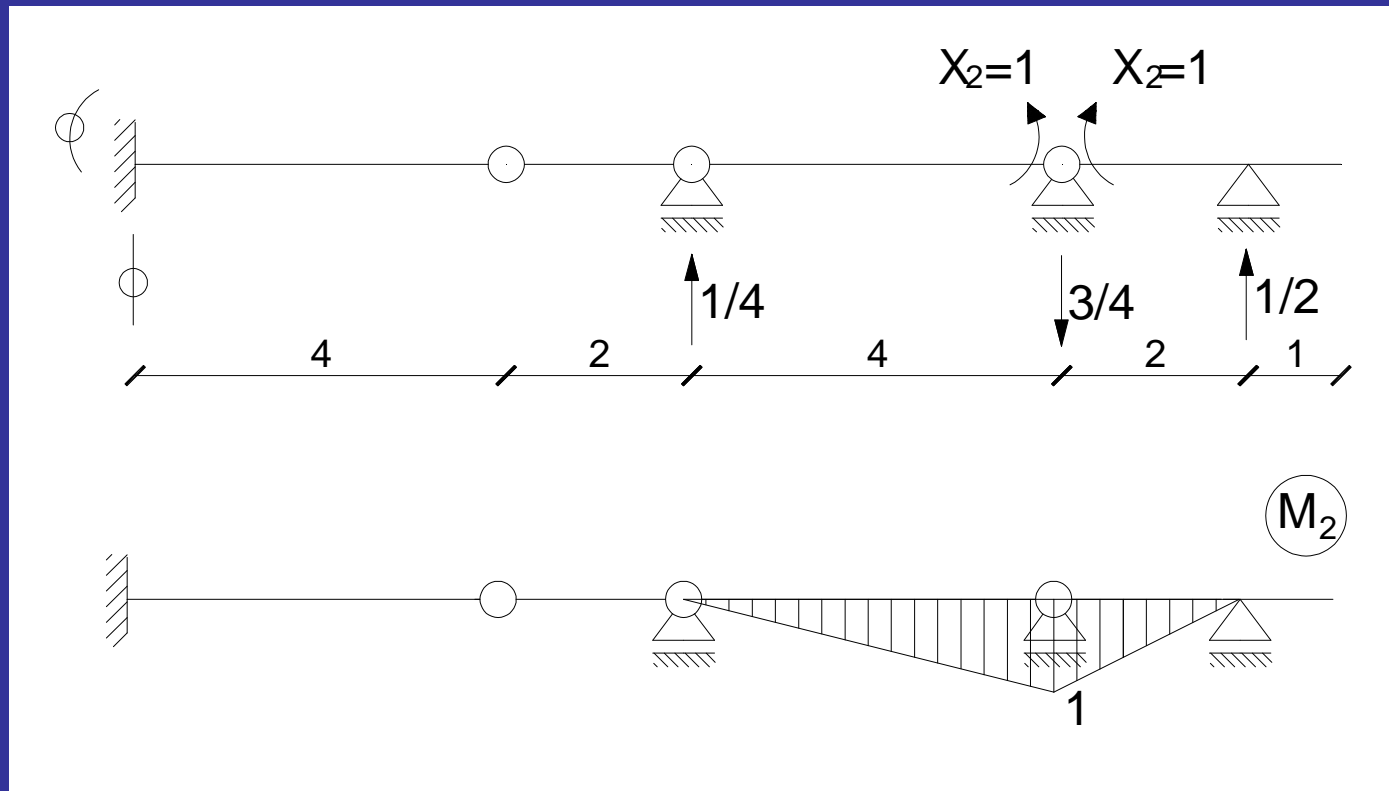


Wykres $X_1=1$



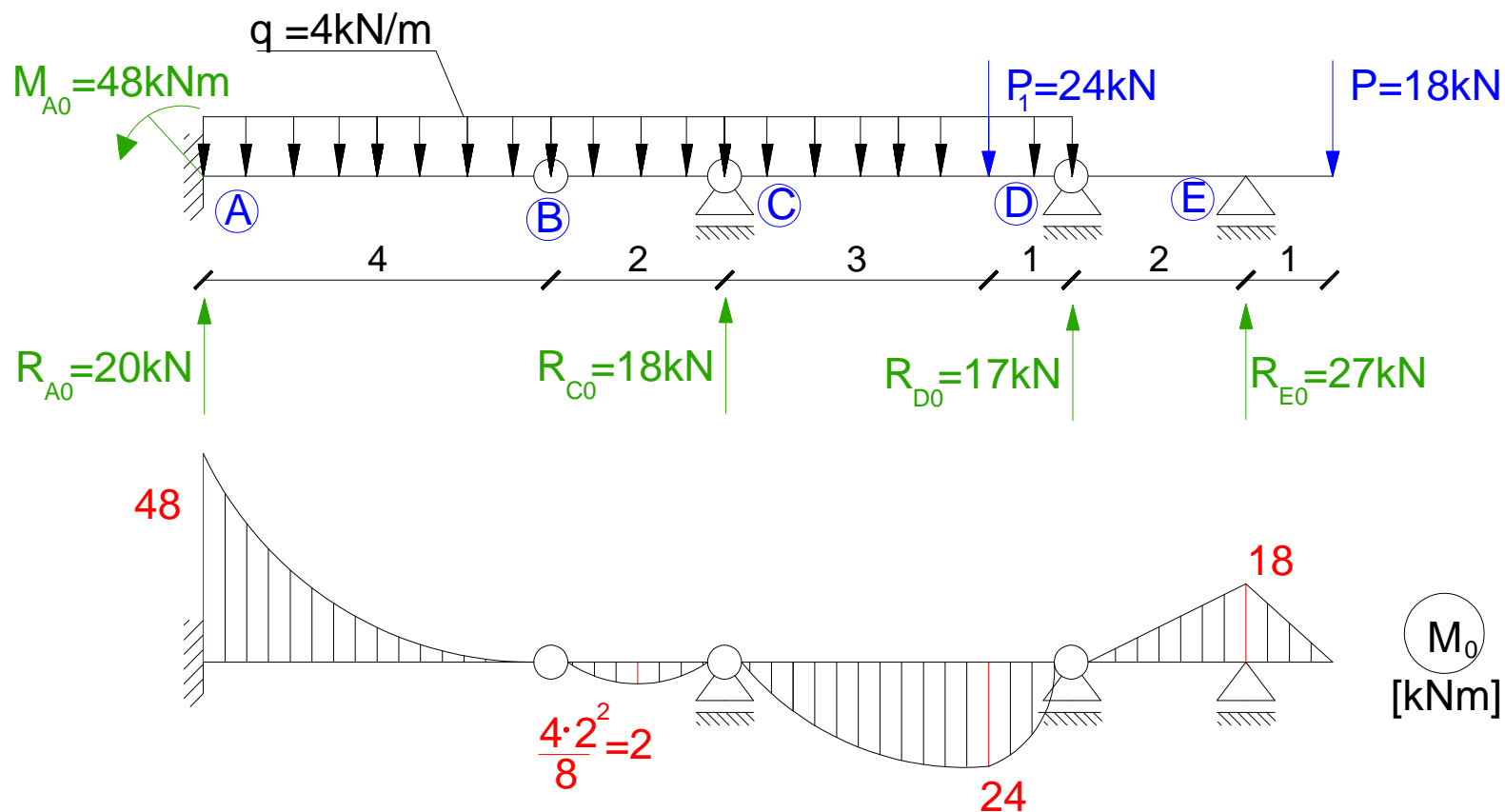
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Wykres $X_2=1$



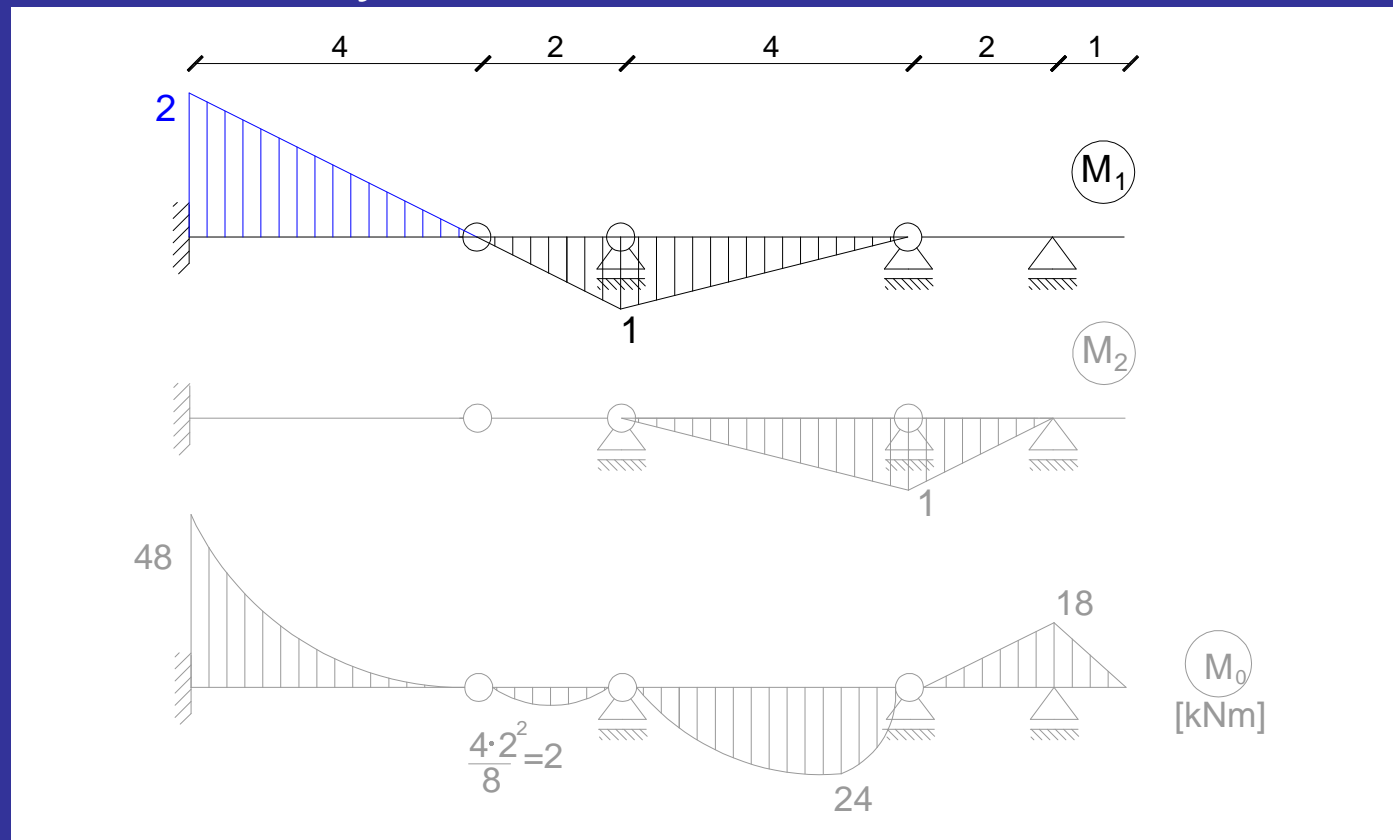
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Wykres od obciążenia zewnętrznego



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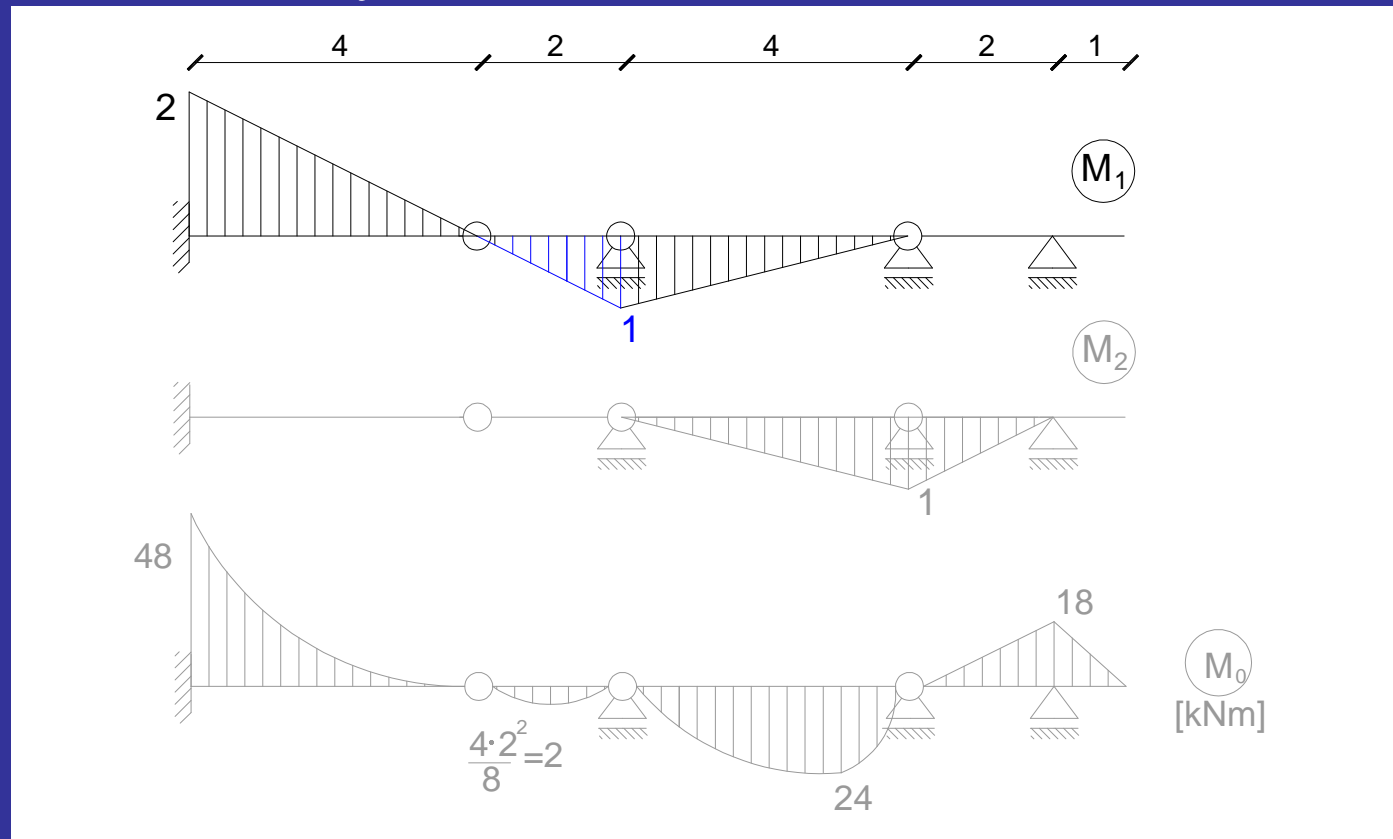
Całkowanie wykresów:



$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \cdot 2 \cdot 4 \cdot \frac{2}{3} \cdot 2 \right)$$

dr inż. Hanna Weber

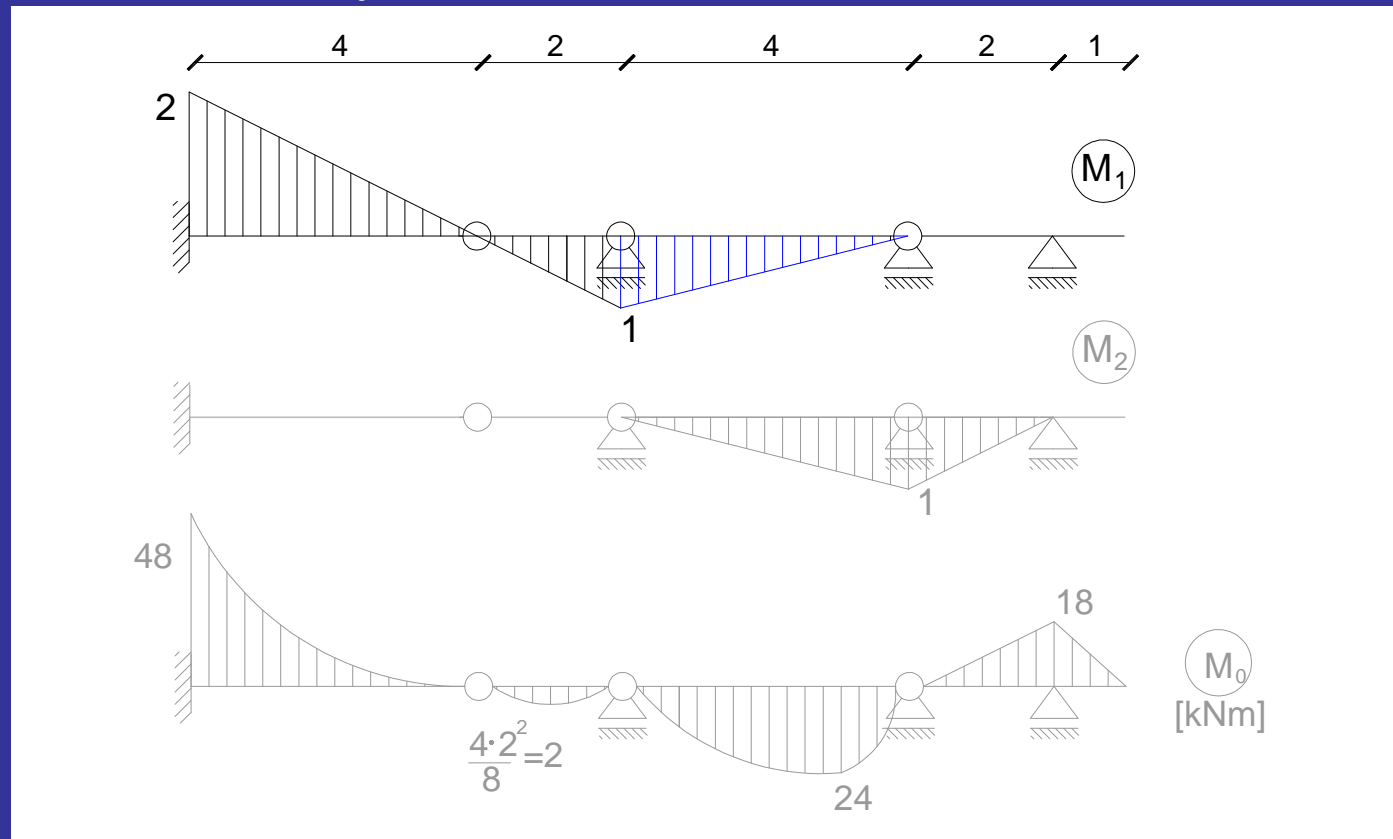
Całkowanie wykresów:



$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \cdot 2 \cdot 4 \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 \cdot \frac{2}{3} \cdot 1 \right)$$

dr inż. Hanna Weber

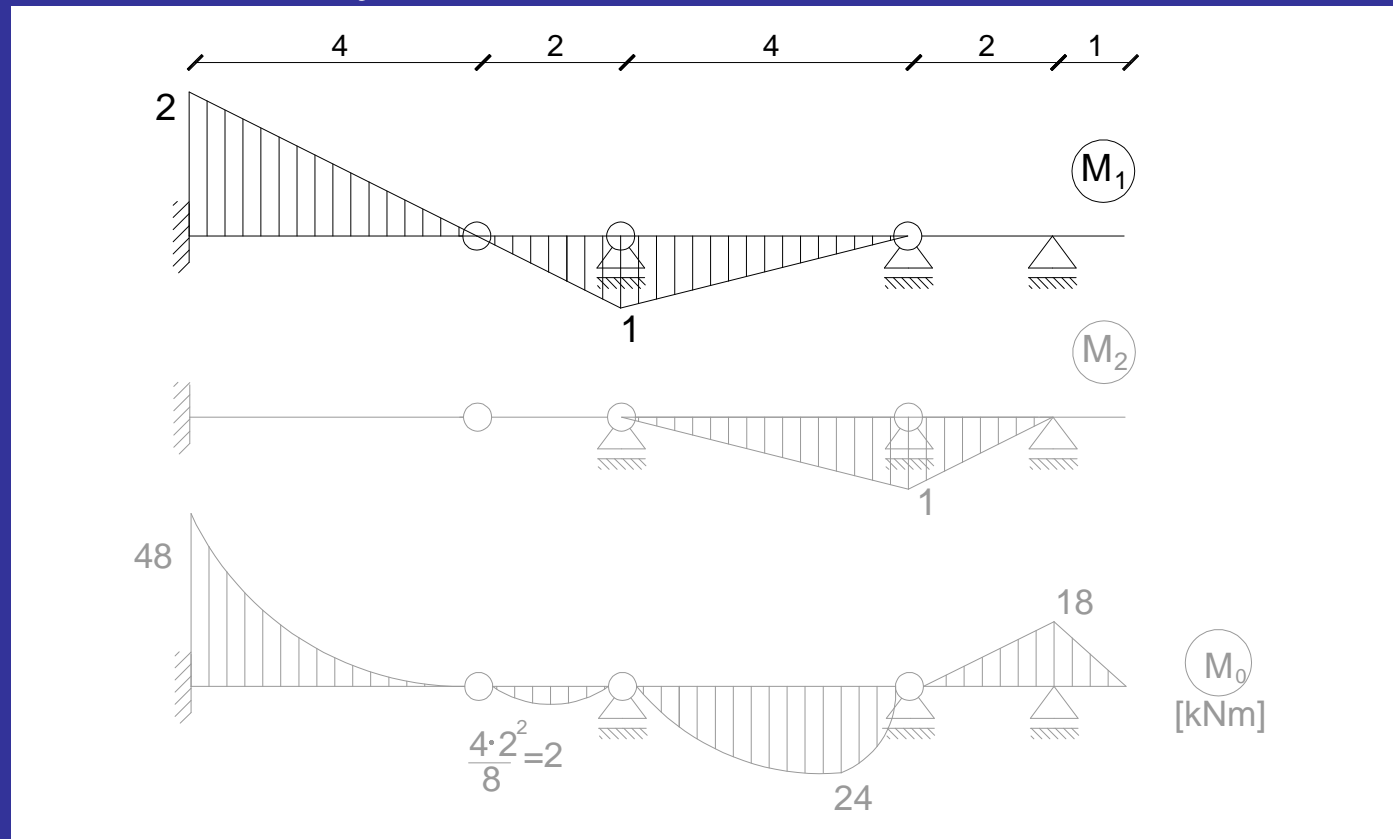
Całkowanie wykresów:



$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \cdot 2 \cdot 4 \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 \right)$$

dr inż. Hanna Weber

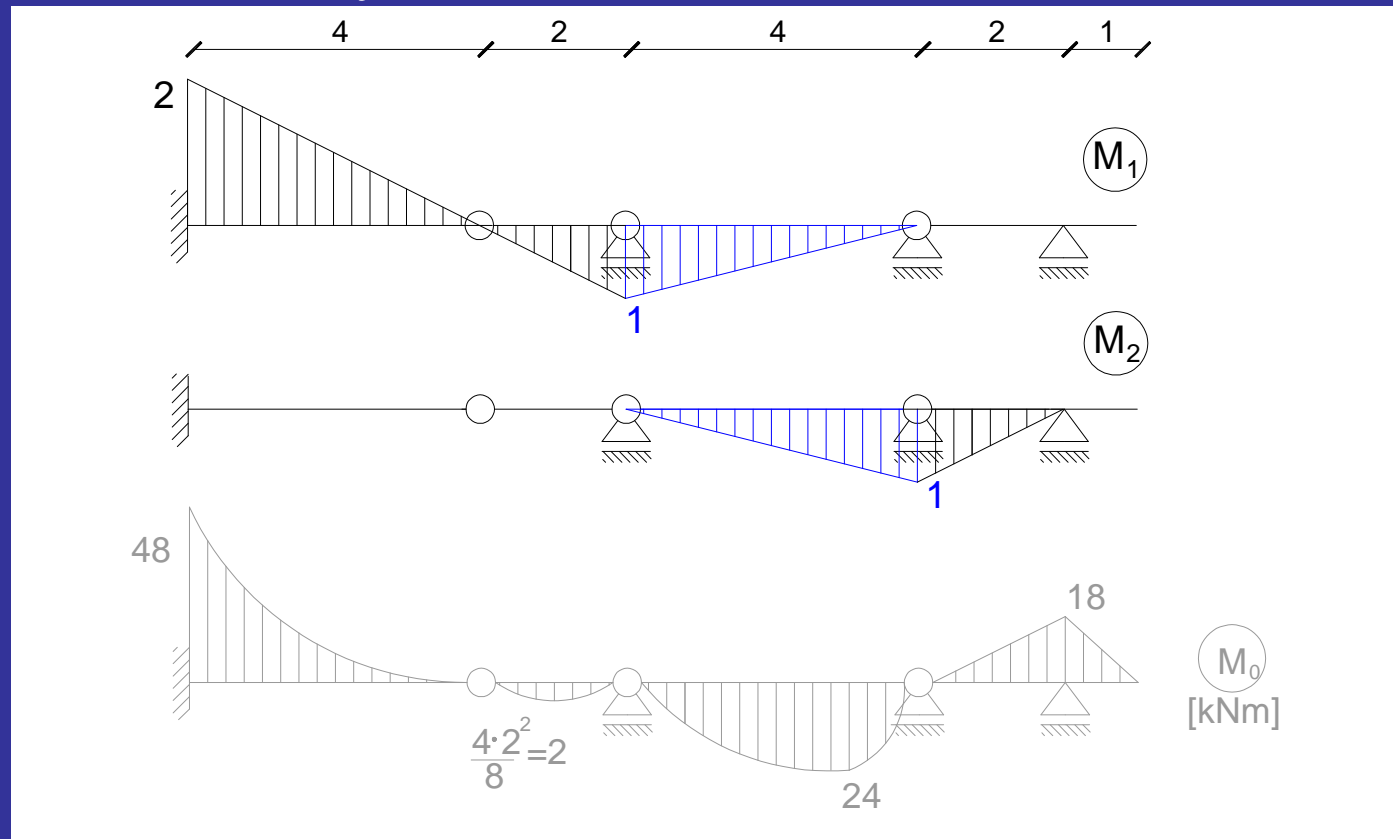
Całkowanie wykresów:



$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \cdot 2 \cdot 4 \cdot \frac{2}{3} \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 \right) = \frac{22}{3EI}$$

dr inż. Hanna Weber

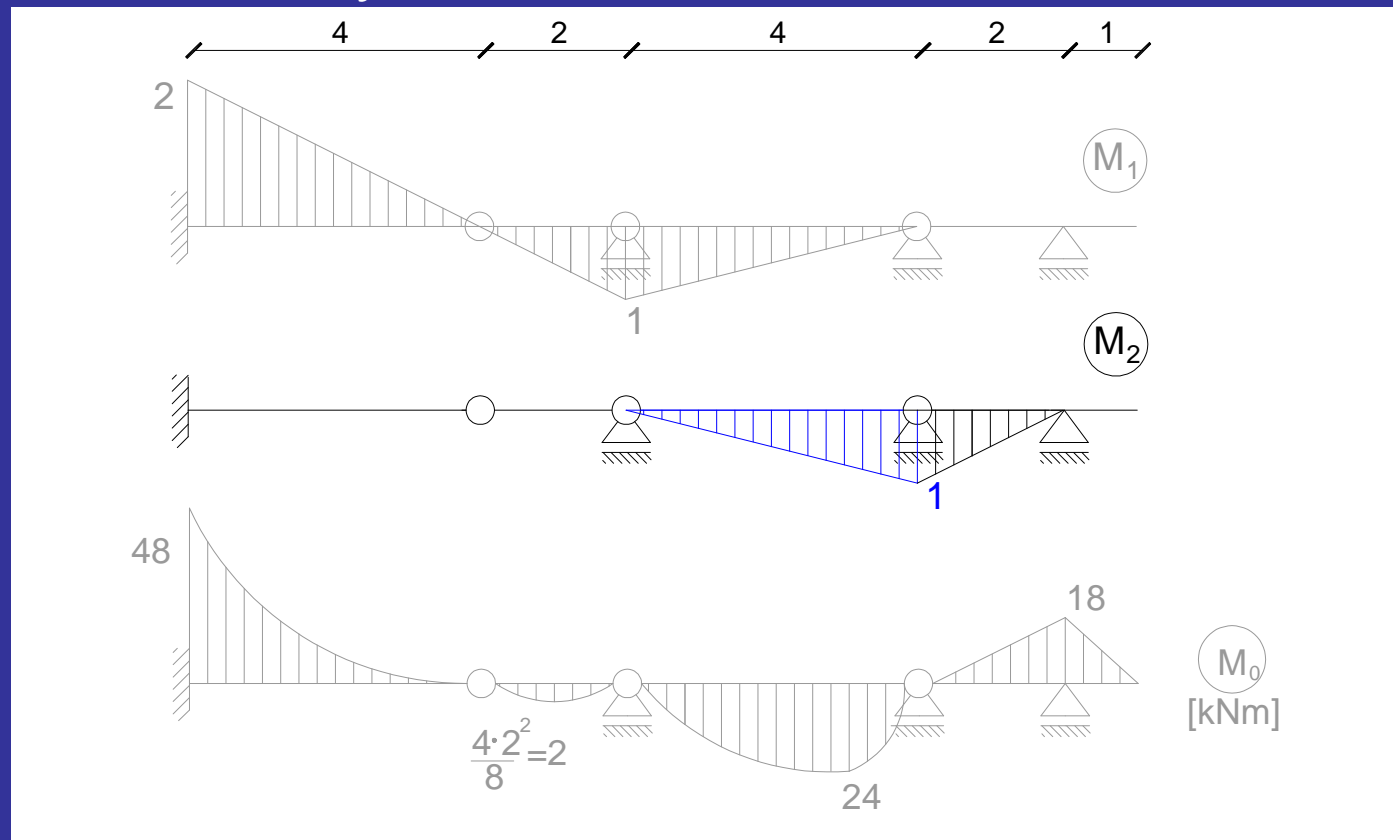
Całkowanie wykresów:



$$\delta_{12} = \frac{1}{EI} \left(\frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{1}{3} \cdot 1 \right) = \frac{2}{3EI}$$

dr inż. Hanna Weber

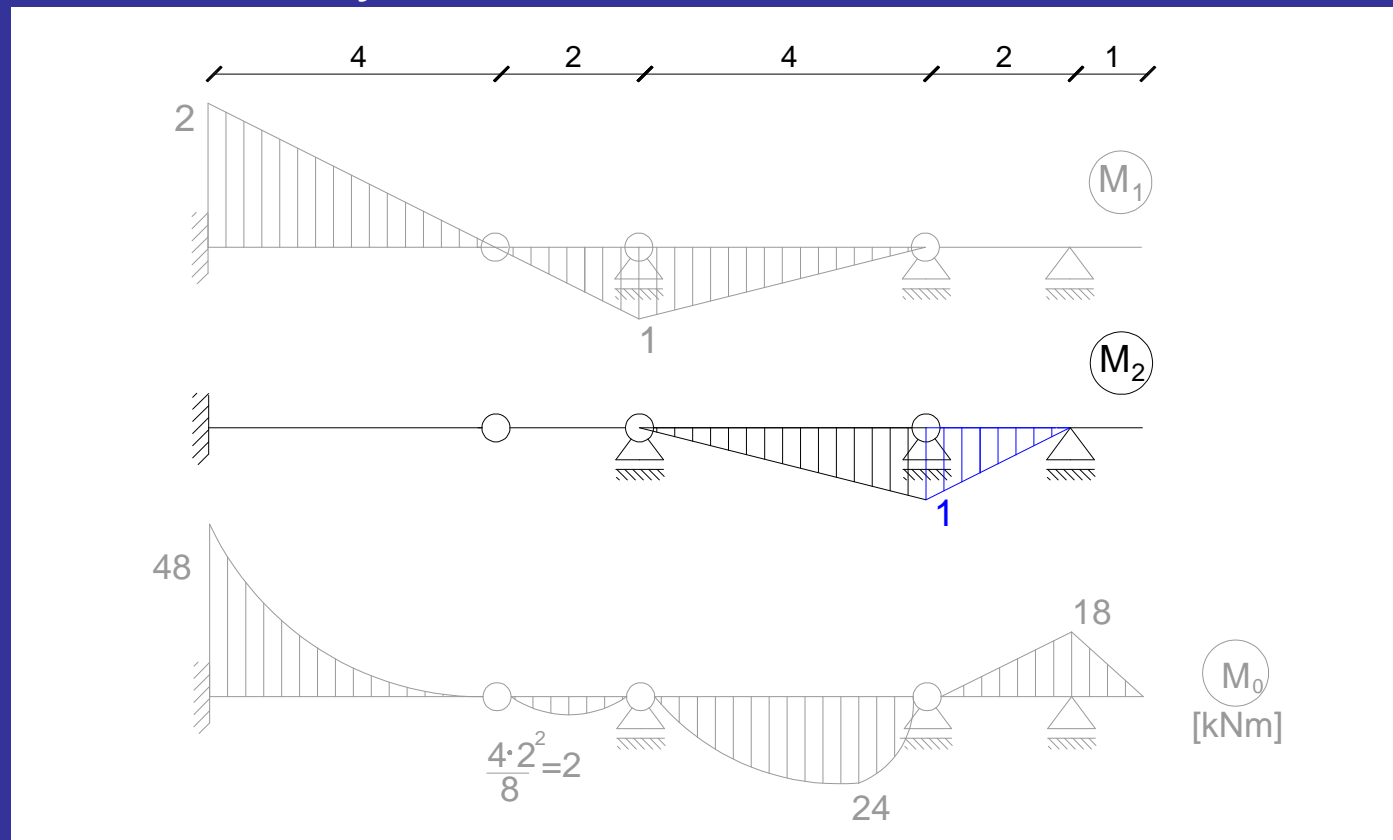
Całkowanie wykresów:



$$\delta_{22} = \frac{1}{EI} \left(\frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 \right)$$

dr inż. Hanna Weber

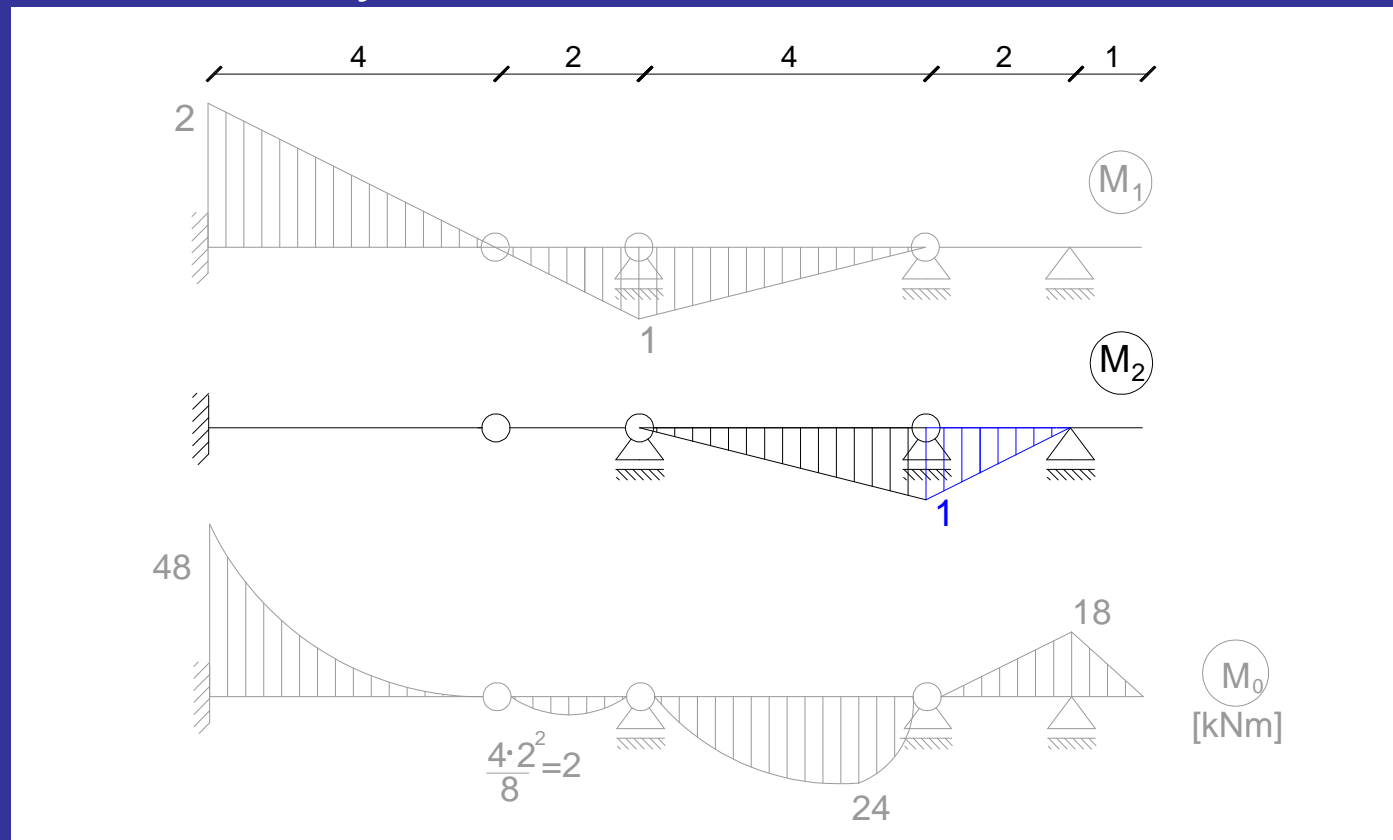
Całkowanie wykresów:



$$\delta_{22} = \frac{1}{EI} \left(\frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 1 \cdot 2 \cdot \frac{2}{3} \cdot 1 \right)$$

dr inż. Hanna Weber

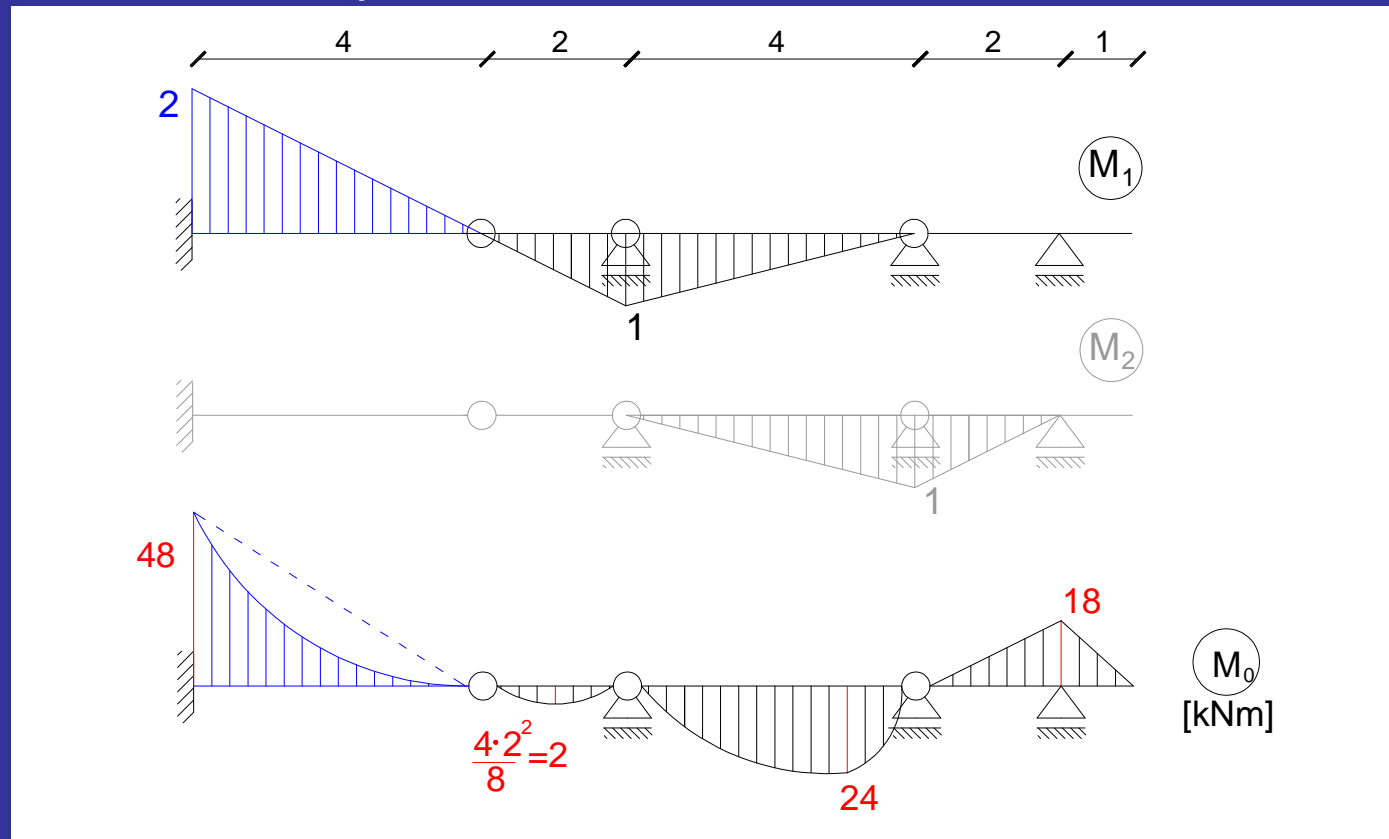
Całkowanie wykresów:



$$\delta_{22} = \frac{1}{EI} \left(\frac{1}{2} \cdot 1 \cdot 4 \cdot \frac{2}{3} \cdot 1 + \frac{1}{2} \cdot 1 \cdot 2 \cdot \frac{2}{3} \cdot 1 \right) = \frac{2}{EI}$$

dr inż. Hanna Weber

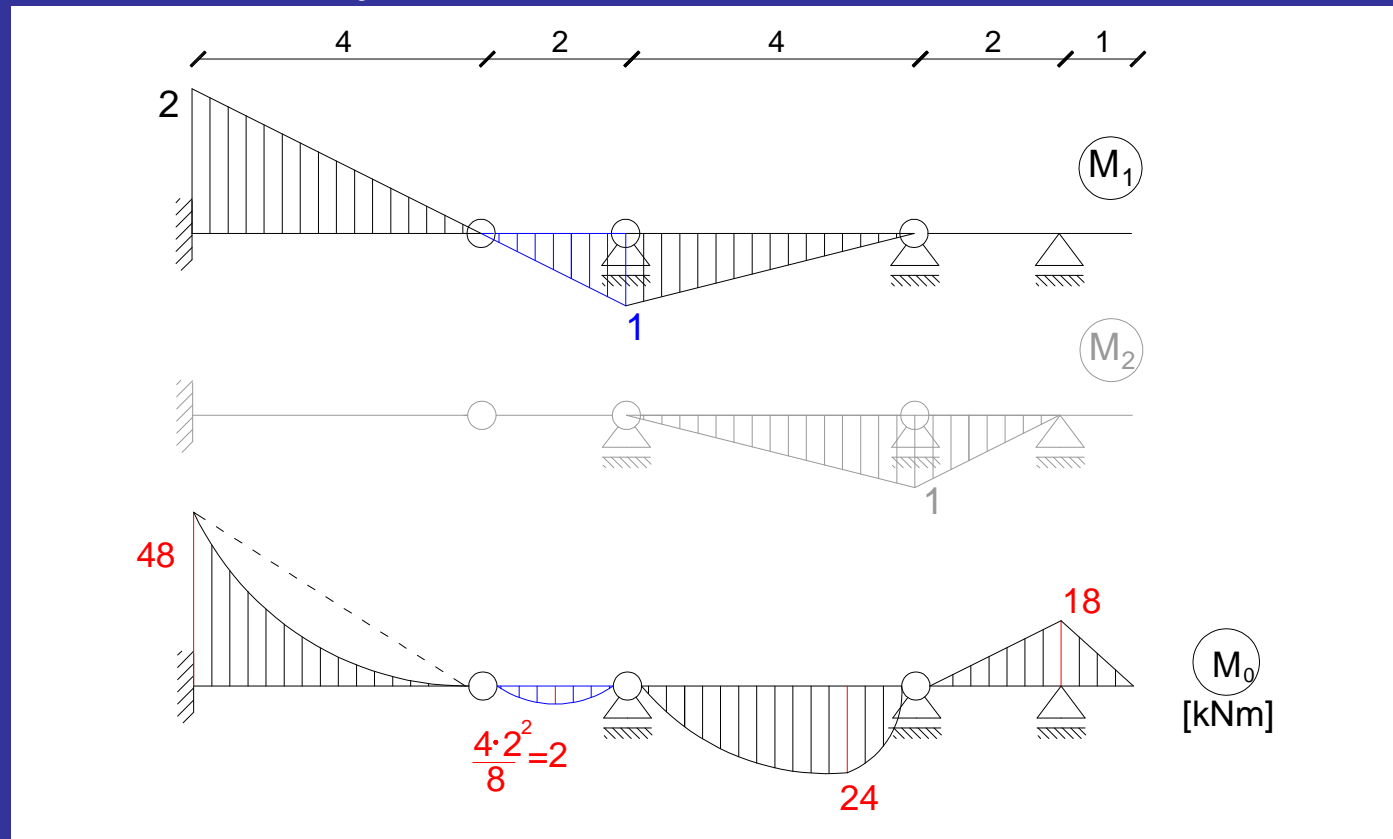
Całkowanie wykresów:



$$\delta_{10} = \frac{1}{EI} \left(\frac{1}{2} \cdot 48 \cdot 4 \cdot \frac{2}{3} \cdot 2 - \frac{2}{3} \cdot \frac{4 \cdot 4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 2 \right)$$

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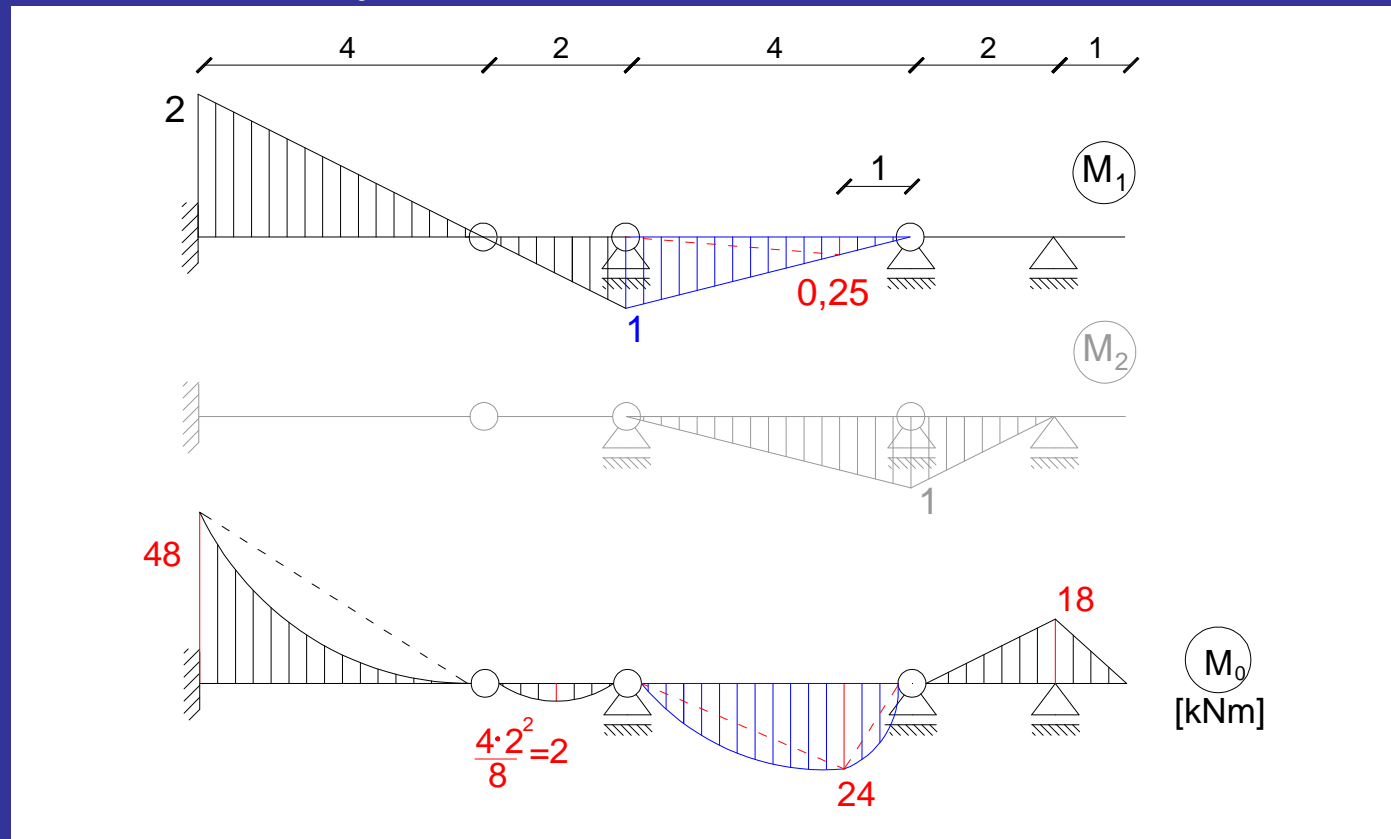
Całkowanie wykresów:



$$\delta_{10} = \frac{1}{EI} \left(\frac{1}{2} \cdot 48 \cdot 4 \cdot \frac{2}{3} \cdot 2 - \frac{2}{3} \cdot \frac{4 \cdot 4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 2 + \frac{2}{3} \cdot \frac{4 \cdot 2^2}{8} \cdot 2 \cdot \frac{1}{2} \cdot 1 \right)$$

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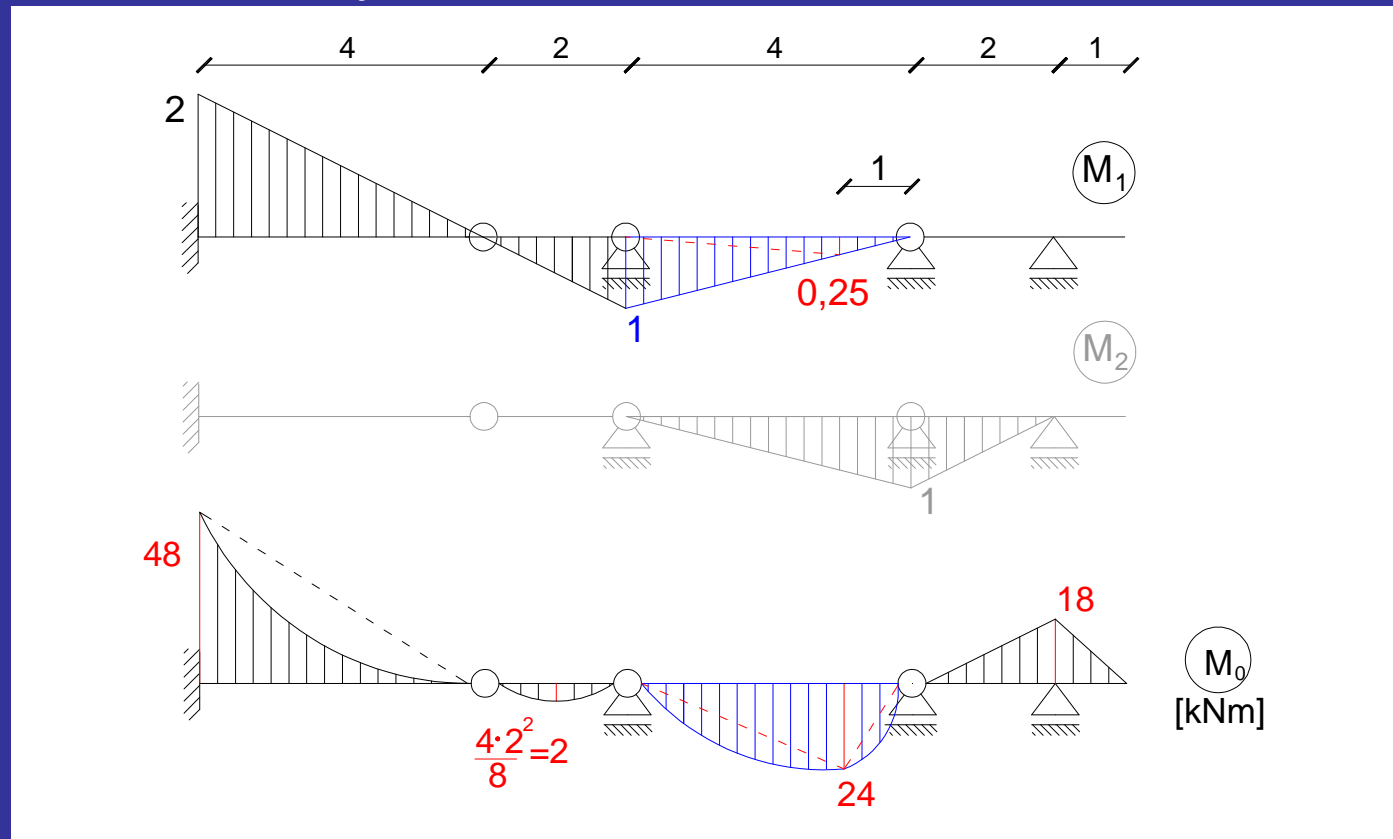
Całkowanie wykresów:



$$\delta_{10} = \frac{1}{EI} \left(\frac{1}{2} \cdot 48 \cdot 4 \cdot \frac{2}{3} \cdot 2 - \frac{2}{3} \cdot \frac{4 \cdot 4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 2 + \frac{2}{3} \cdot \frac{4 \cdot 2^2}{8} \cdot 2 \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 24 \cdot 3 \cdot \left(\frac{2}{3} \cdot 0,25 + \frac{1}{3} \cdot 1 \right) \right) + \frac{1}{EI} \left(\frac{2}{3} \cdot \frac{4 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot (1 + 0,25) + \frac{1}{2} \cdot 24 \cdot 1 \cdot \frac{2}{3} \cdot 0,25 \right)$$

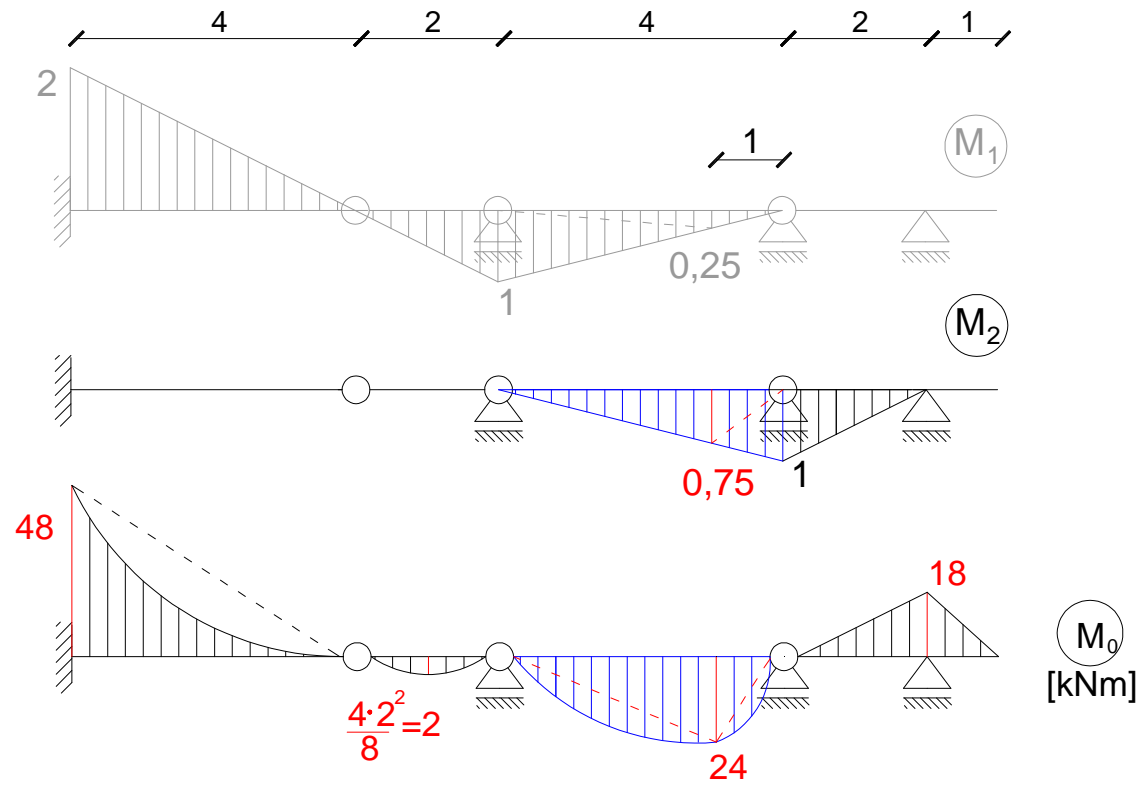
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Całkowanie wykresów:



$$\delta_{10} = \frac{1}{EI} \left(\frac{1}{2} \cdot 48 \cdot 4 \cdot \frac{2}{3} \cdot 2 - \frac{2}{3} \cdot \frac{4 \cdot 4^2}{8} \cdot 4 \cdot \frac{1}{2} \cdot 2 + \frac{2}{3} \cdot \frac{4 \cdot 2^2}{8} \cdot 2 \cdot \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 24 \cdot 3 \cdot \left(\frac{2}{3} \cdot 0,25 + \frac{1}{3} \cdot 1 \right) \right) + \frac{1}{EI} \left(\frac{2}{3} \cdot \frac{4 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot (1 + 0,25) + \frac{1}{2} \cdot 24 \cdot 1 \cdot \frac{2}{3} \cdot 0,25 + \frac{2}{3} \cdot \frac{4 \cdot 1^2}{8} \cdot 1 \cdot \frac{1}{2} \cdot 0,25 \right) = \frac{401}{3EI}$$

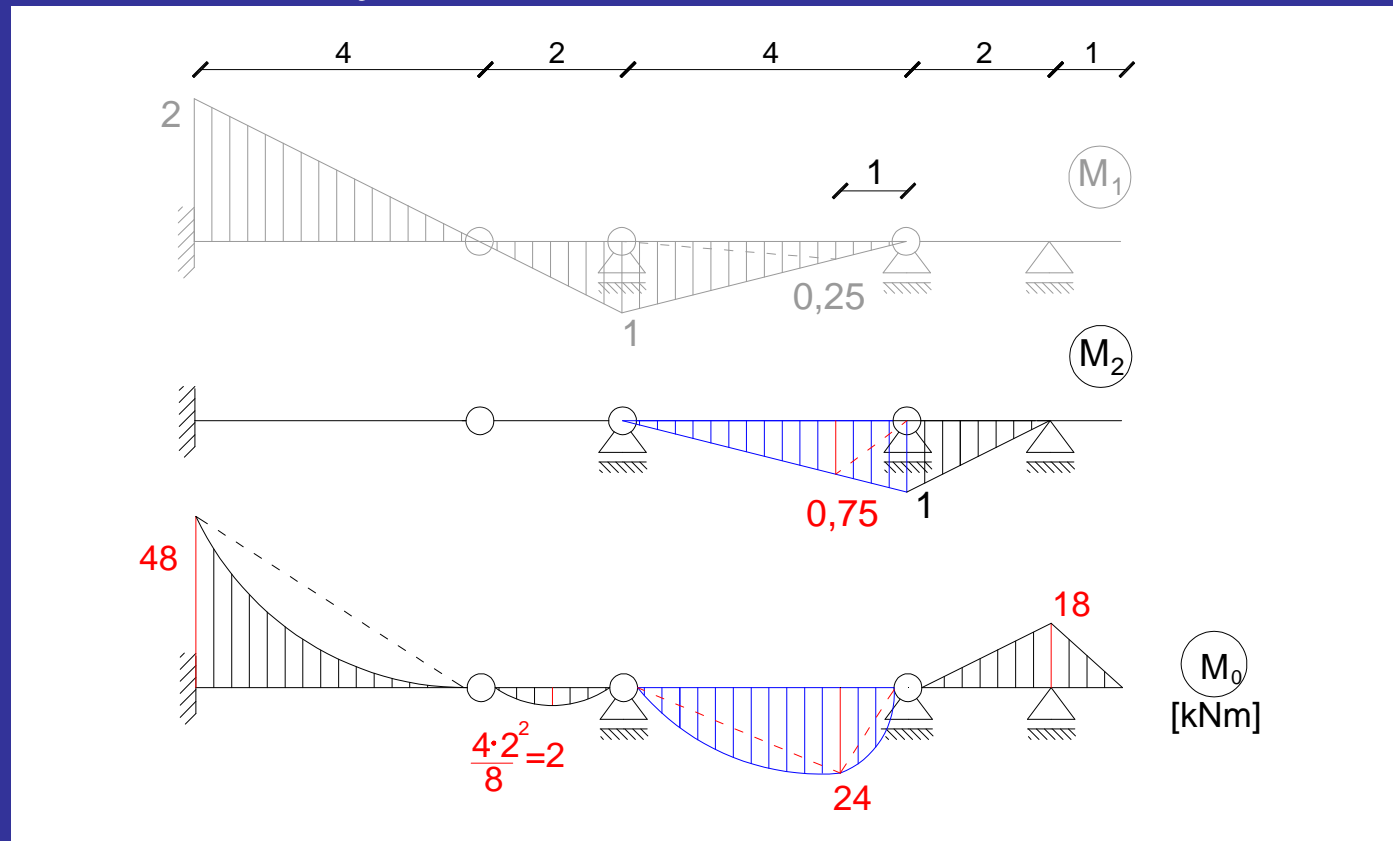
Całkowanie wykresów:



$$\delta_{20} = \frac{1}{EI} \left(\frac{1}{2} \cdot 24 \cdot 3 \cdot \frac{2}{3} \cdot 0,75 + \frac{2}{3} \cdot \frac{4 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot 0,75 \right)$$

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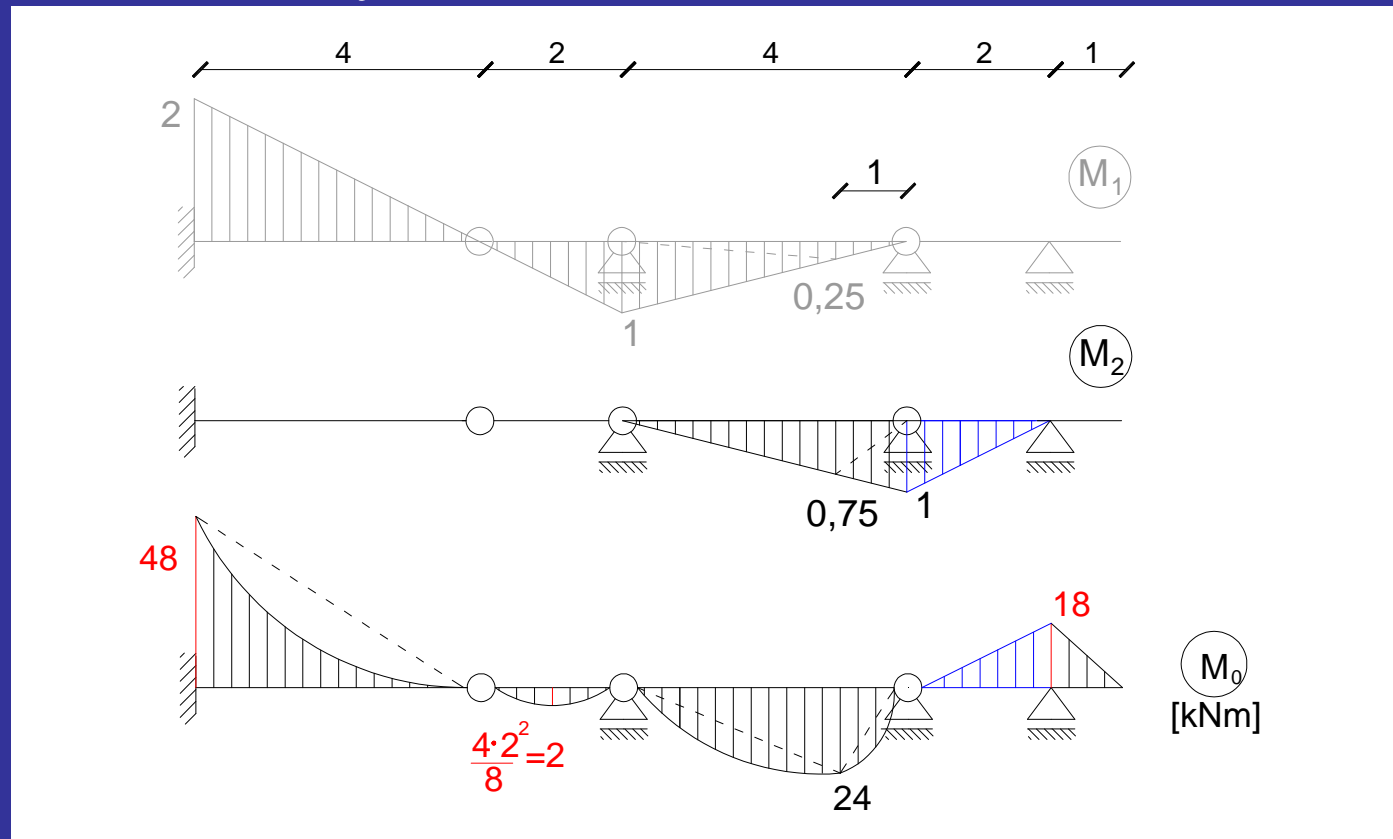
Całkowanie wykresów:



$$\delta_{20} = \frac{1}{EI} \left(\frac{1}{2} \cdot 24 \cdot 3 \cdot \frac{2}{3} \cdot 0,75 + \frac{2}{3} \cdot \frac{4 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot 0,75 + \frac{1}{2} \cdot 24 \cdot 1 \cdot \left(\frac{2}{3} \cdot 0,75 + \frac{1}{3} \cdot 1 \right) + \frac{2}{3} \cdot \frac{4 \cdot 1^2}{8} \cdot 1 \cdot \frac{1}{2} \cdot (0,75 + 1) \right)$$

dr inż. Hanna Weber

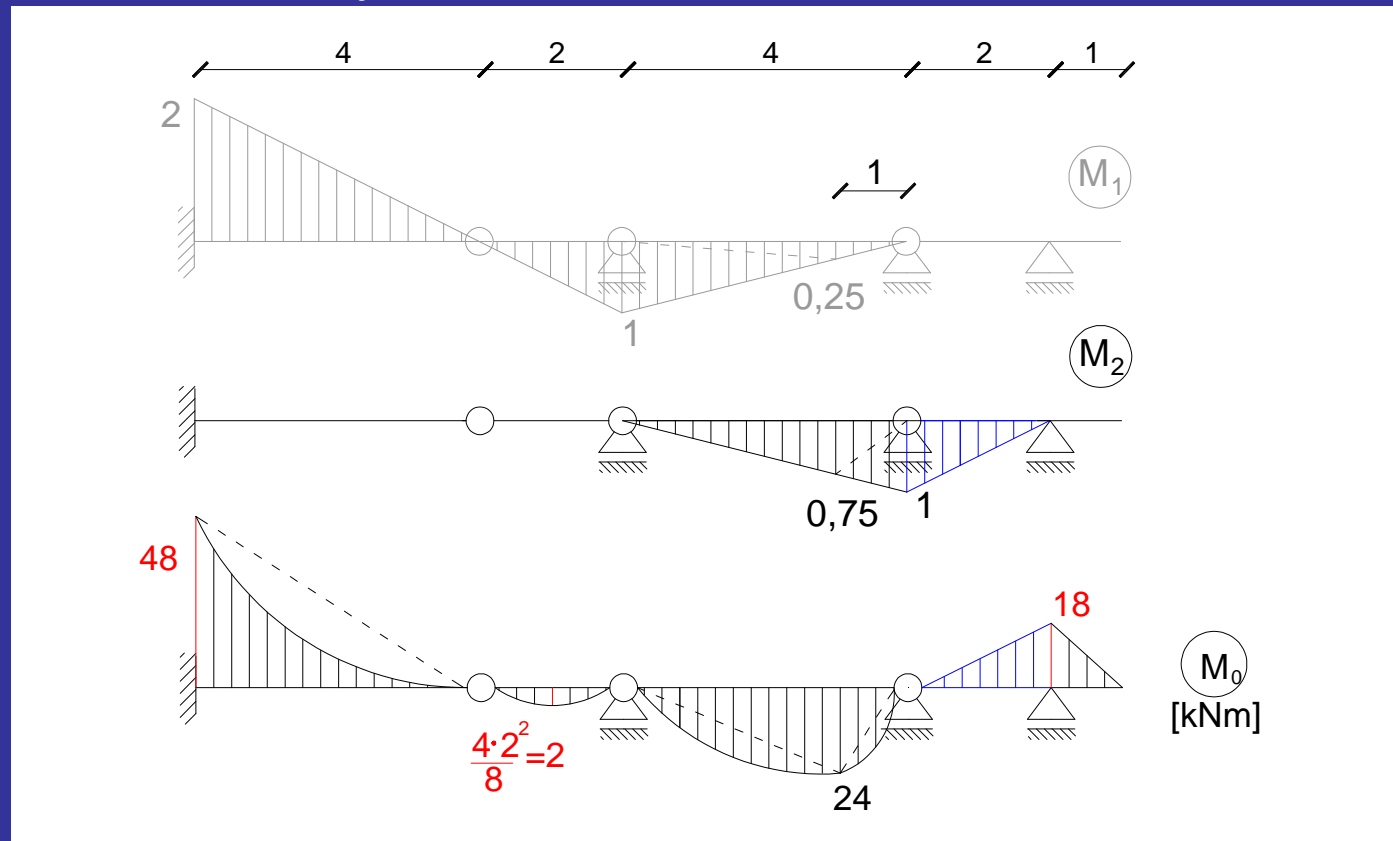
Całkowanie wykresów:



$$\delta_{20} = \frac{1}{EI} \left(\frac{1}{2} \cdot 24 \cdot 3 \cdot \frac{2}{3} \cdot 0,75 + \frac{2}{3} \cdot \frac{4 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot 0,75 + \frac{1}{2} \cdot 24 \cdot 1 \cdot \left(\frac{2}{3} \cdot 0,75 + \frac{1}{3} \cdot 1 \right) + \frac{2}{3} \cdot \frac{4 \cdot 1^2}{8} \cdot 1 \cdot \frac{1}{2} \cdot (0,75 + 1) \right) - \frac{1}{EI} \frac{1}{2} \cdot 18 \cdot 2 \cdot \frac{1}{3} \cdot 1$$

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Całkowanie wykresów:



$$\delta_{20} = \frac{1}{EI} \left(\frac{1}{2} \cdot 24 \cdot 3 \cdot \frac{2}{3} \cdot 0,75 + \frac{2}{3} \cdot \frac{4 \cdot 3^2}{8} \cdot 3 \cdot \frac{1}{2} \cdot 0,75 + \frac{1}{2} \cdot 24 \cdot 1 \cdot \left(\frac{2}{3} \cdot 0,75 + \frac{1}{3} \cdot 1 \right) + \frac{2}{3} \cdot \frac{4 \cdot 1^2}{8} \cdot 1 \cdot \frac{1}{2} \cdot (0,75 + 1) \right) - \frac{1}{EI} \frac{1}{2} \cdot 18 \cdot 2 \cdot \frac{1}{3} \cdot 1 = \frac{77}{3EI}$$

dr inż. Hanna Weber

Układ równań kanonicznych metody sił
dla schematu dwukrotnie statycznie
niewyznaczalnego:

$$\begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{10} = 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{20} = 0 \end{cases}$$

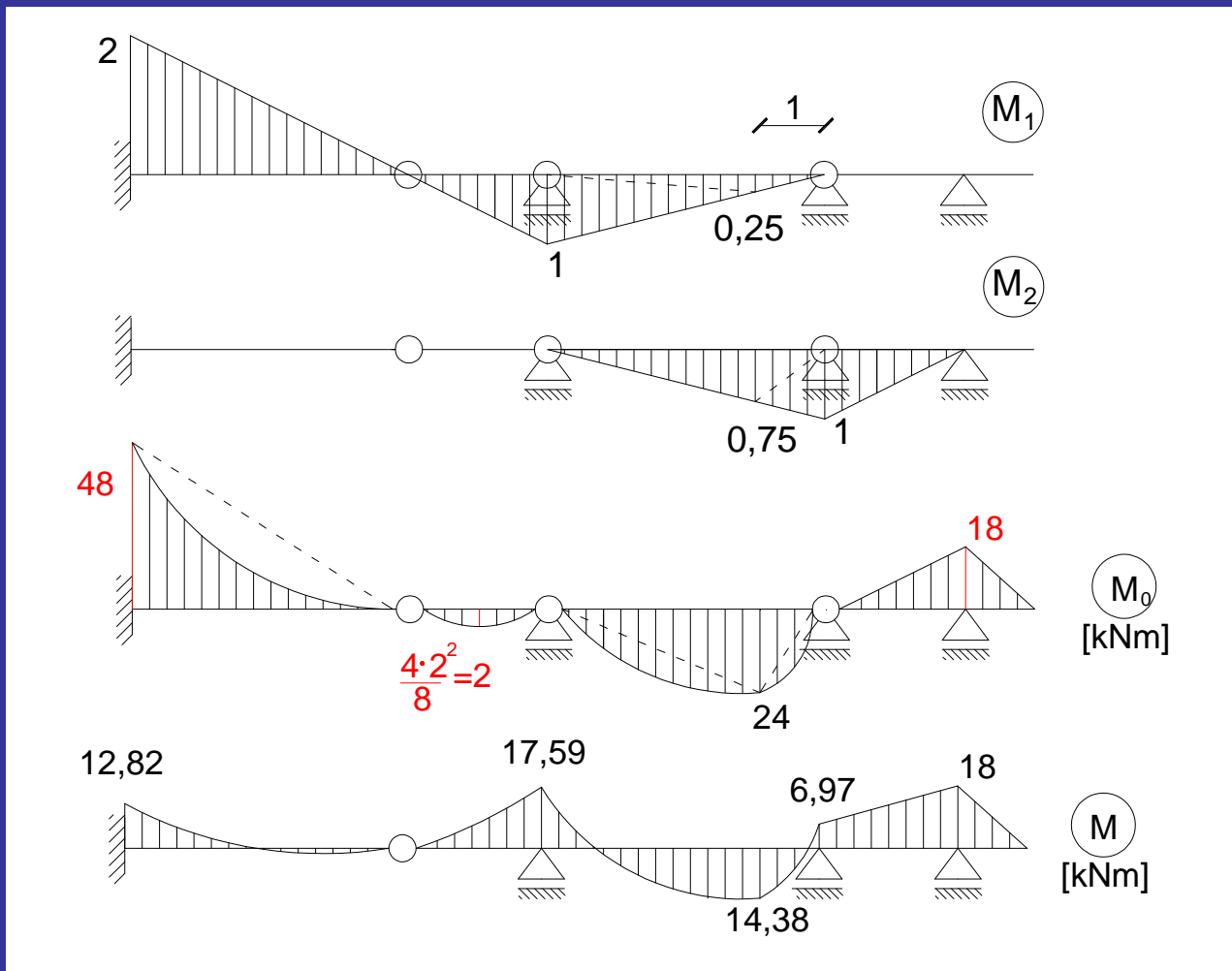
$$\begin{cases} \frac{22}{3EI} \cdot X_1 + \frac{2}{3EI} \cdot X_2 + \frac{401}{3EI} = 0 \\ \frac{2}{3EI} \cdot X_1 + \frac{2}{EI} \cdot X_2 + \frac{77}{3EI} = 0 \end{cases}$$

↓

$$X_1 = -17,59kNm$$

$$X_2 = -6,97kNm$$

Tworzenie ostatecznego wykresu momentów:

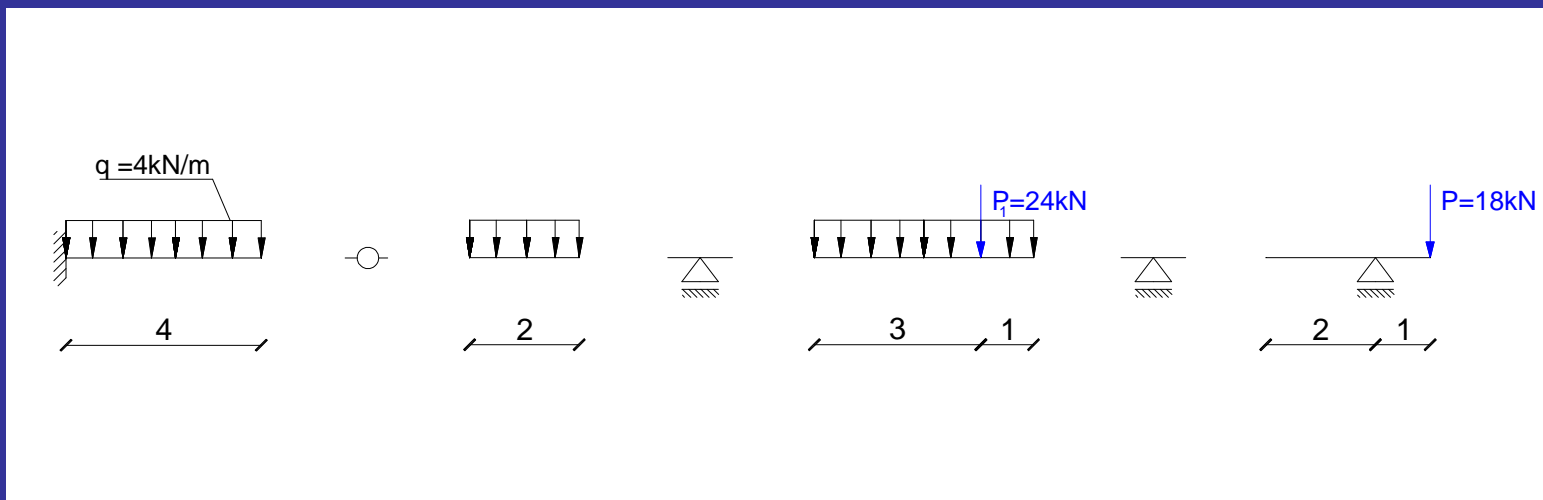
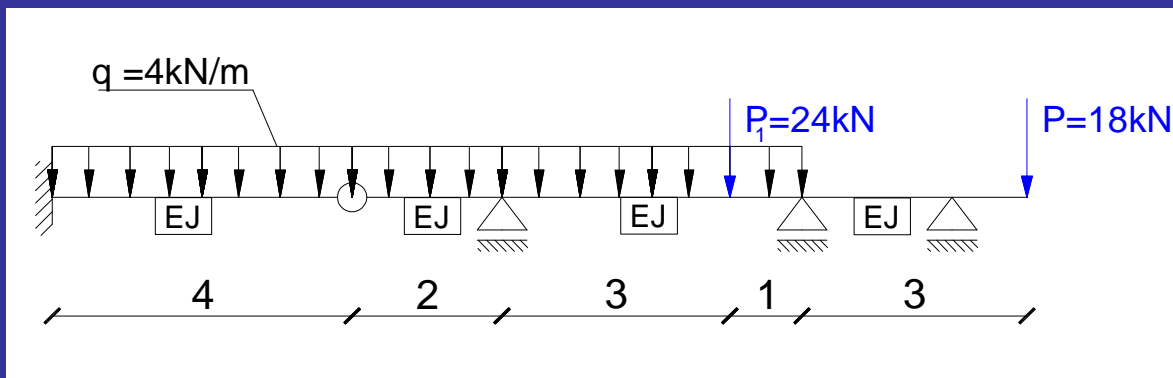


$$X_1 = -17,59 \text{ kNm}$$

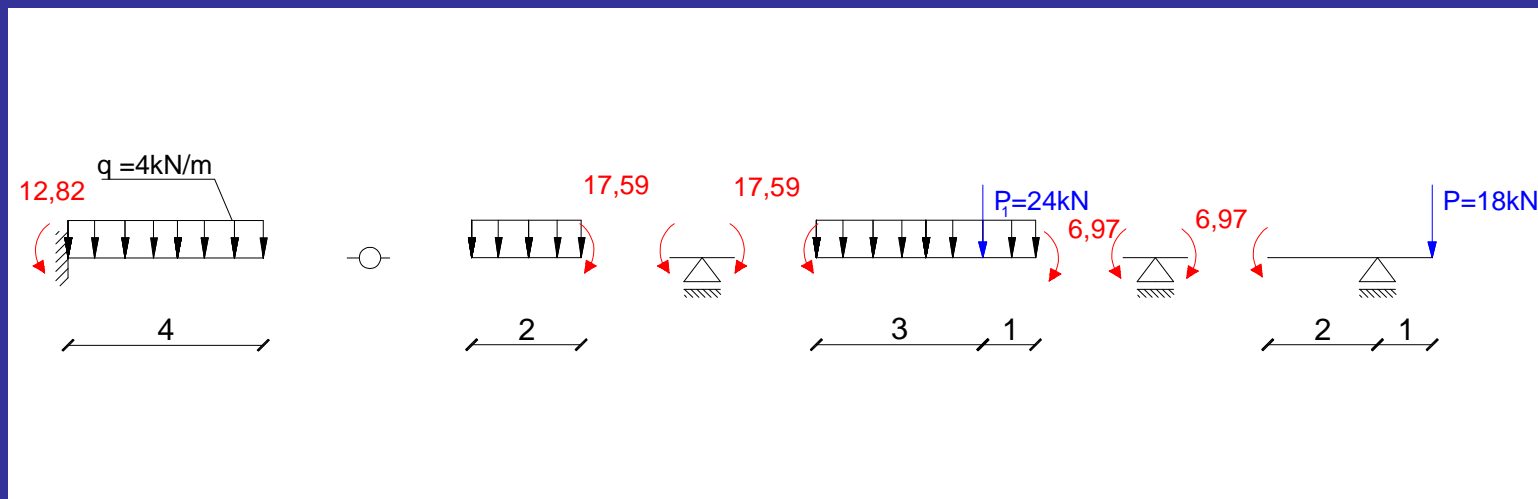
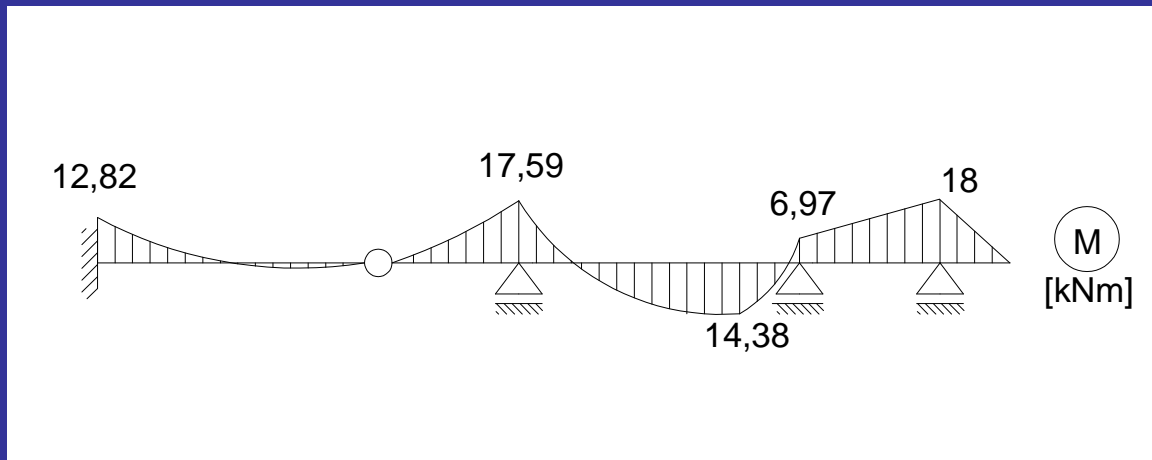
$$X_2 = -6,97 \text{ kNm}$$

$$M_i = M_{i1} \cdot X_1 + M_{i2} \cdot X_2 + M_{i0}$$

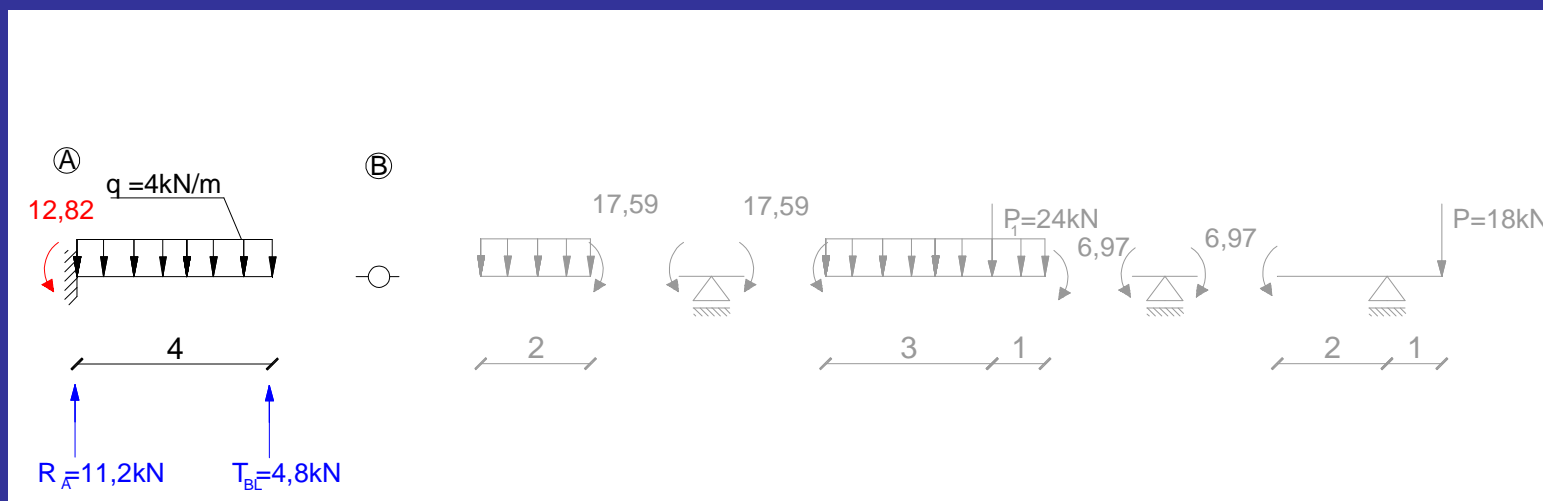
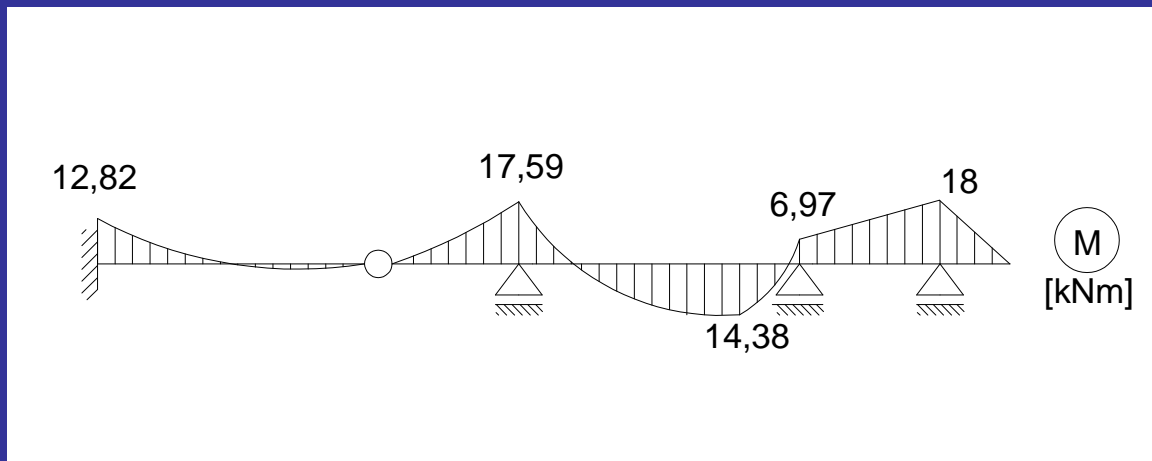
Wyznaczenie wartości sił tnących:



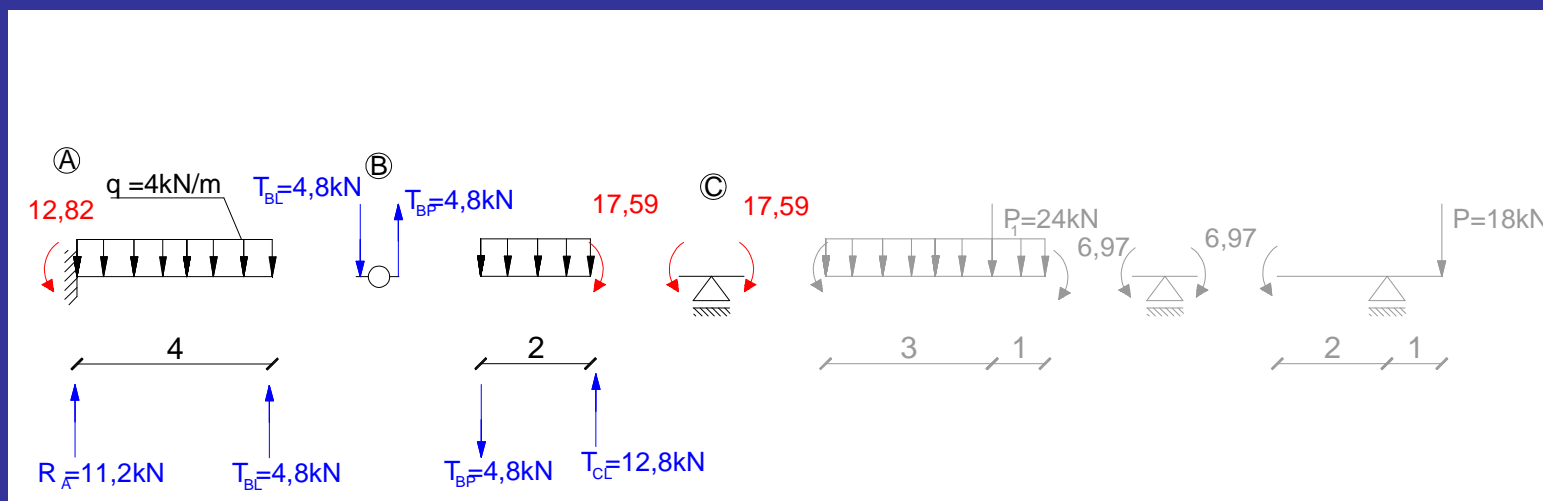
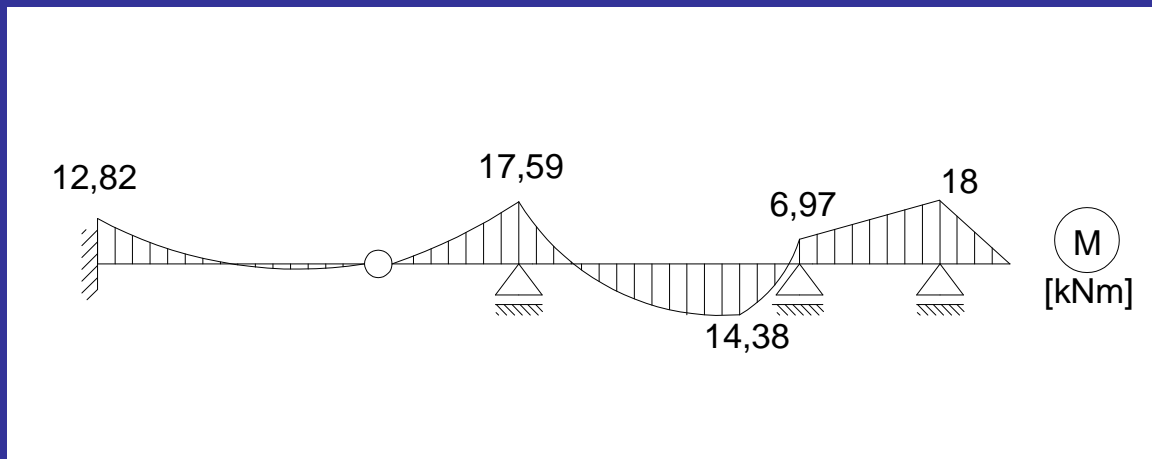
Wyznaczenie wartości sił tnących:



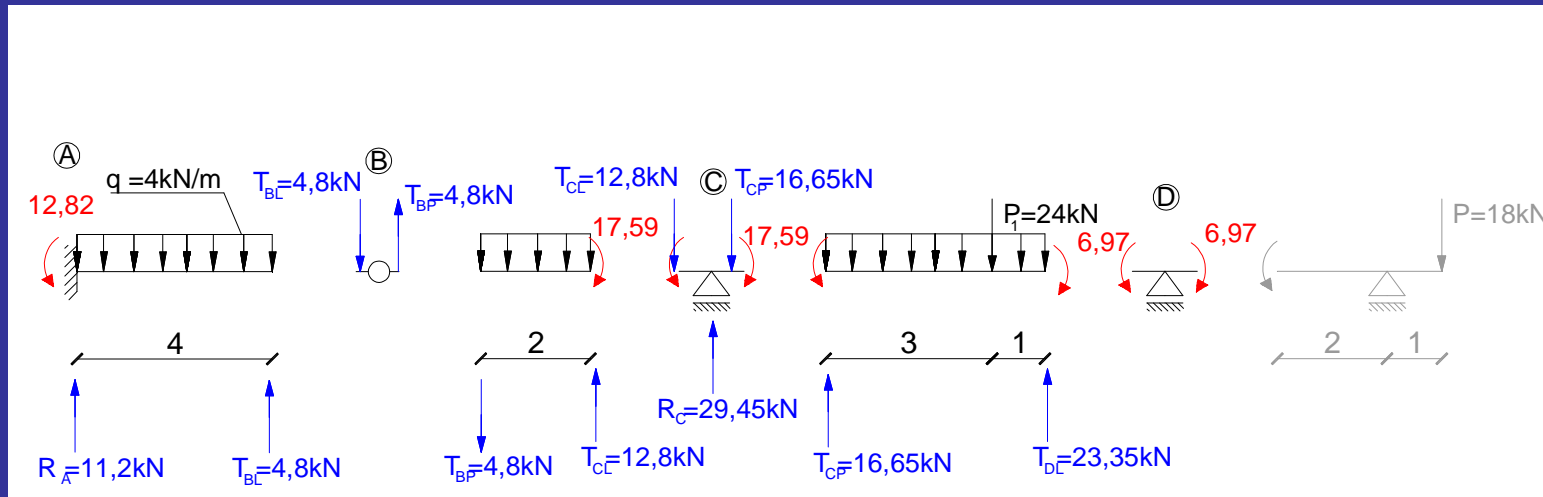
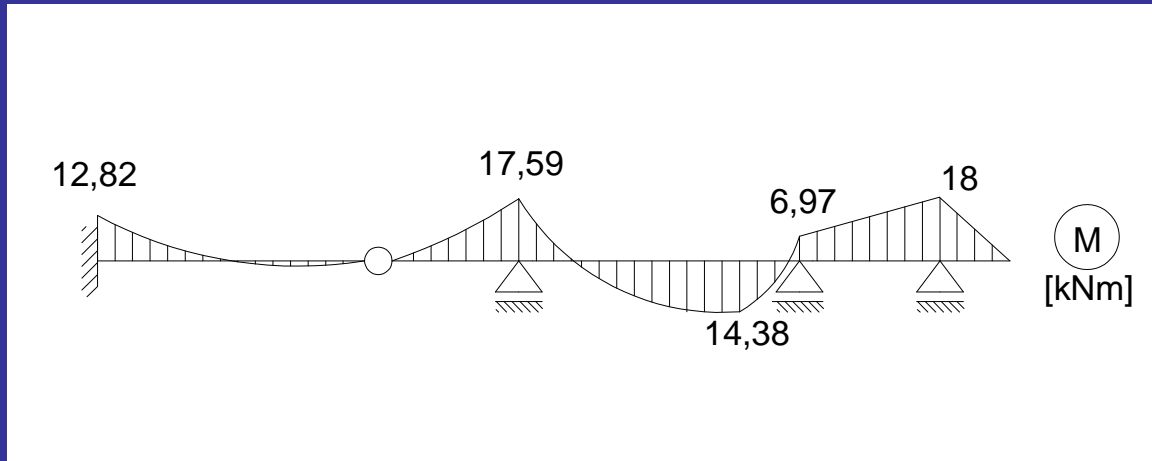
Wyznaczenie wartości sił tnących:



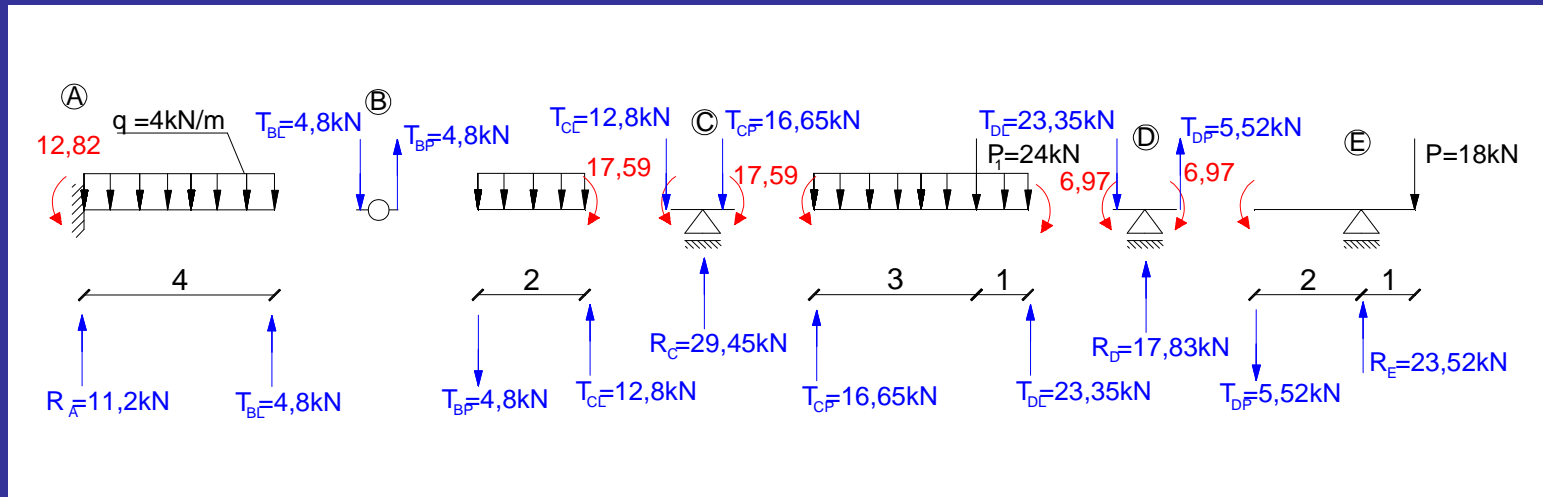
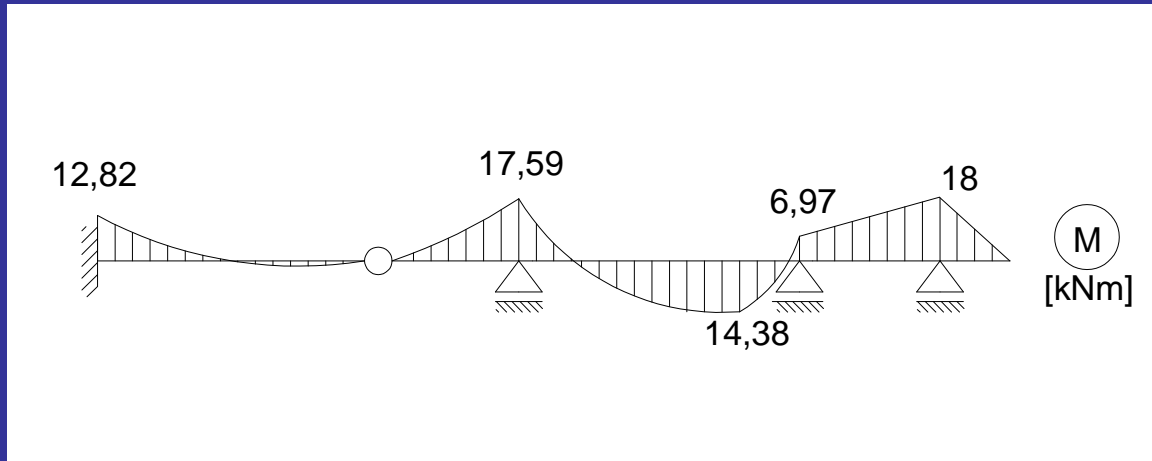
Wyznaczenie wartości sił tnących:



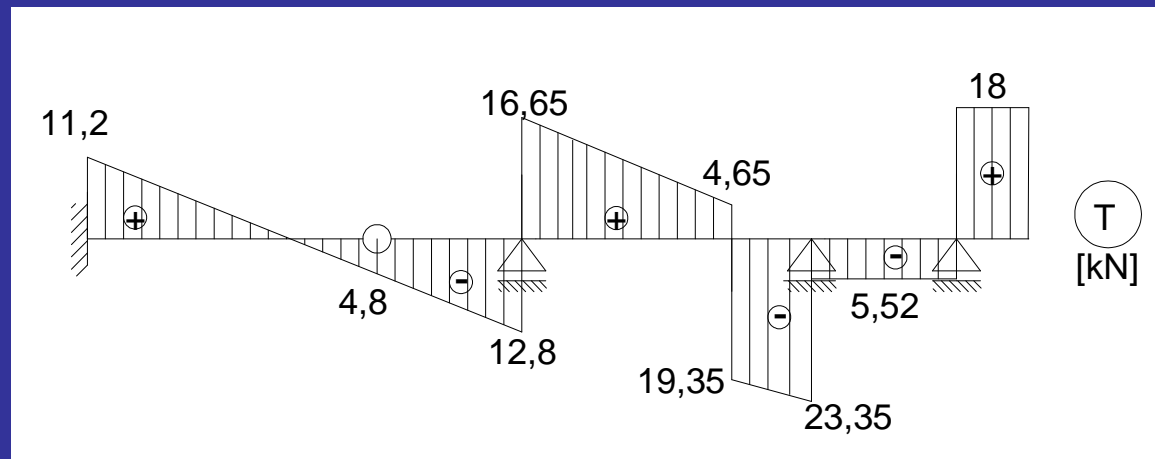
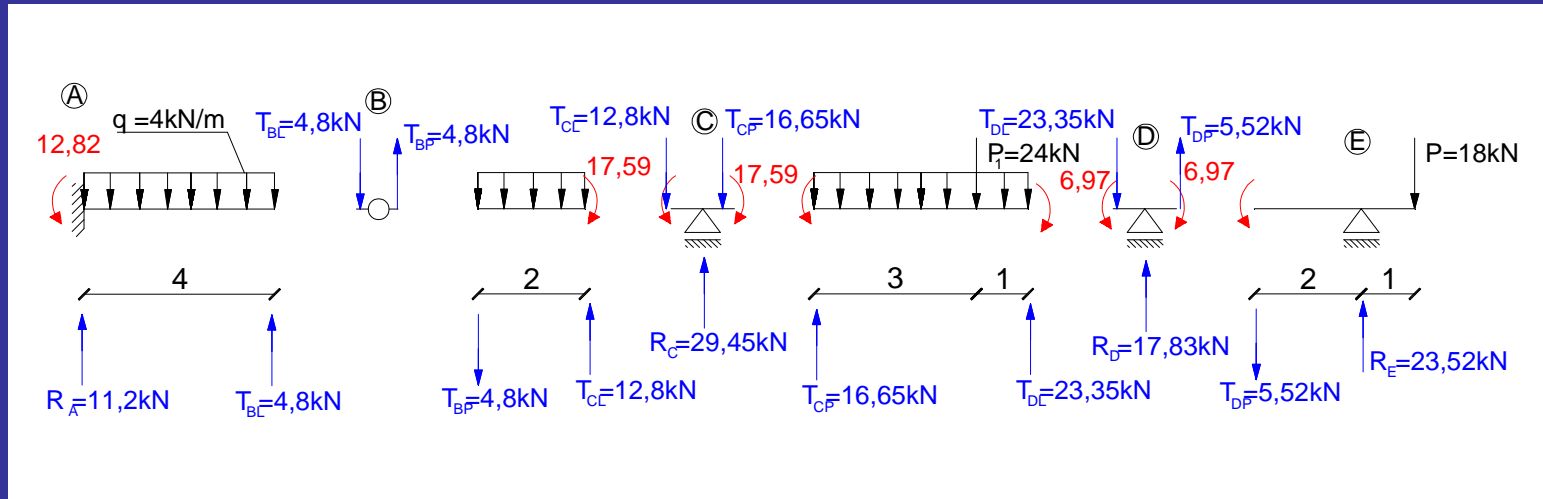
Wyznaczenie wartości sił tnących:



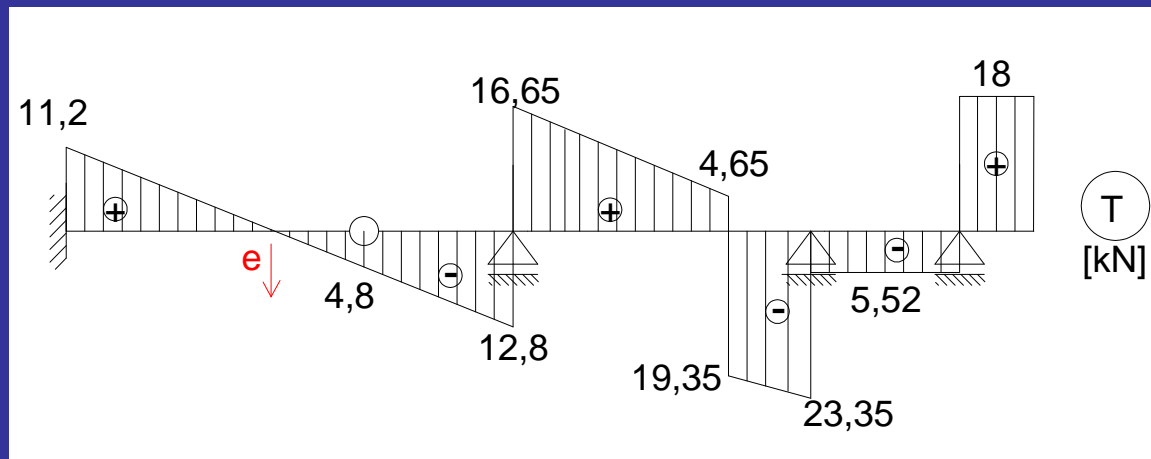
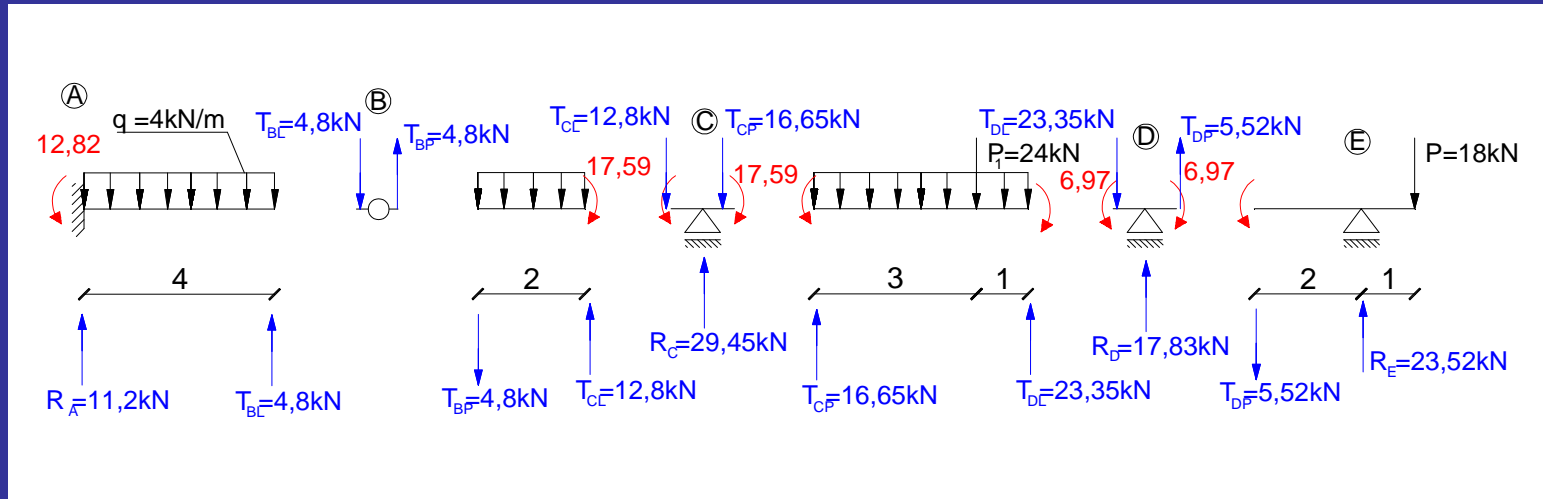
Wyznaczenie wartości sił tnących:



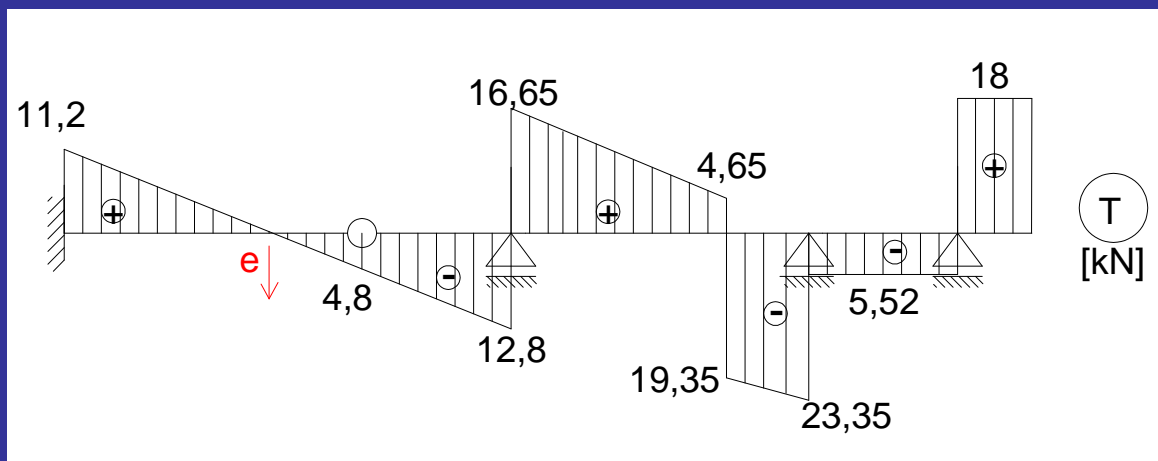
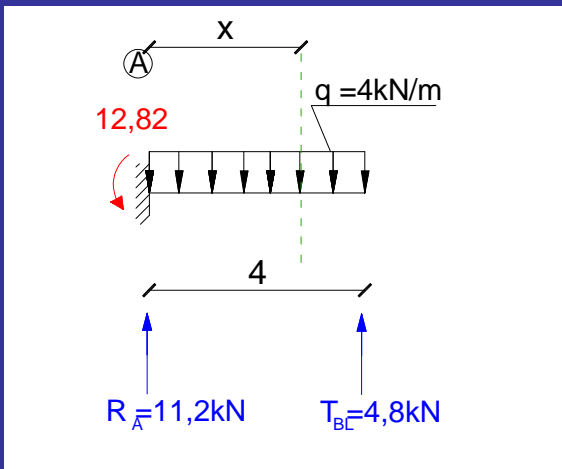
Rysowanie wykresu sił tnących:



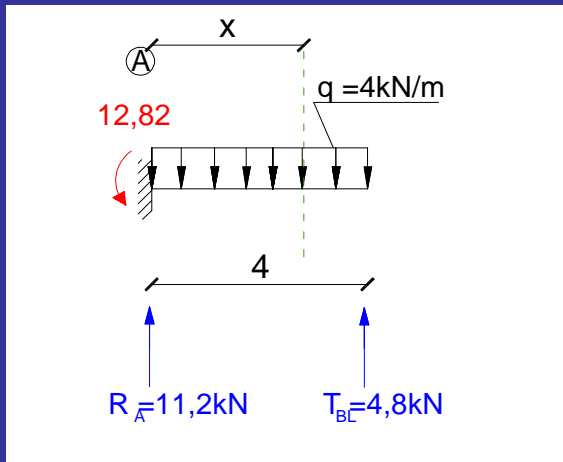
Wyznaczanie ekstremum:



Wyznaczanie ekstremum:



Wyznaczanie ekstremum:

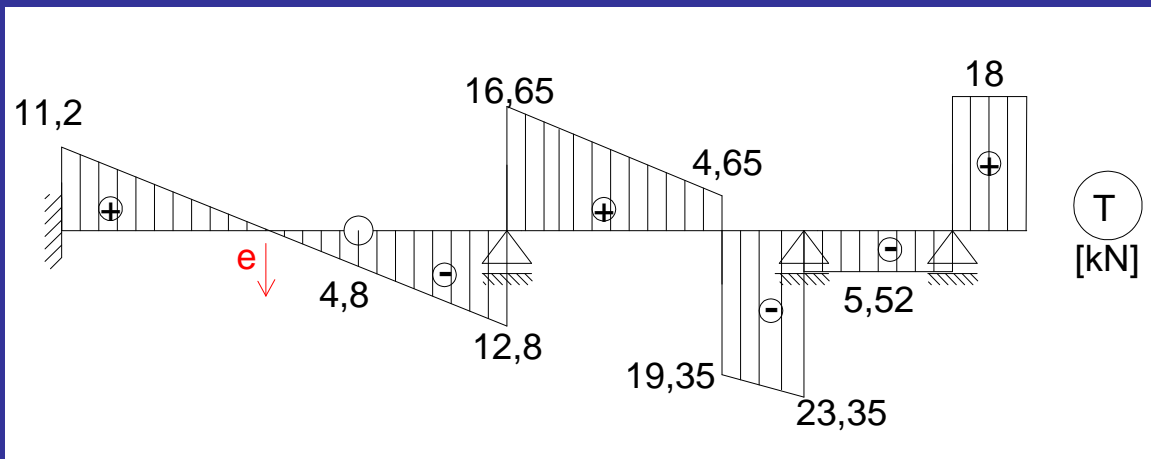


$$T(x) = 11,2 - 4x = 0$$

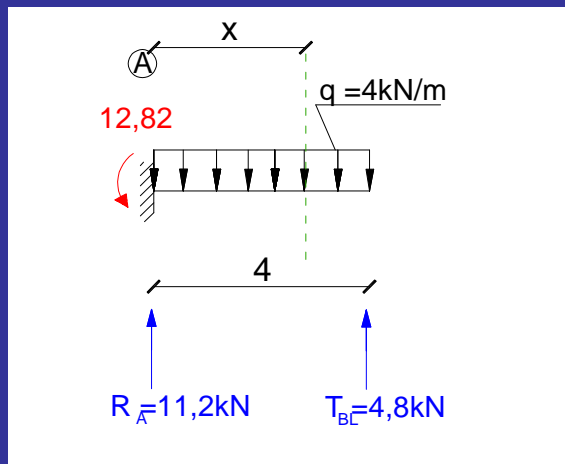
↓

$$x = \frac{11,2}{4} = 2,8m$$

$$M(x) = -12,82 + 11,2 \cdot 2,8 - \frac{4 \cdot 2,8^2}{2} = 2,86kNm$$



Wyznaczanie ekstremum:

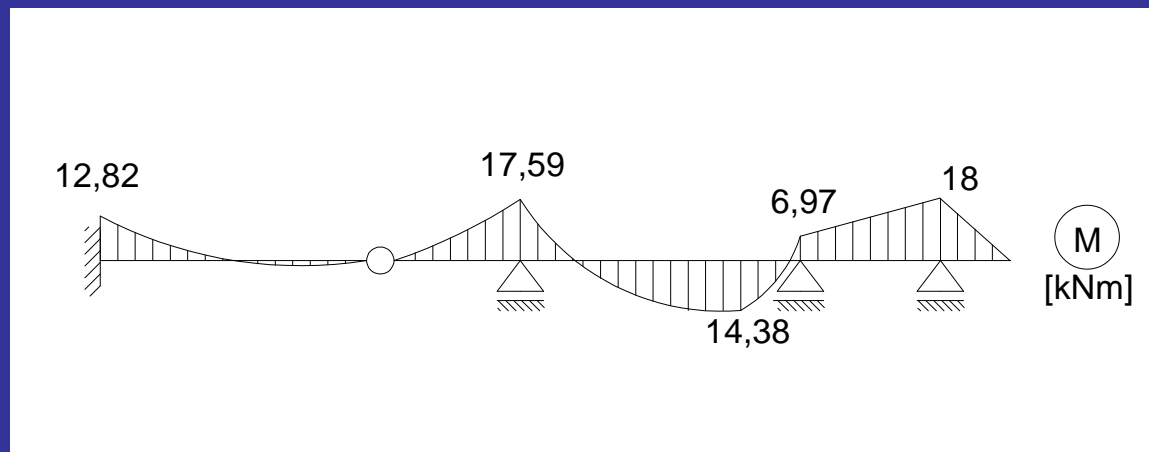


$$T(x) = 11,2 - 4x = 0$$

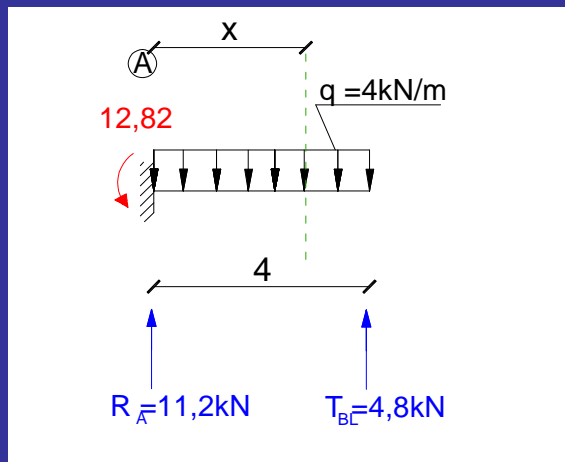
↓

$$x = \frac{11,2}{4} = 2,8 \text{ m}$$

$$M(x) = -12,82 + 11,2 \cdot 2,8 - \frac{4 \cdot 2,8^2}{2} = 2,86 \text{ kNm}$$



Wyznaczanie ekstremum:



$$T(x) = 11,2 - 4x = 0$$

↓

$$x = \frac{11,2}{4} = 2,8m$$

$$M(x) = -12,82 + 11,2 \cdot 2,8 - \frac{4 \cdot 2,8^2}{2} = 2,86kNm$$

