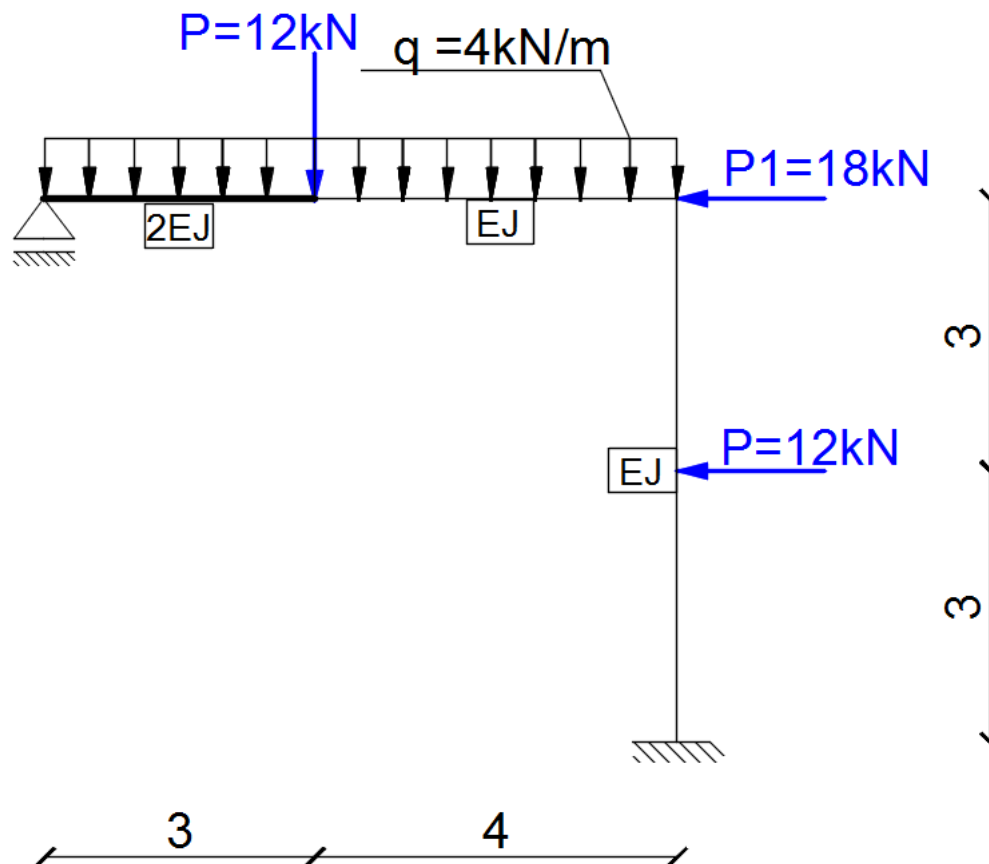


Mechanika Budowli 2 – sem. IV N1

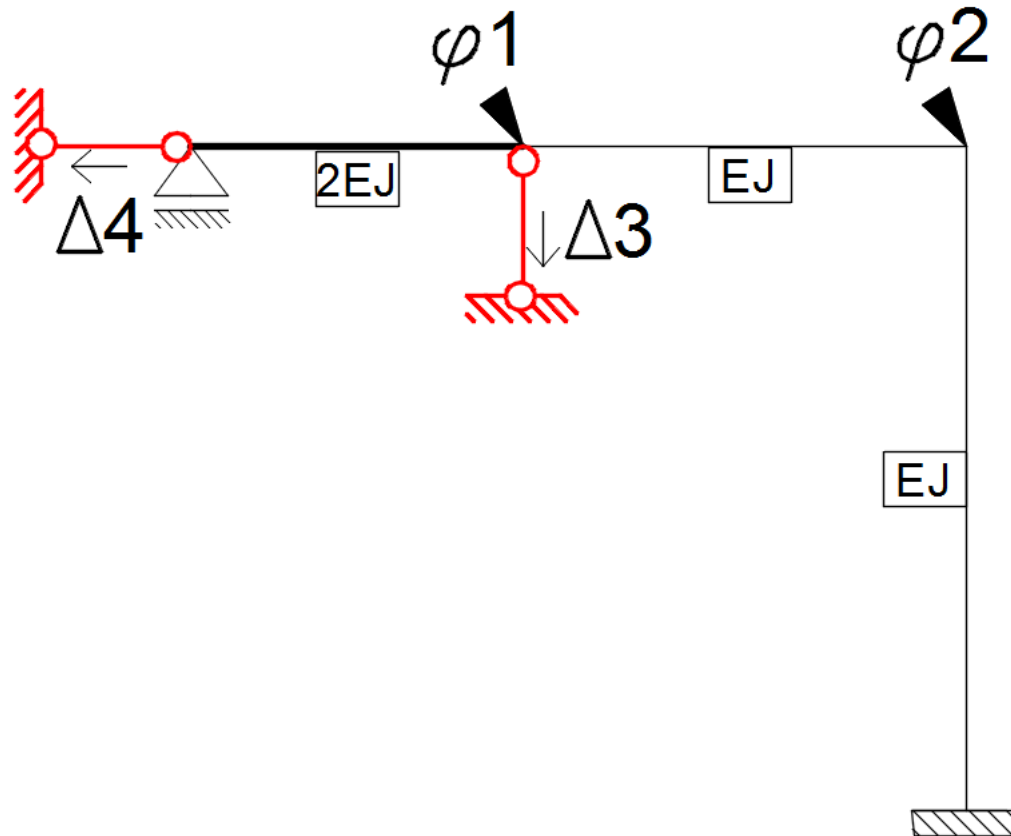
Wykład nr 3:

Obliczanie ramy przesuwnej metodą przemieszczeń

Zadanie: Wyznaczyć współczynniki układu równań metody przemieszczeń.
Zadanie rozwiązać w minimalnej bazie niewiadomych.



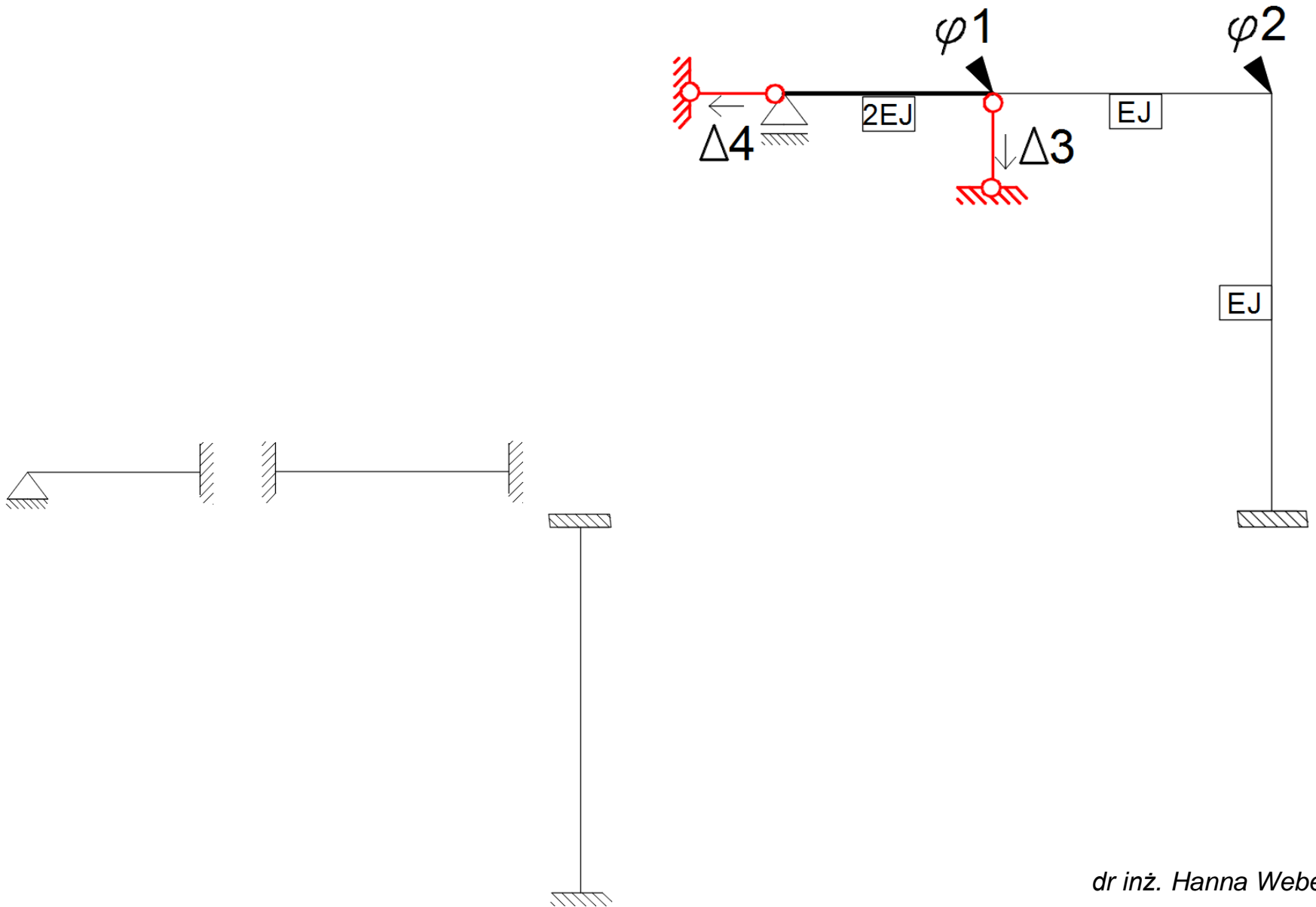
Schemat podstawowy geometrycznie wyznaczalny



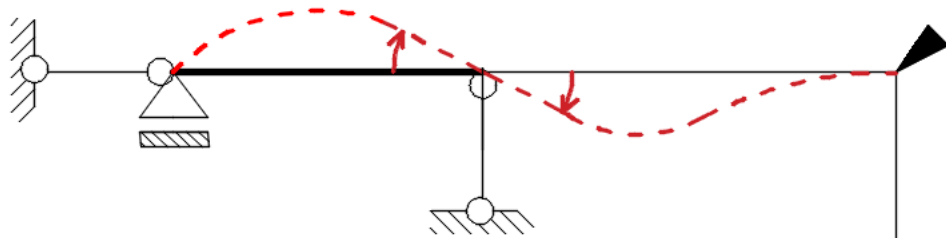
Układ czterokrotnie geometrycznie niewyznaczalny

$$n_g = 4(\varphi 1, \varphi 2, \Delta 3, \Delta 4)$$

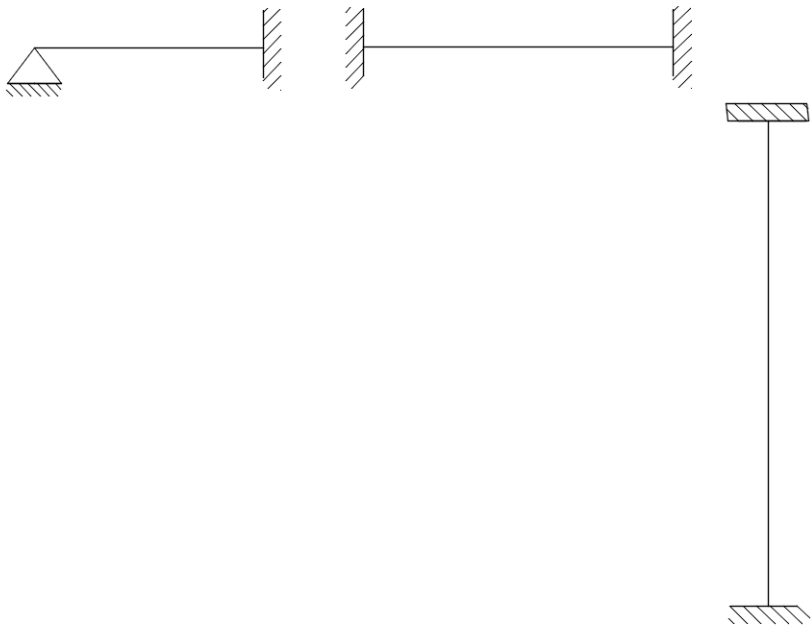
Schemat podstawowy geometrycznie wyznaczalny



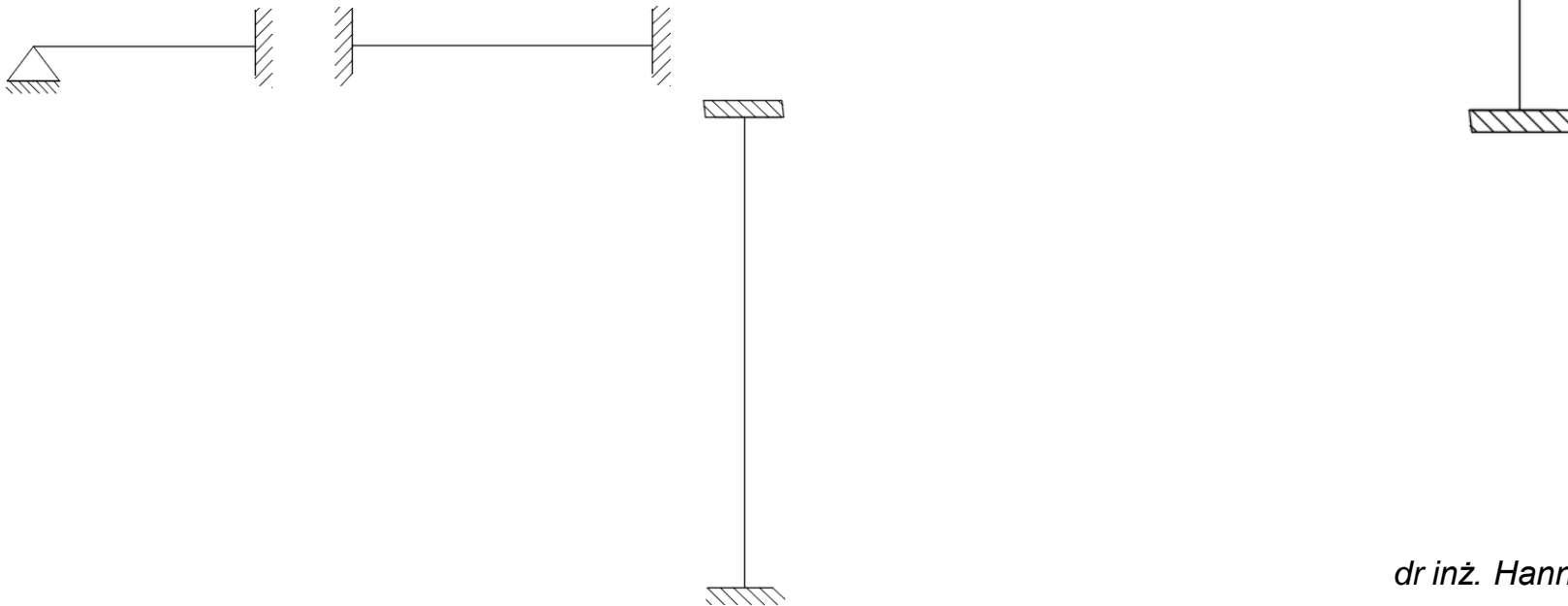
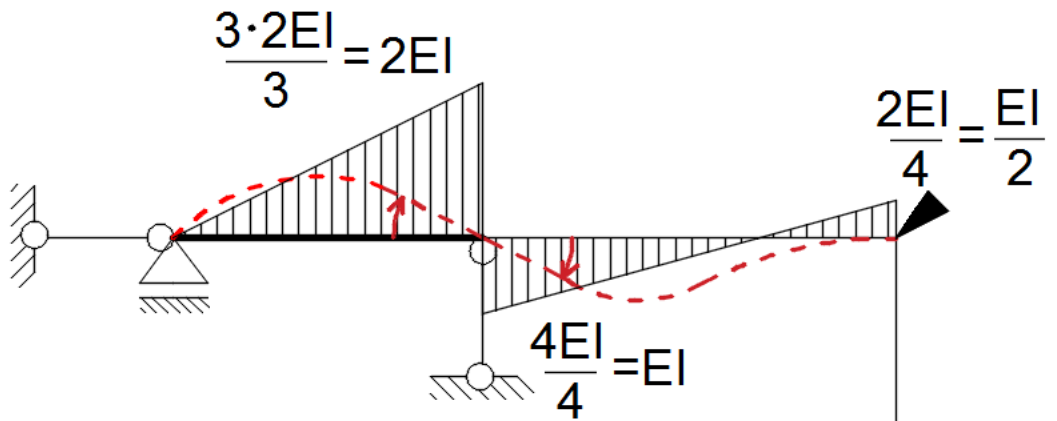
Stan $\varphi_1=1$



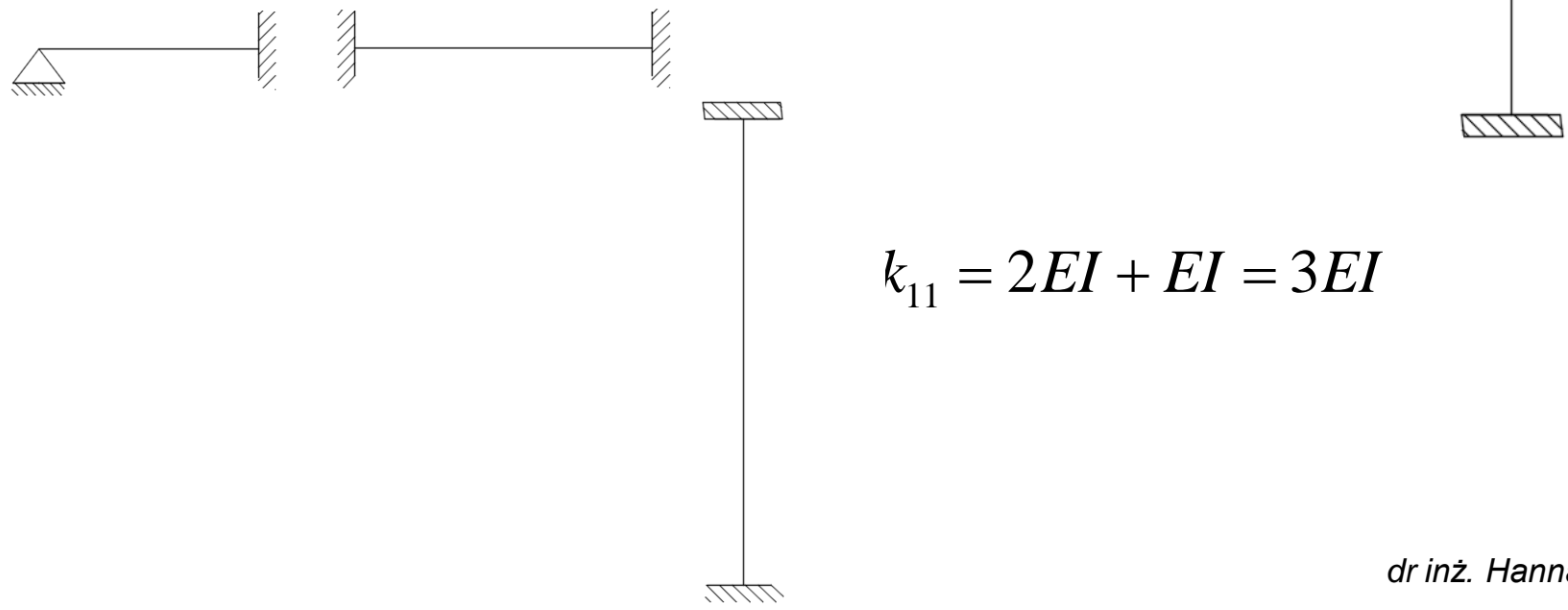
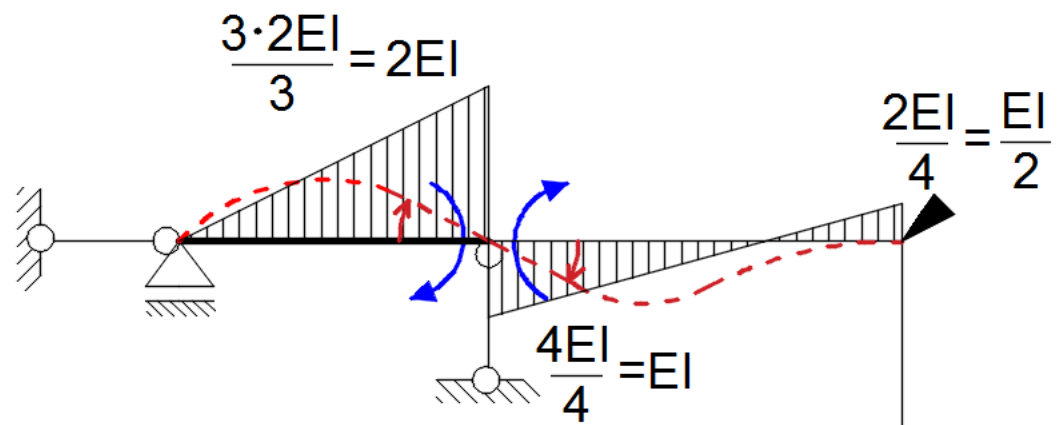
EJ



Stan $\varphi_1=1$

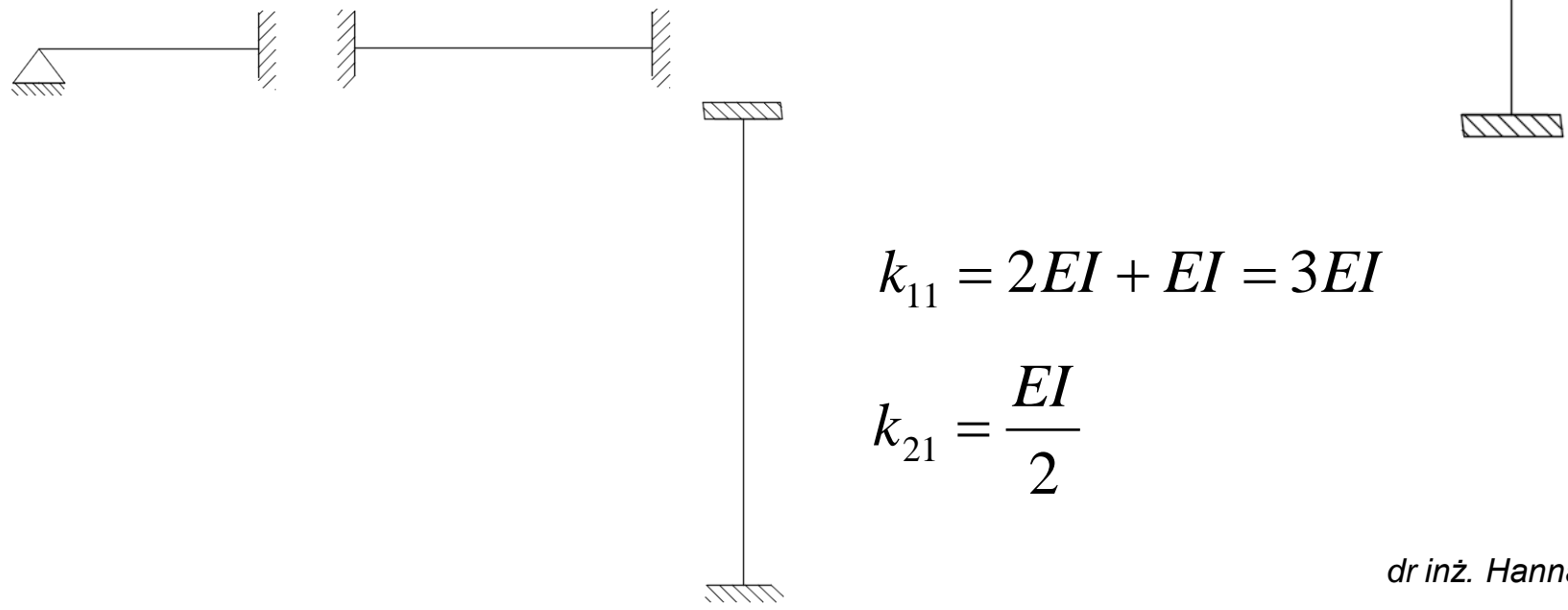
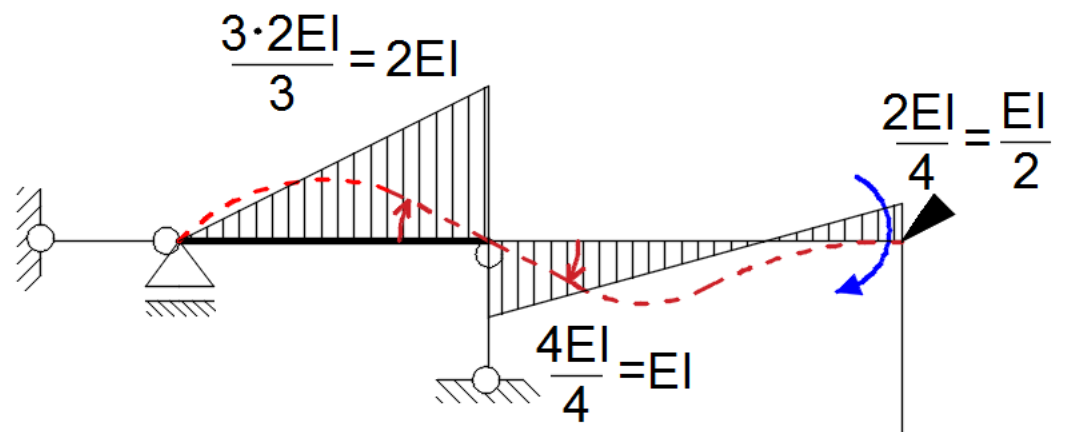


Stan $\varphi_1=1$



$$k_{11} = 2EI + EI = 3EI$$

Stan $\varphi_1=1$



$$k_{11} = 2EI + EI = 3EI$$

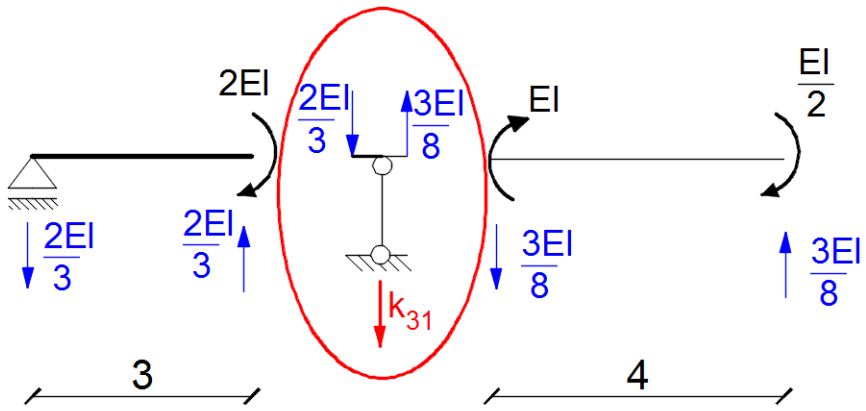
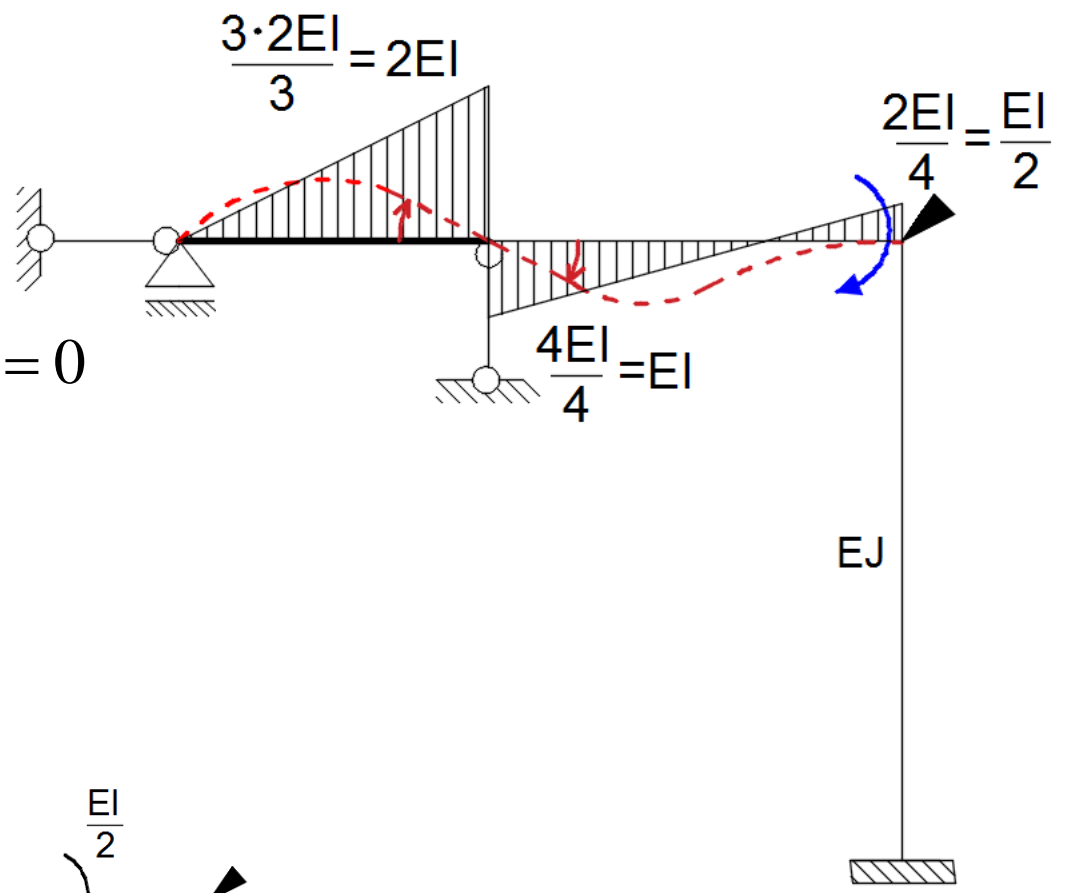
$$k_{21} = \frac{EI}{2}$$

Stan $\varphi_1=1$

$$\sum R_y = -\frac{2EI}{3} + \frac{3EI}{8} - k_{31} = 0$$

↓

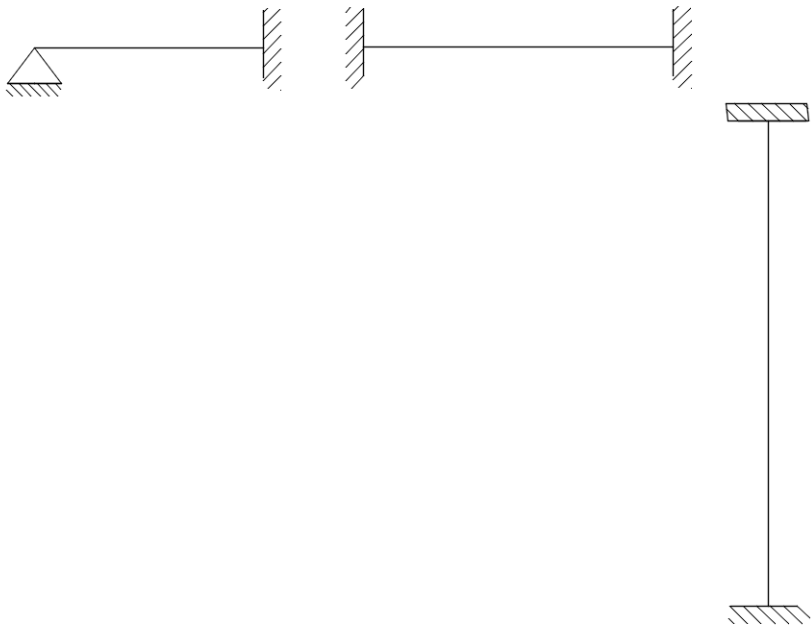
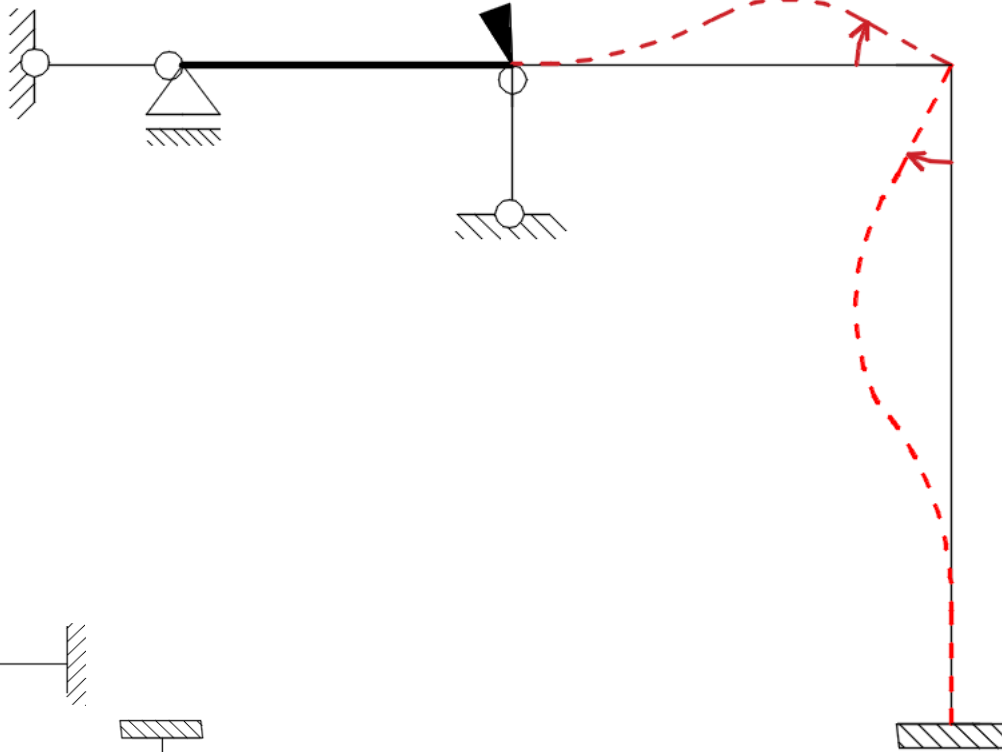
$$k_{31} = -\frac{7EI}{24}$$



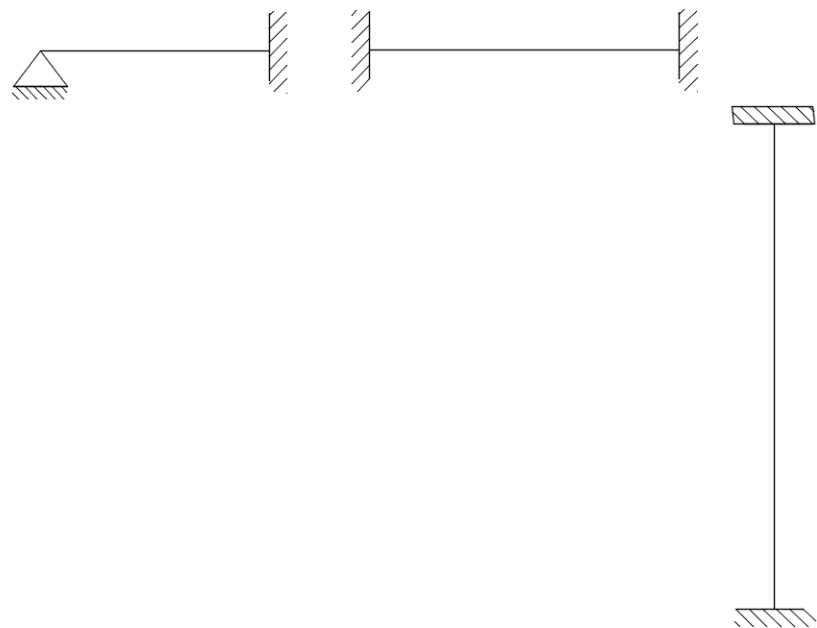
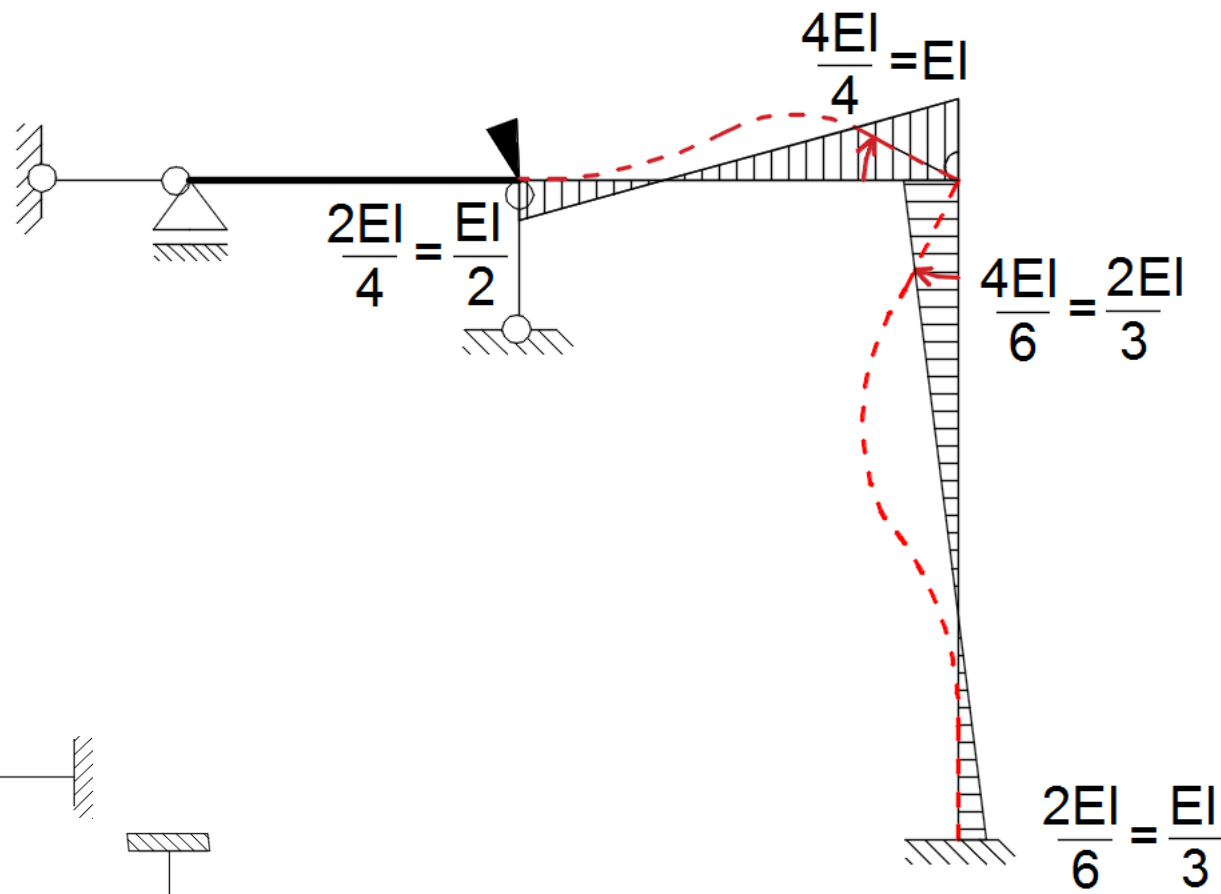
$$k_{11} = 2EI + EI = 3EI$$

$$k_{21} = \frac{EI}{2}$$

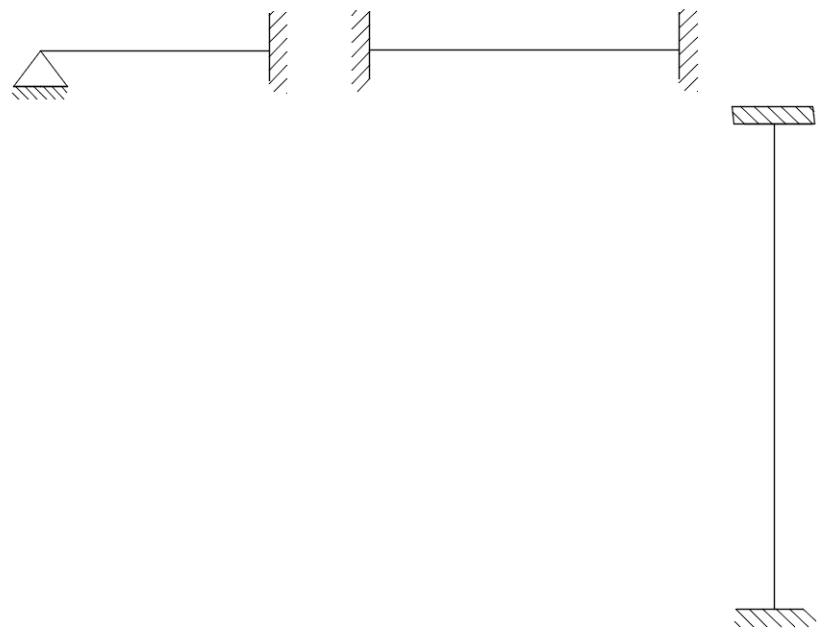
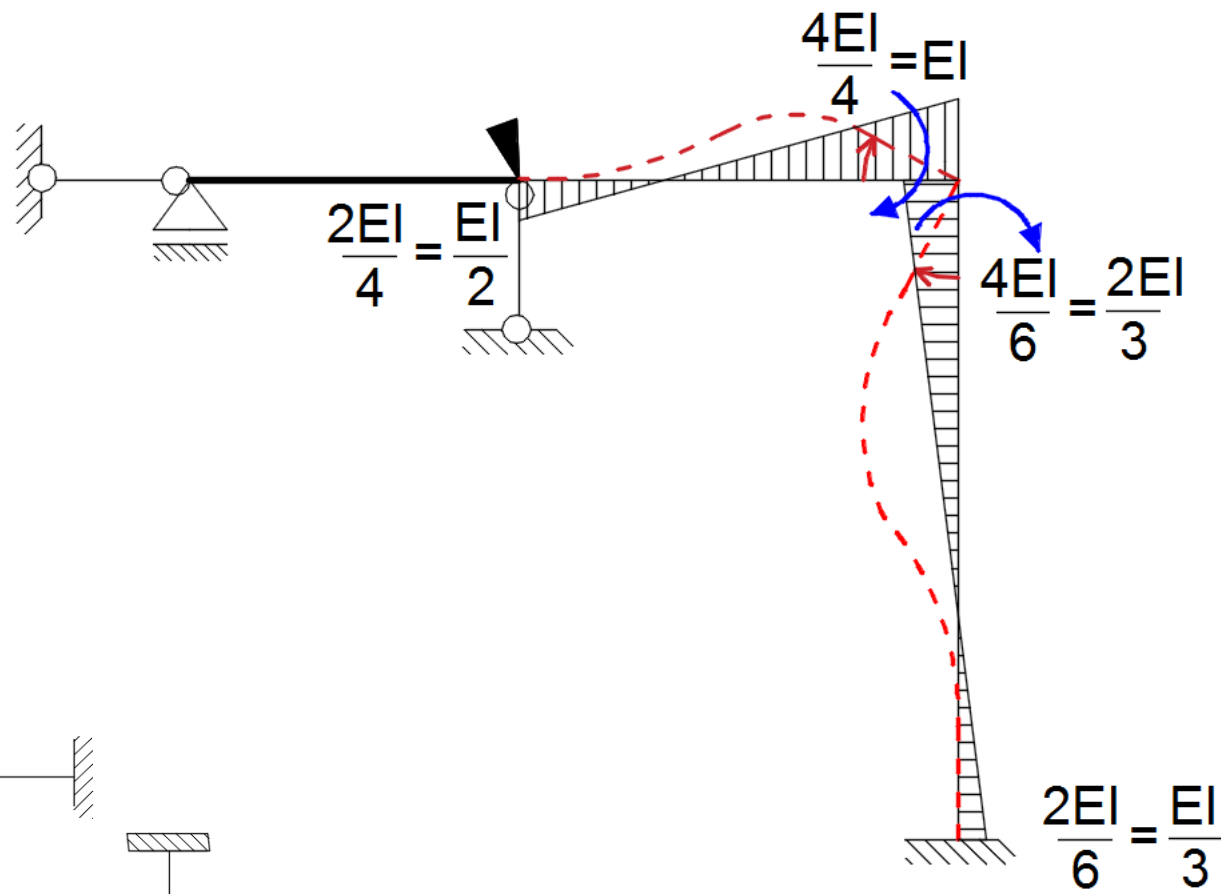
Stan $\varphi_2=1$



Stan $\varphi_2=1$

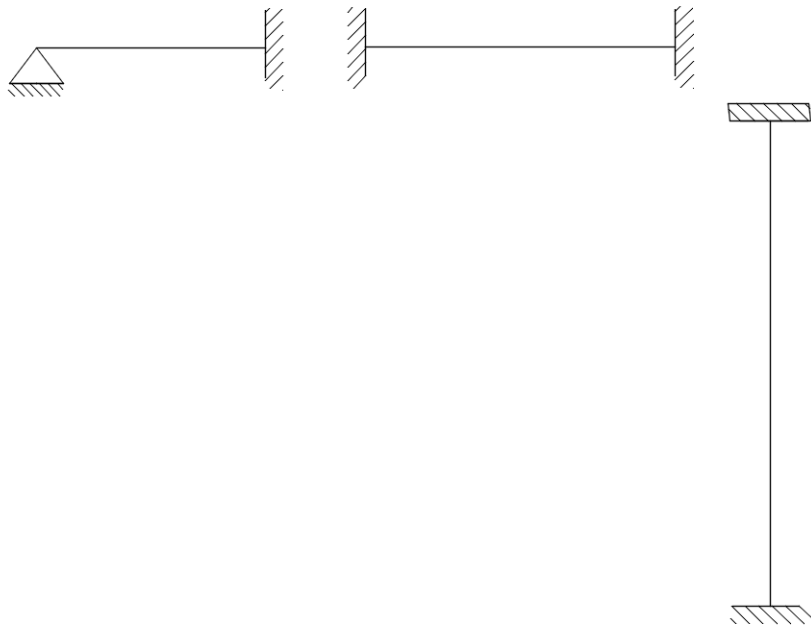
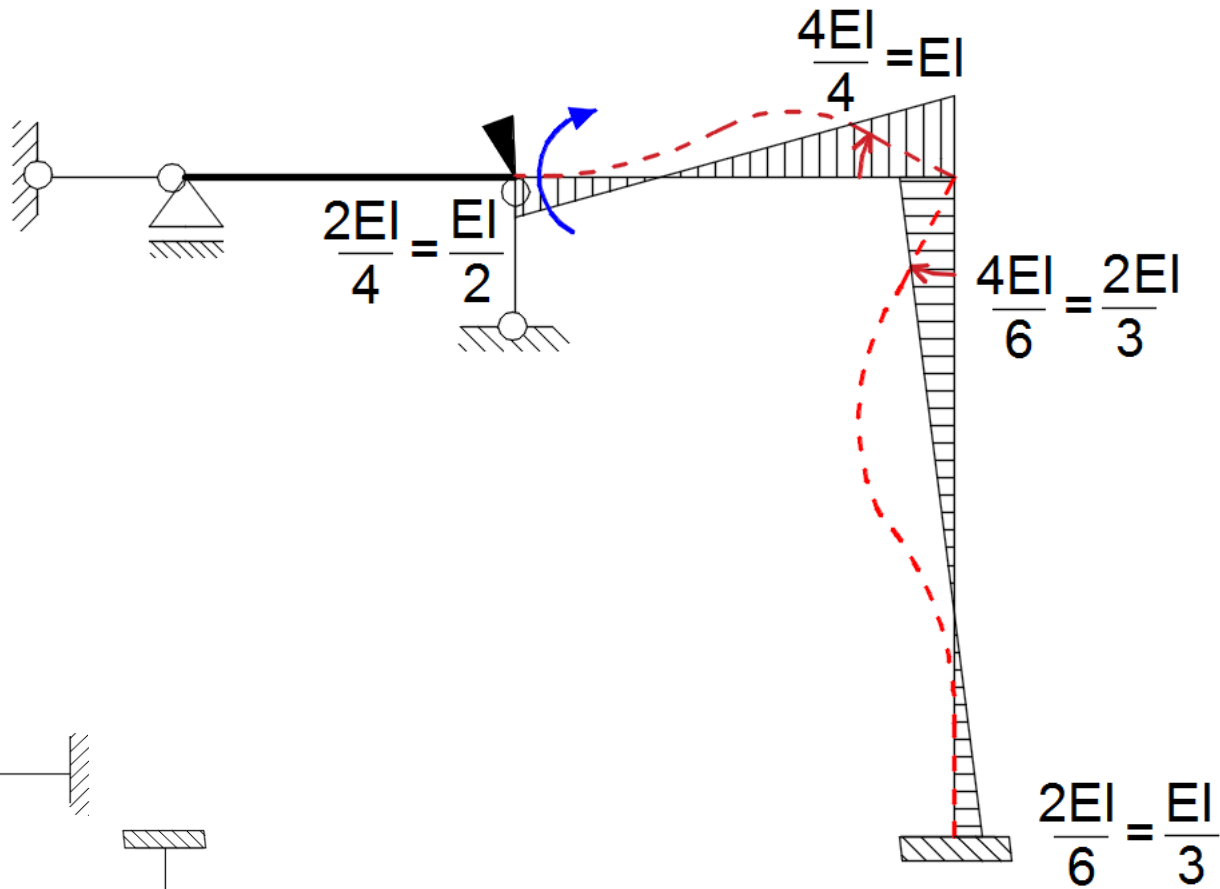


Stan $\varphi_2=1$



$$k_{22} = EI + \frac{2EI}{3} = \frac{5EI}{3}$$

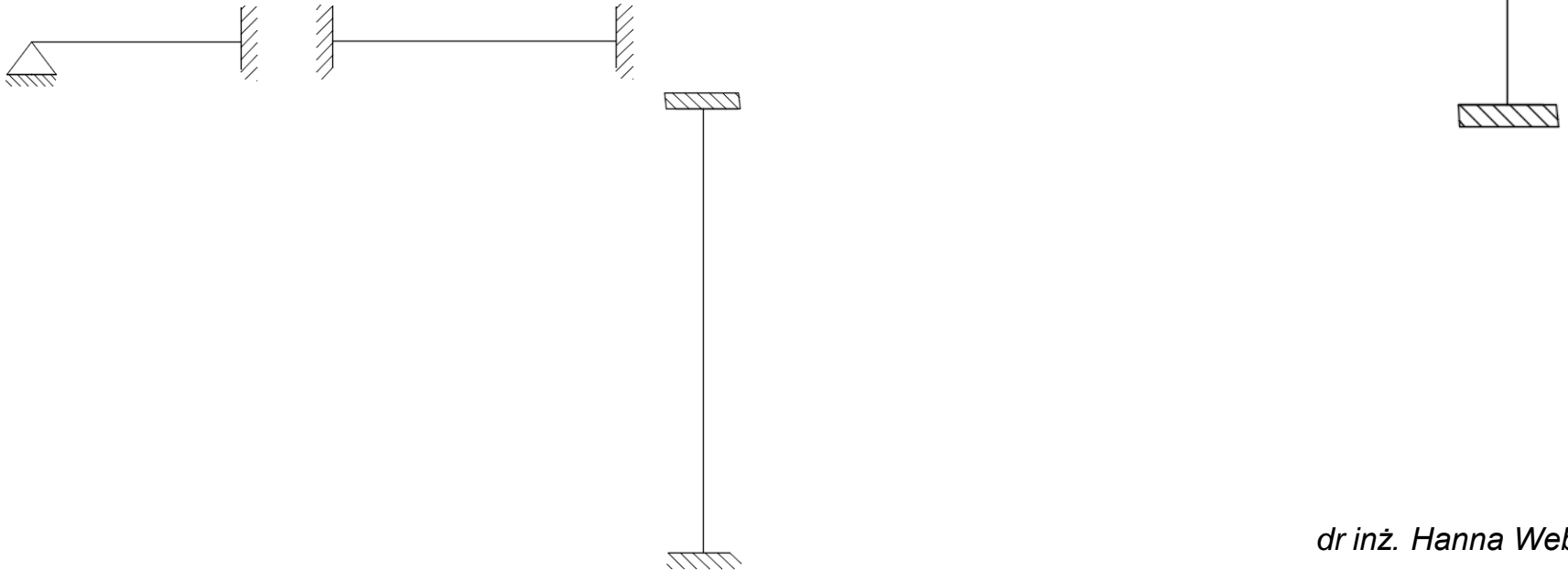
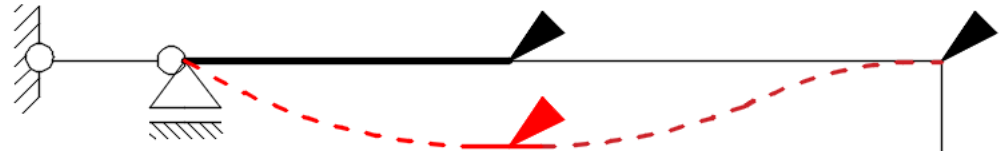
Stan $\varphi_2=1$



$$k_{22} = EI + \frac{2EI}{3} = \frac{5EI}{3}$$

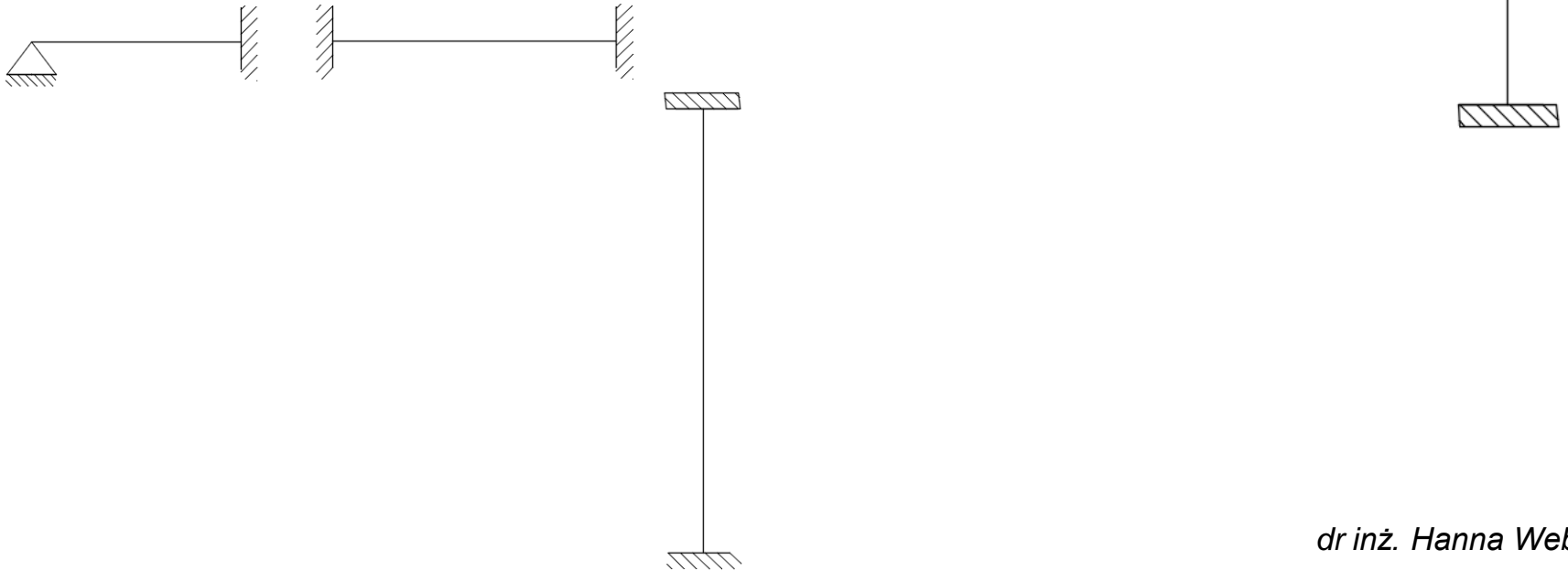
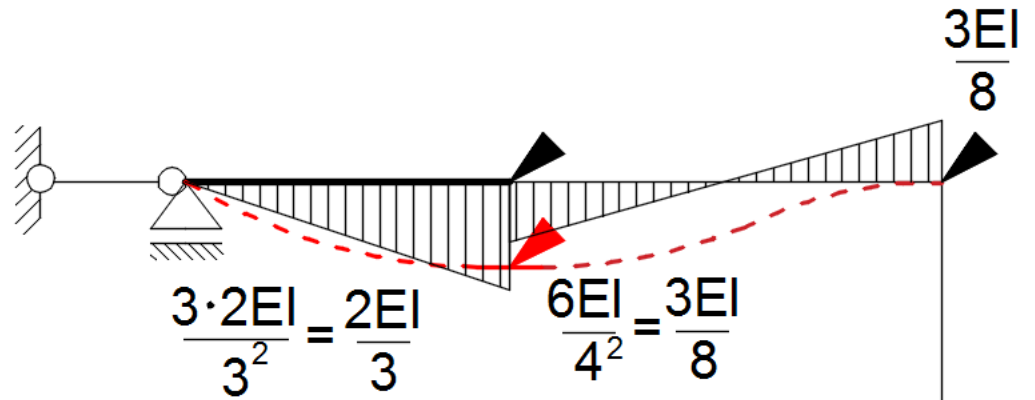
$$k_{12} = \frac{EI}{2}$$

Stan $\Delta_3=1$

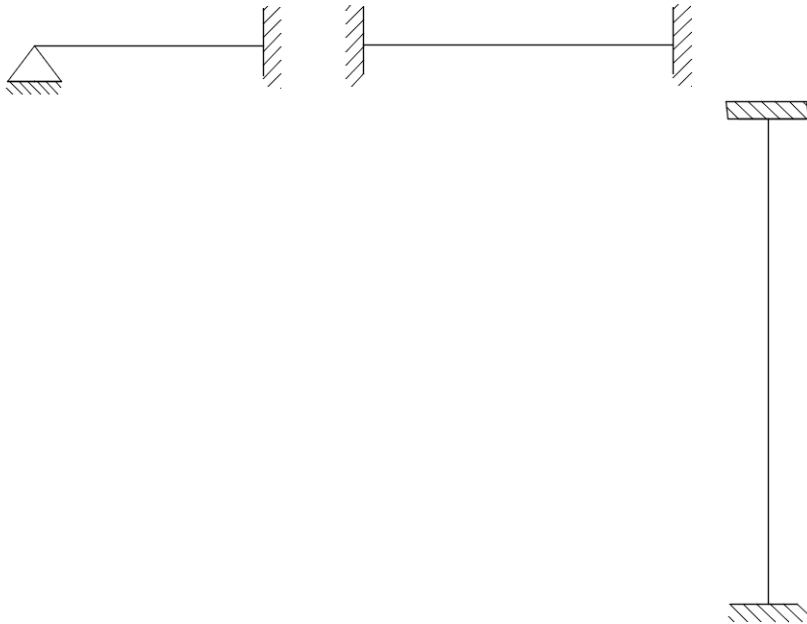
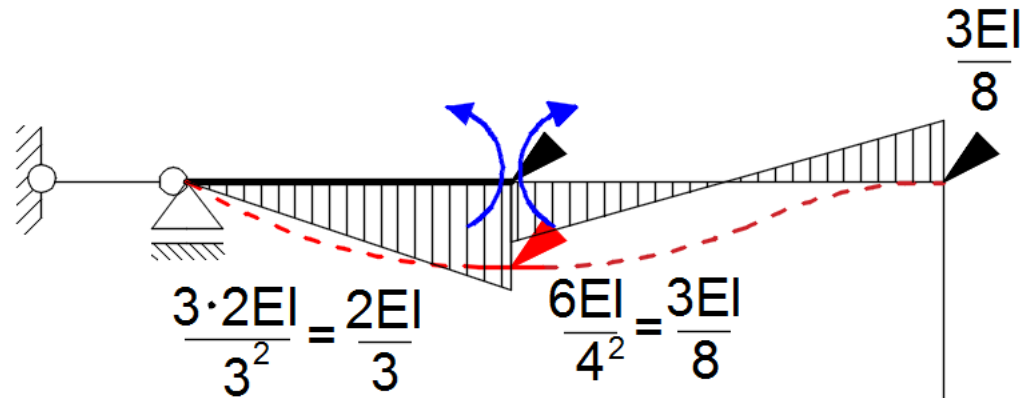


dr inž. Hanna Weber

Stan $\Delta_3=1$

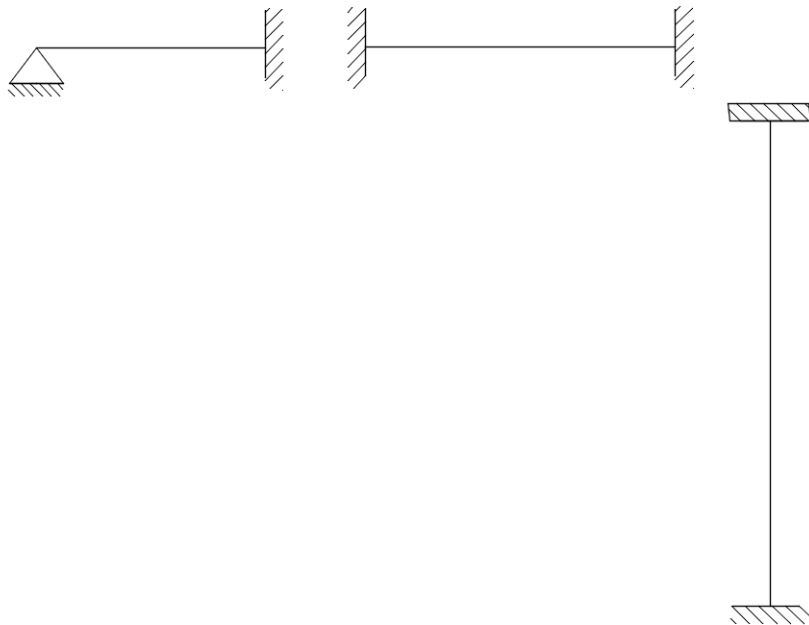
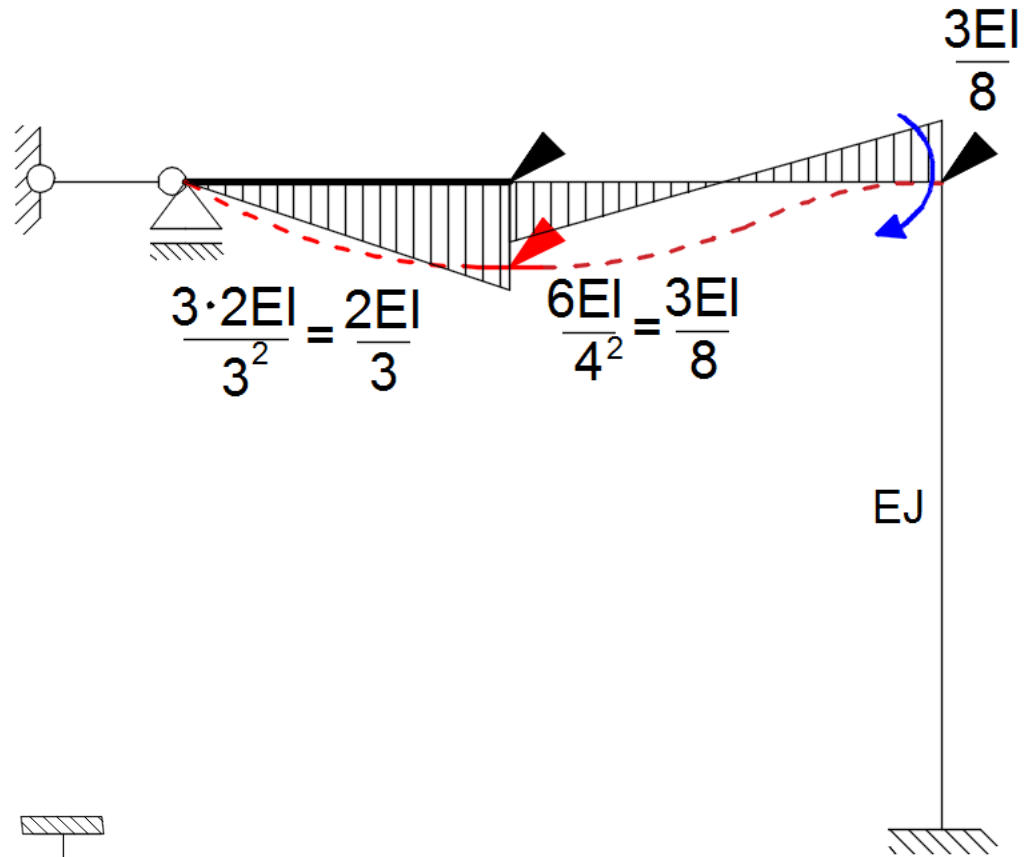


Stan $\Delta_3=1$



$$k_{13} = \frac{3EI}{8} - \frac{2EI}{3} = -\frac{7EI}{24} = k_{31}$$

Stan $\Delta_3=1$



$$k_{13} = \frac{3EI}{8} - \frac{2EI}{3} = -\frac{7EI}{24} = k_{31}$$

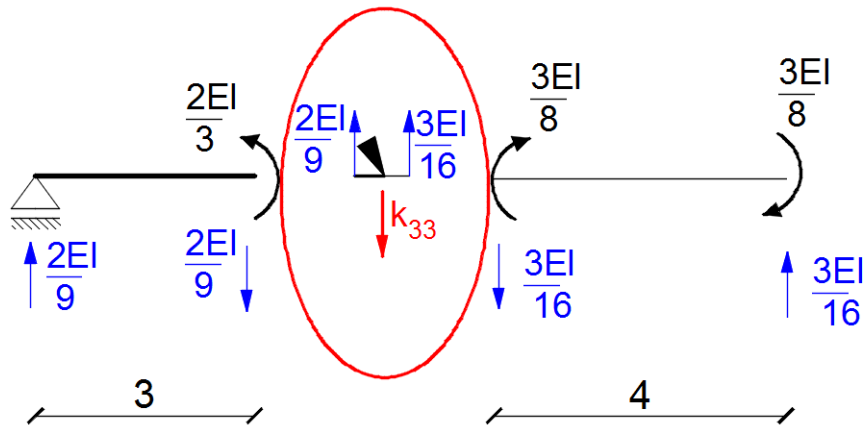
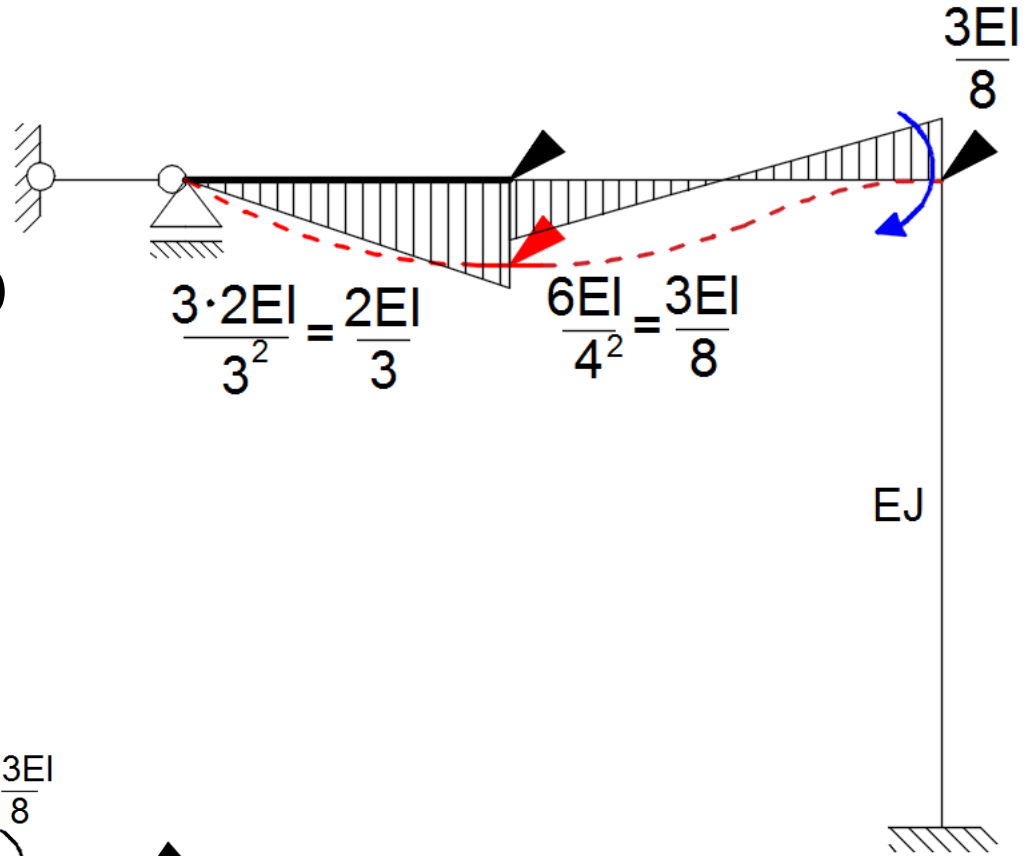
$$k_{23} = \frac{3EI}{8} = k_{32}$$

Stan $\Delta_3=1$

$$\sum R_y = \frac{2EI}{9} + \frac{3EI}{16} - k_{33} = 0$$

↓

$$k_{33} = \frac{59EI}{144}$$



$$k_{13} = \frac{3EI}{8} - \frac{2EI}{3} = -\frac{7EI}{24} = k_{31}$$

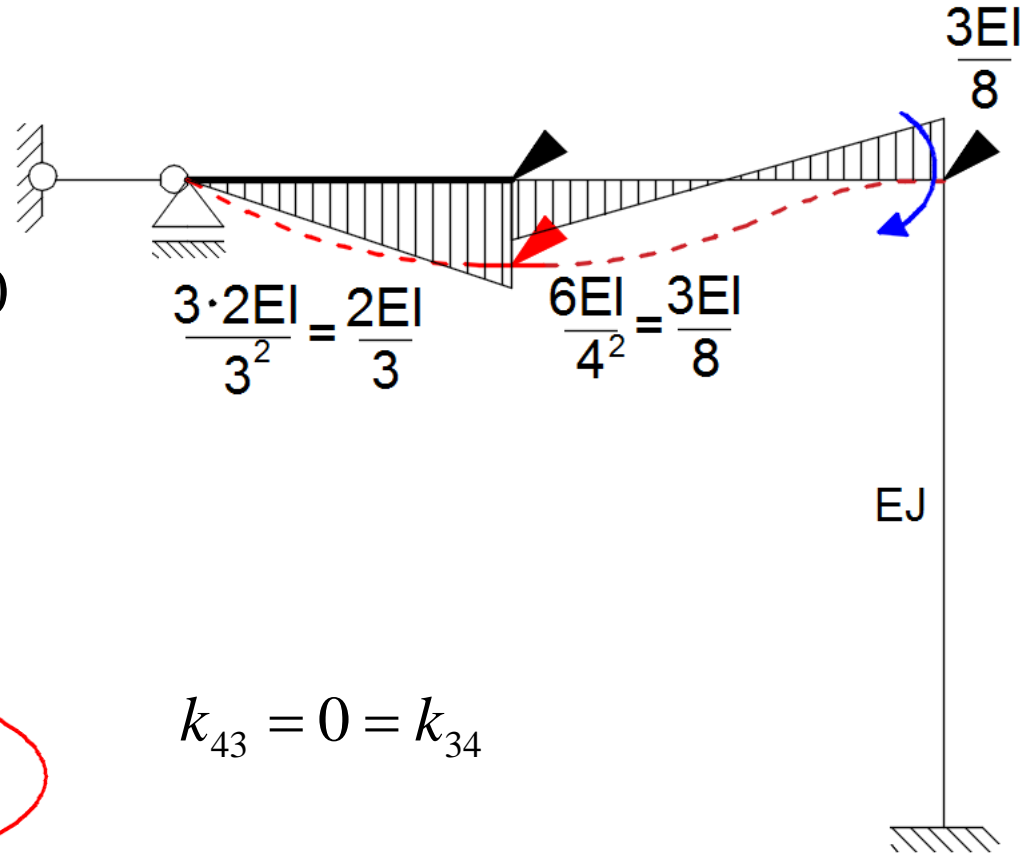
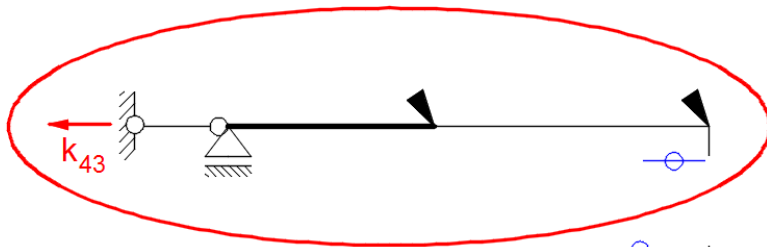
$$k_{23} = \frac{3EI}{8} = k_{32}$$

Stan $\Delta_3=1$

$$\sum R_y = \frac{2EI}{9} + \frac{3EI}{16} - k_{33} = 0$$

↓

$$k_{33} = \frac{59EI}{144}$$

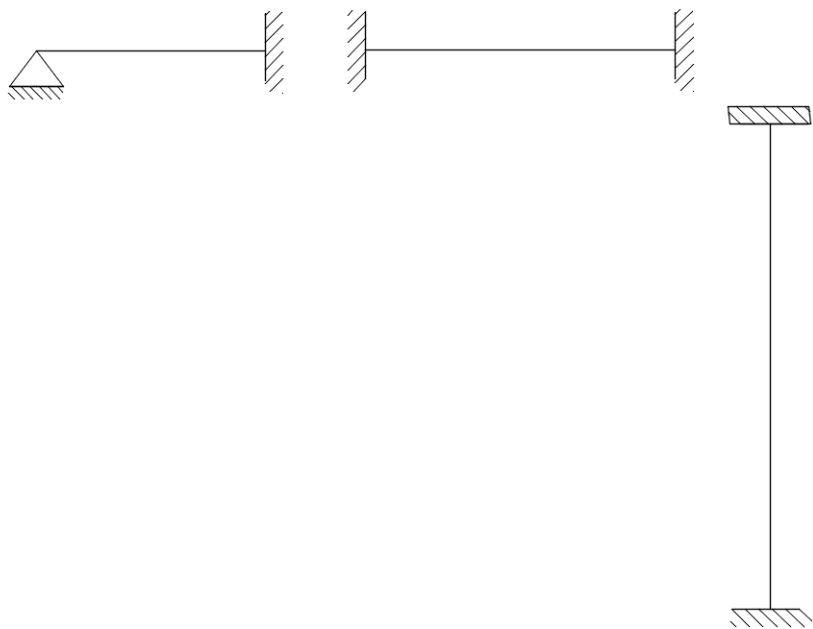
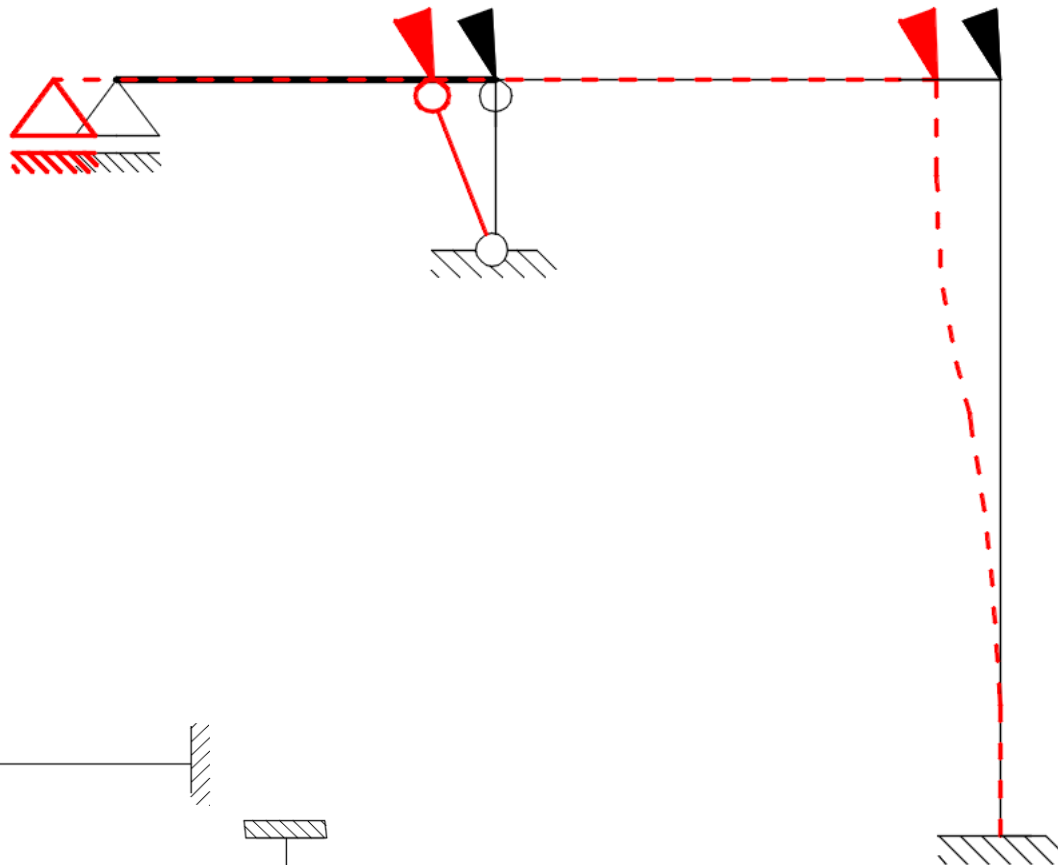


$$k_{43} = 0 = k_{34}$$

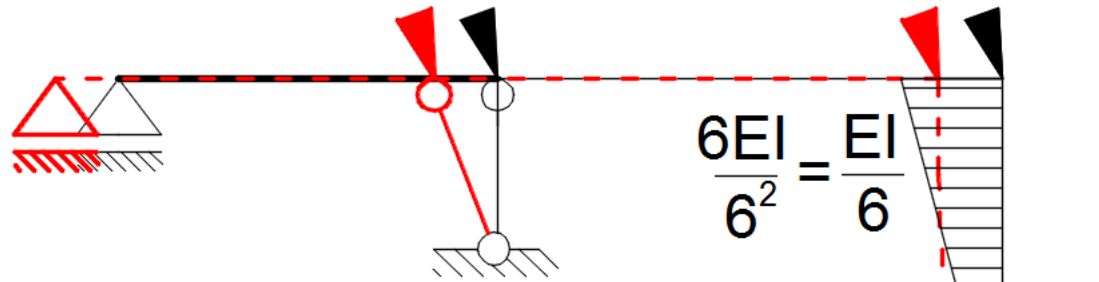
$$k_{13} = \frac{3EI}{8} - \frac{2EI}{3} = -\frac{7EI}{24} = k_{31}$$

$$k_{23} = \frac{3EI}{8} = k_{32}$$

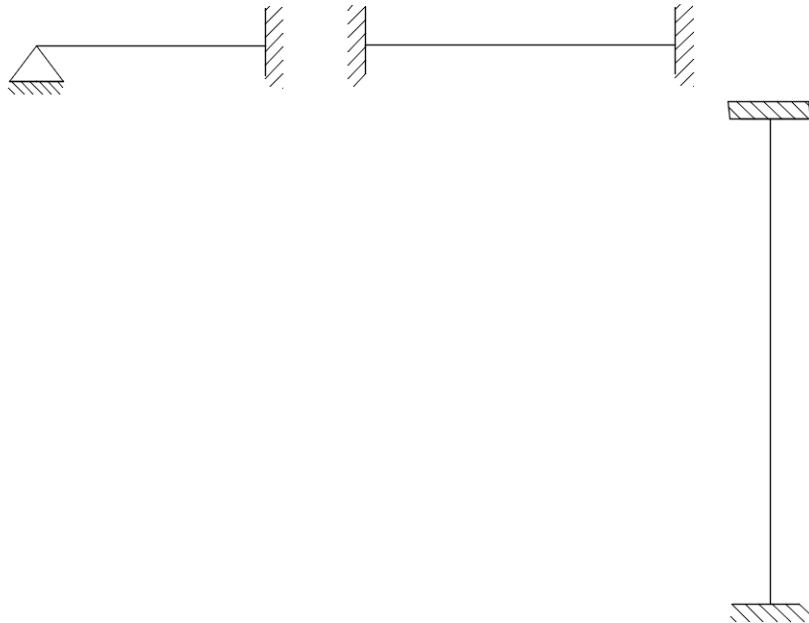
Stan $\Delta_4=1$



Stan $\Delta_4=1$

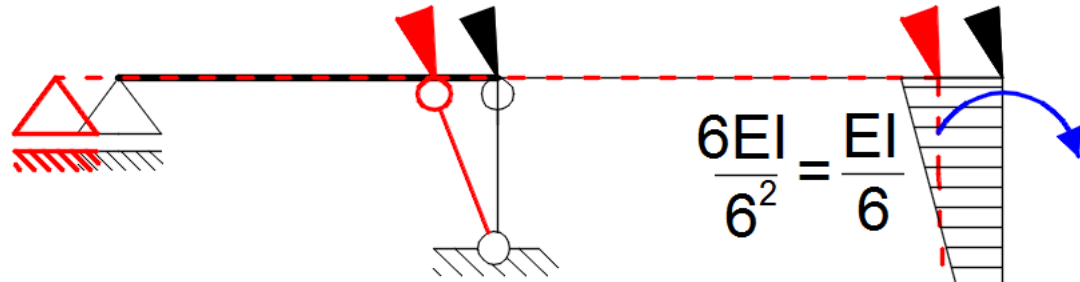


$$\frac{6EI}{6^2} = \frac{EI}{6}$$



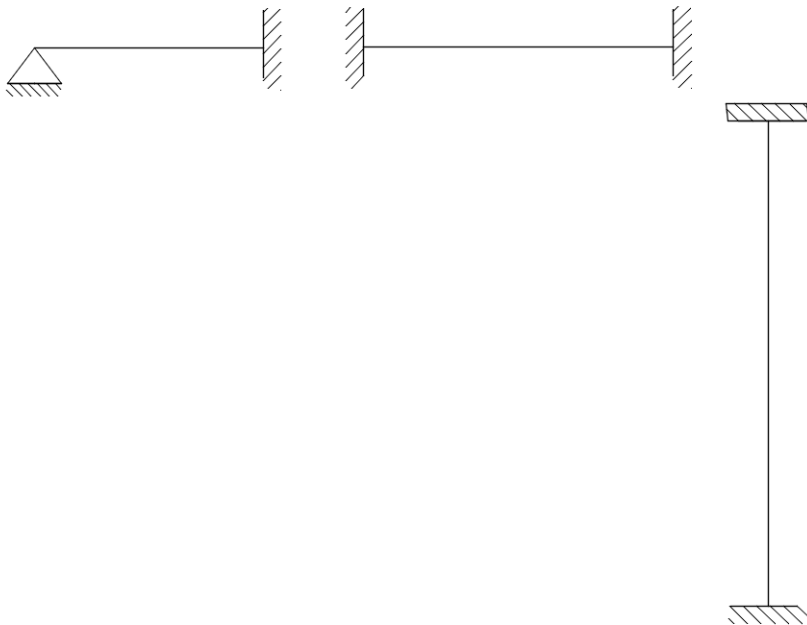
$$\frac{6EI}{6^2} = \frac{EI}{6}$$

Stan $\Delta_4=1$



$$\frac{6EI}{6^2} = \frac{EI}{6}$$

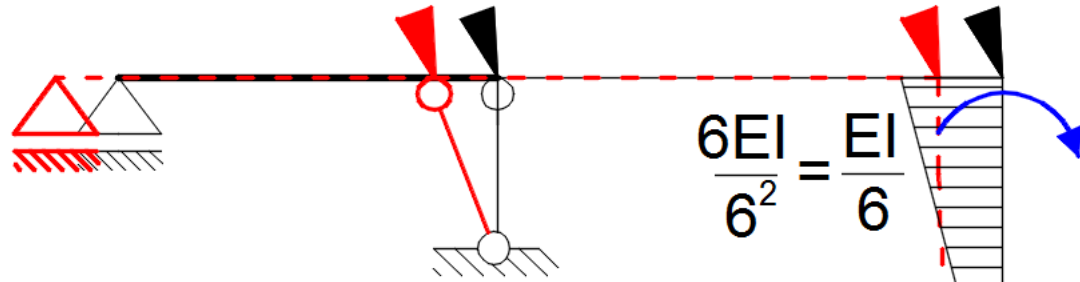
$$\frac{6EI}{6^2} = \frac{EI}{6}$$



$$k_{14} = 0 = k_{41}$$

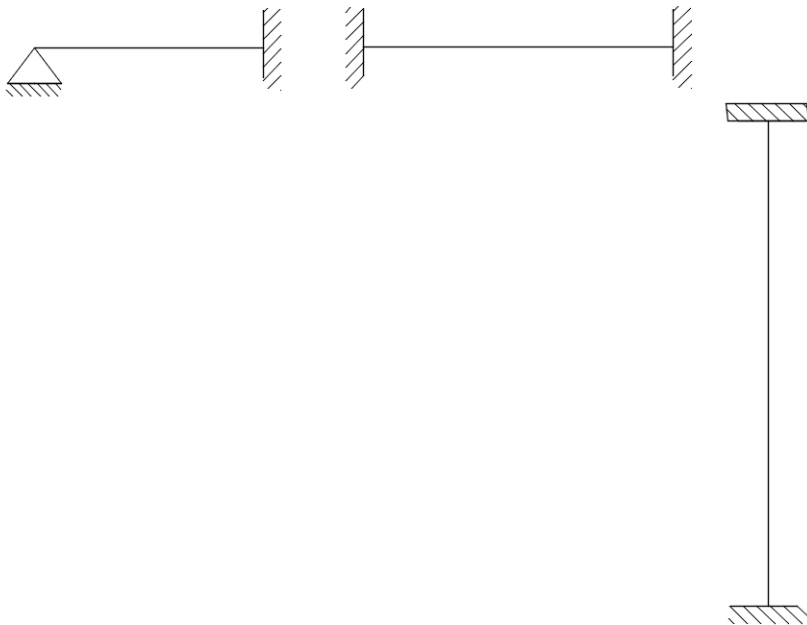
$$k_{24} = \frac{EI}{6} = k_{42}$$

Stan $\Delta_4=1$



$$\frac{6EI}{6^2} = \frac{EI}{6}$$

$$\frac{6EI}{6^2} = \frac{EI}{6}$$



$$k_{14} = 0 = k_{41}$$

$$k_{34} = k_{43} = 0$$

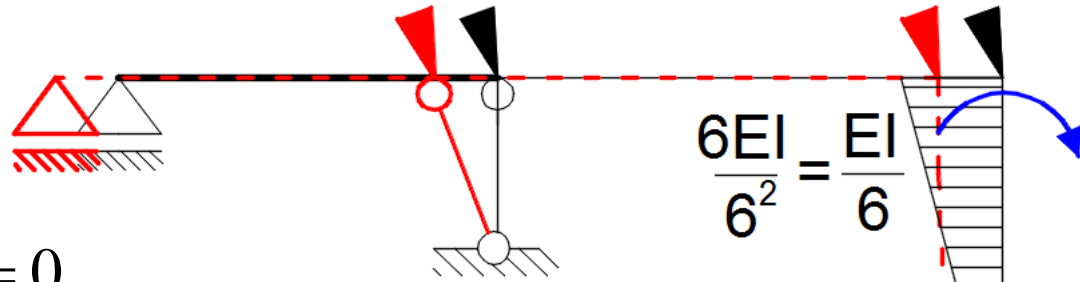
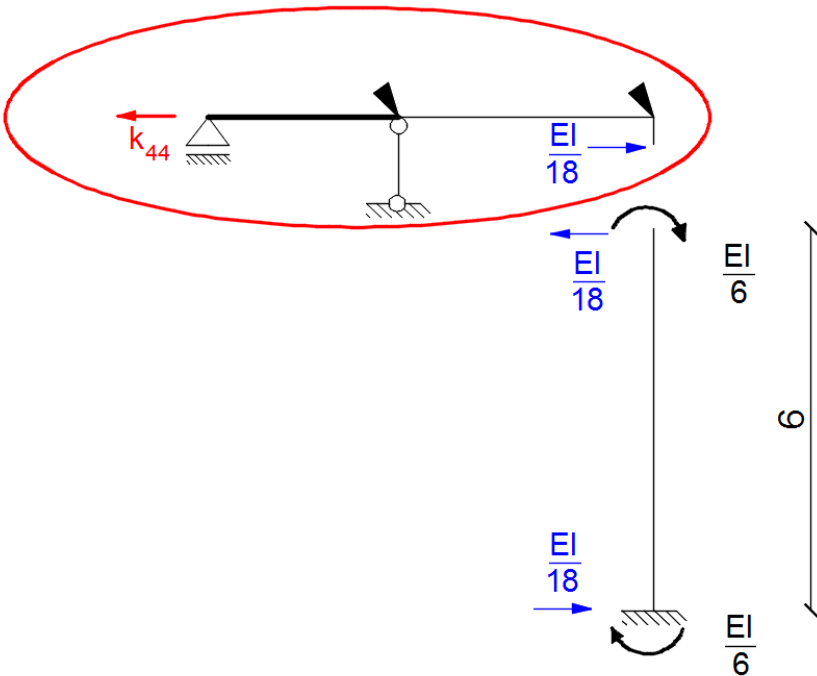
$$k_{24} = \frac{EI}{6} = k_{42}$$

Stan $\Delta_4=1$

$$\sum R_x = -k_{44} + \frac{EI}{18} = 0$$

↓

$$k_{44} = \frac{EI}{18}$$



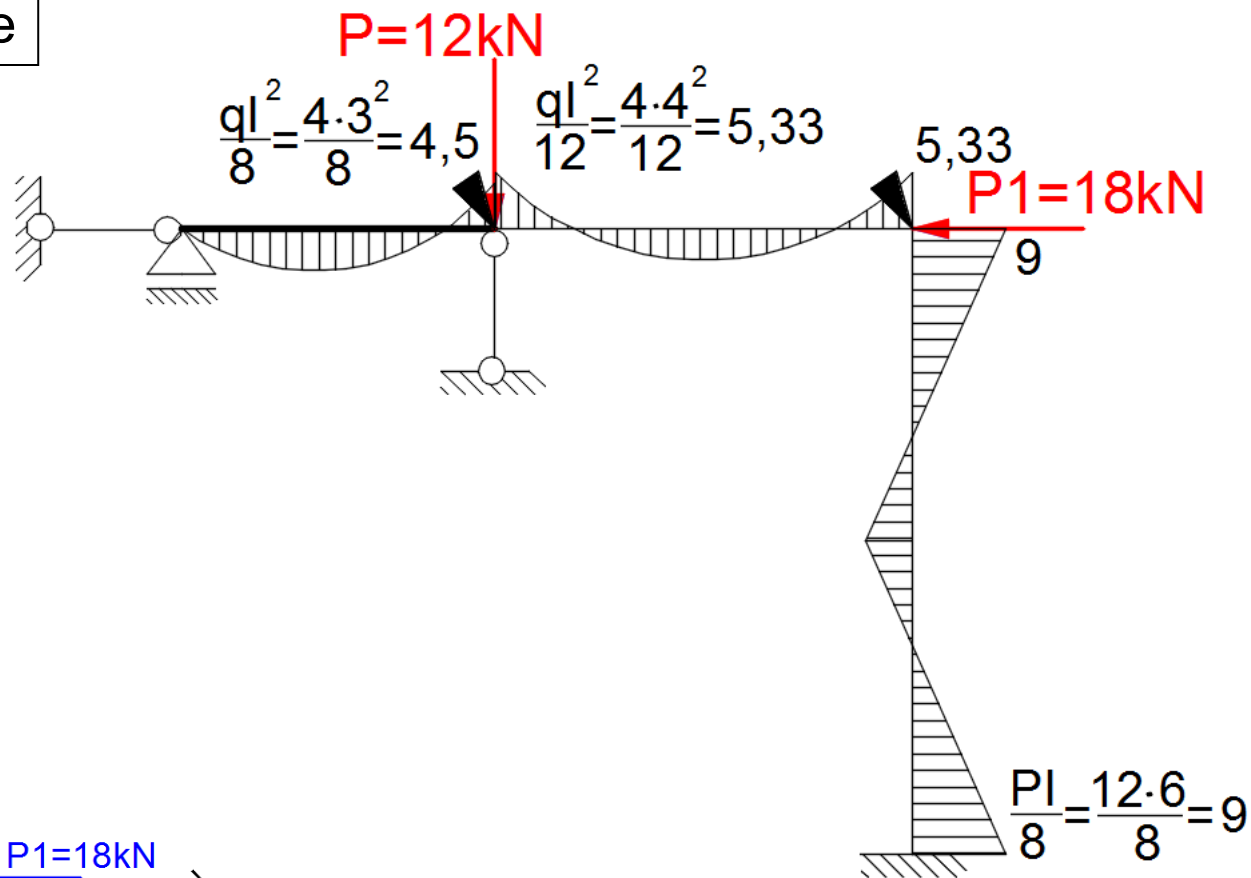
$$\frac{6EI}{6^2} = \frac{EI}{6}$$

$$k_{14} = 0 = k_{41}$$

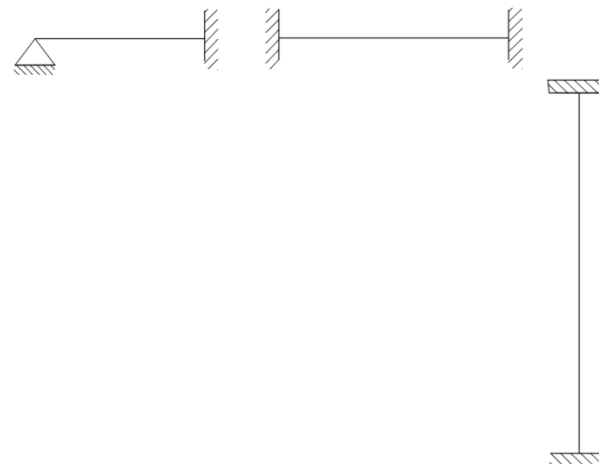
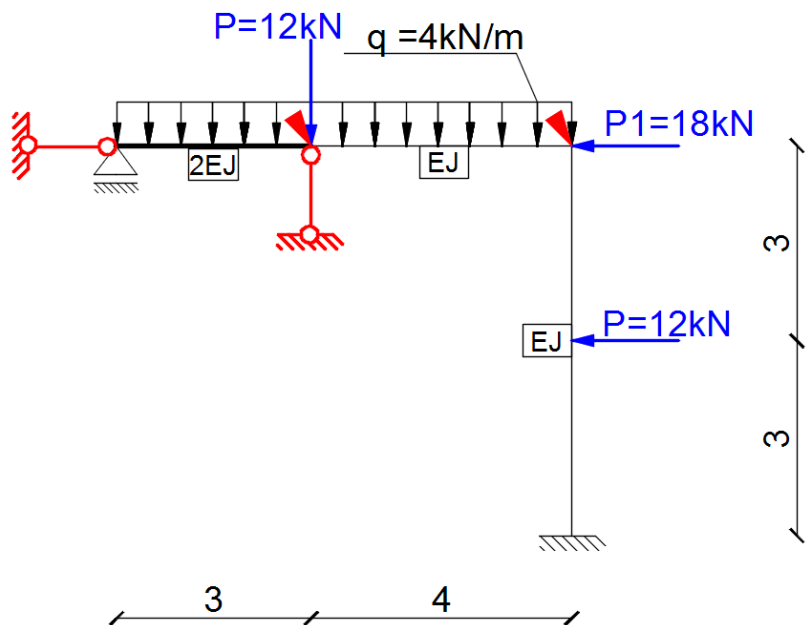
$$k_{34} = k_{43} = 0$$

$$k_{24} = \frac{EI}{6} = k_{42}$$

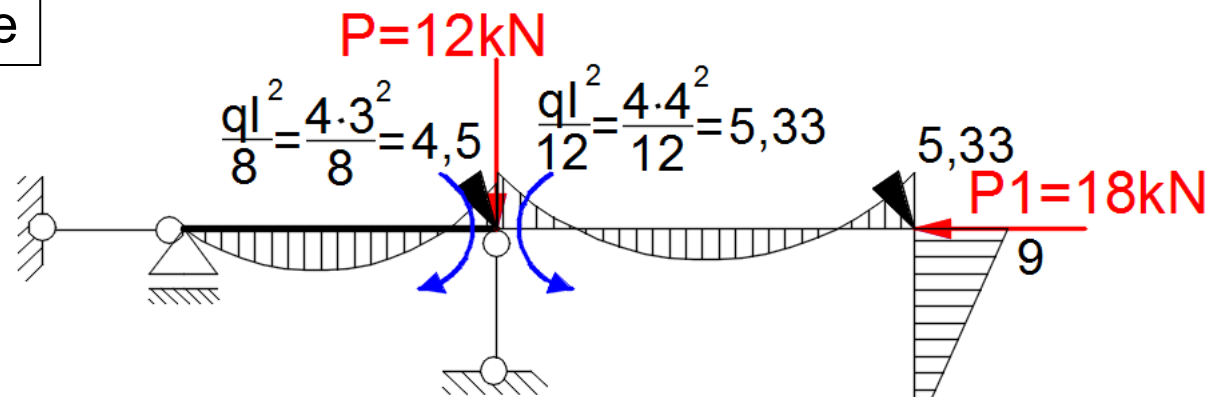
Obciążenie zewnętrzne



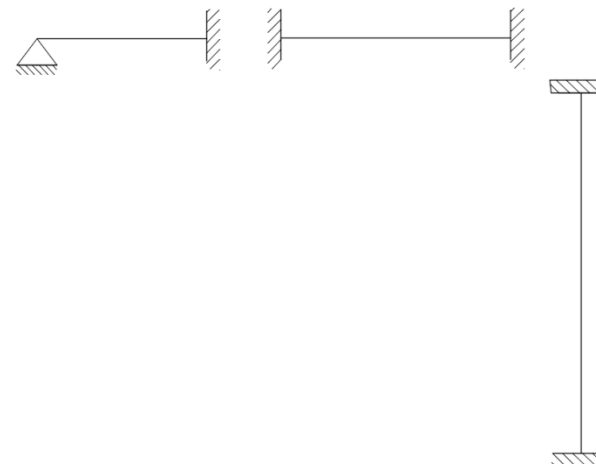
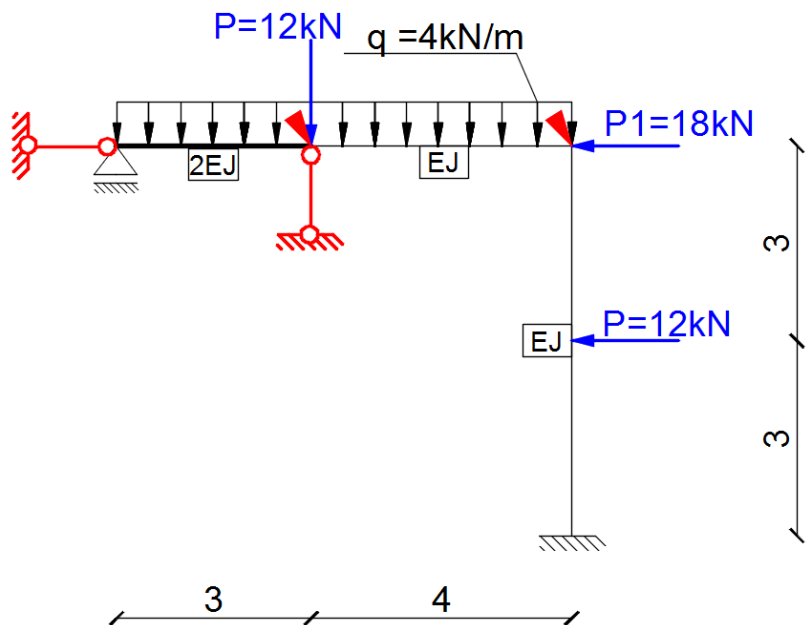
$k_{10} =$



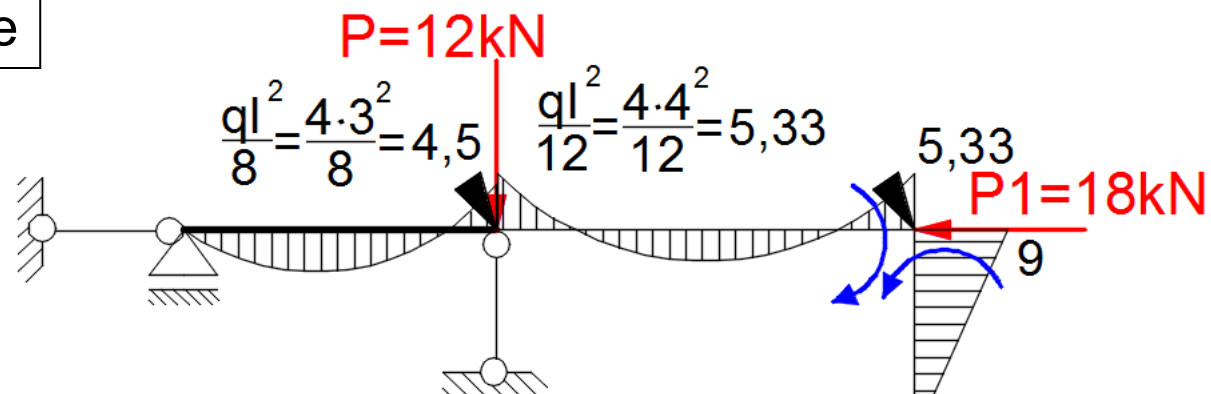
Obciążenie zewnętrzne



$$k_{10} = 4,5 - 5,33 = -0,83 \text{ kNm}$$

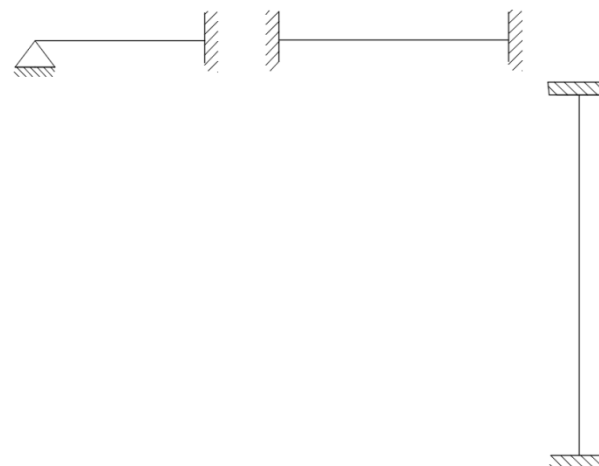
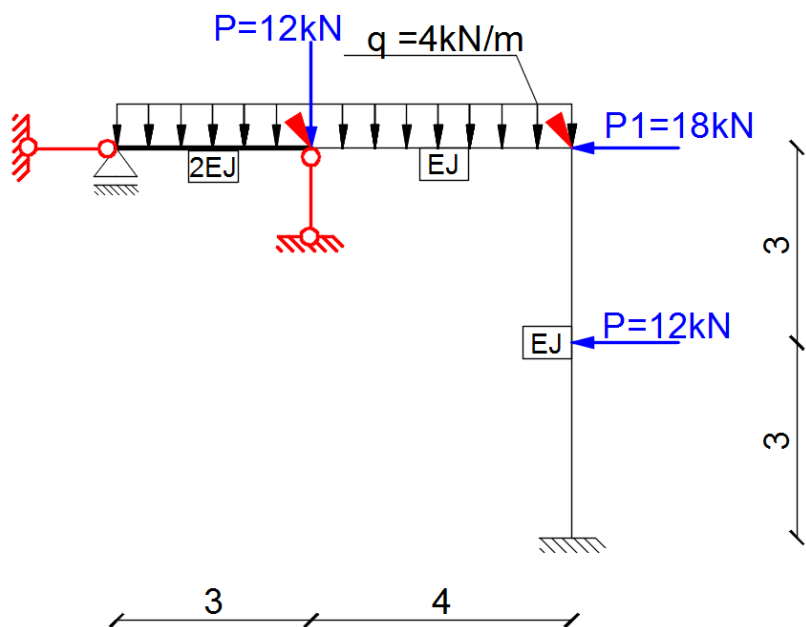


Obciążenie zewnętrzne



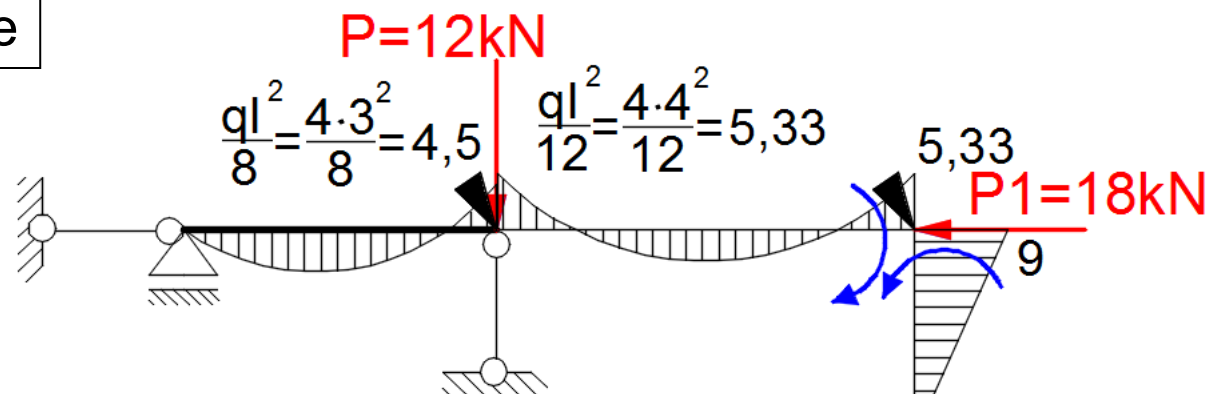
$$k_{10} = 4,5 - 5,33 = -0,83\text{kNm}$$

$$k_{20} = 5,33 - 9 = -3,67\text{kNm}$$



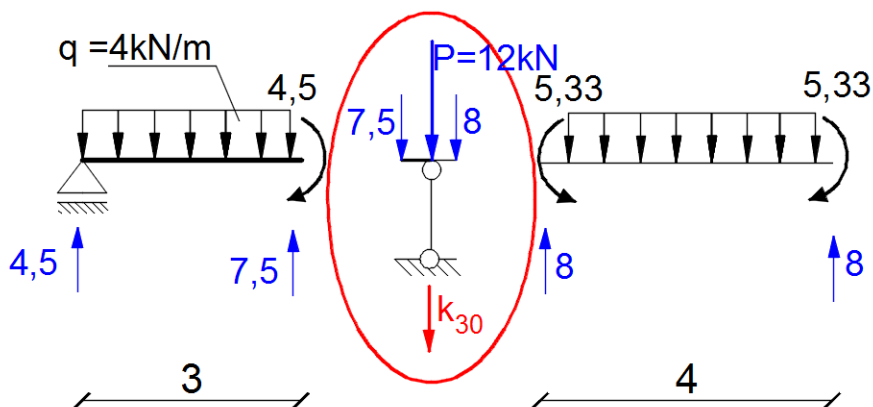
$$\frac{Pl}{8} = \frac{12 \cdot 6}{8} = 9$$

Obciążenie zewnętrzne



$$k_{10} = 4,5 - 5,33 = -0,83\text{kNm}$$

$$k_{20} = 5,33 - 9 = -3,67\text{kNm}$$



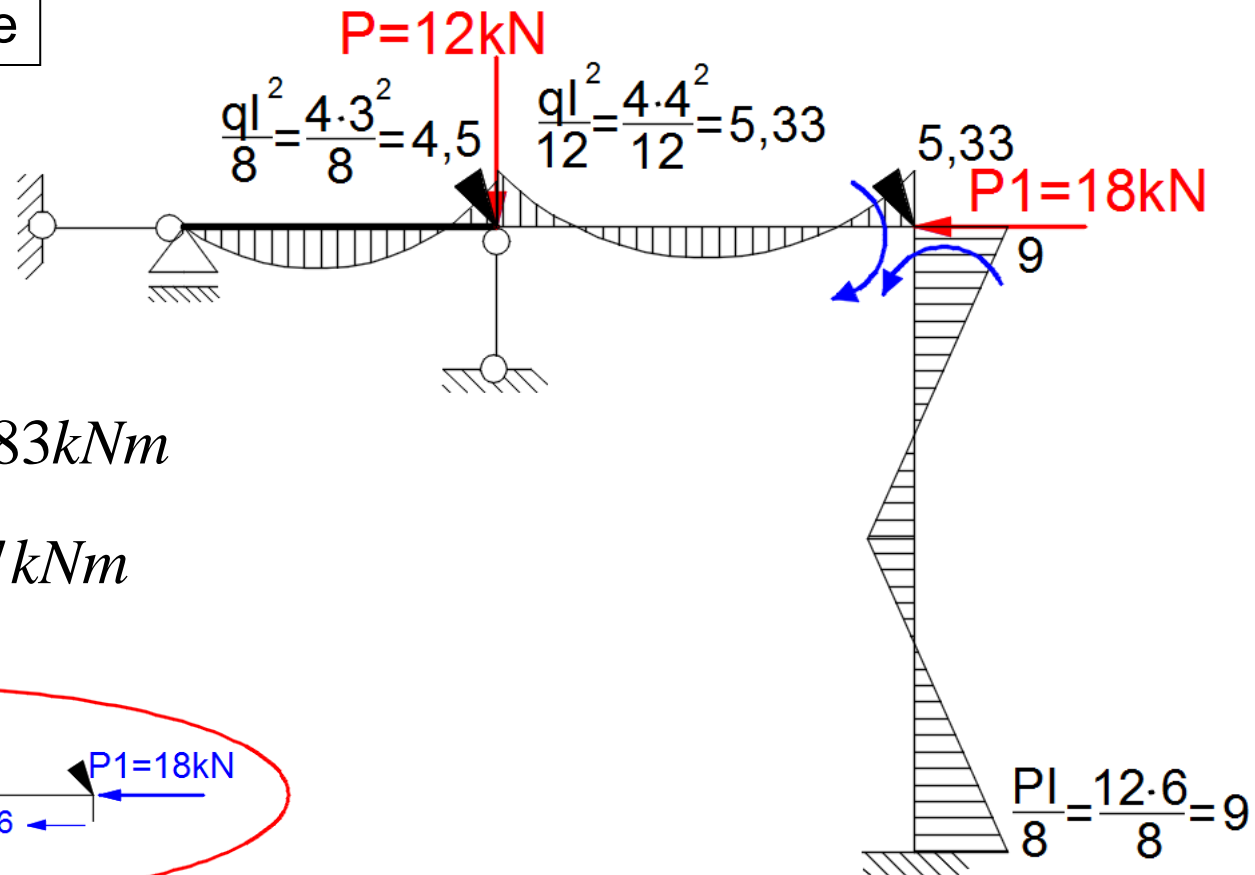
$$\sum R_y = -7,5 - 12 - 8 - k_{30} = 0$$



$$k_{30} = -27,5\text{kN}$$

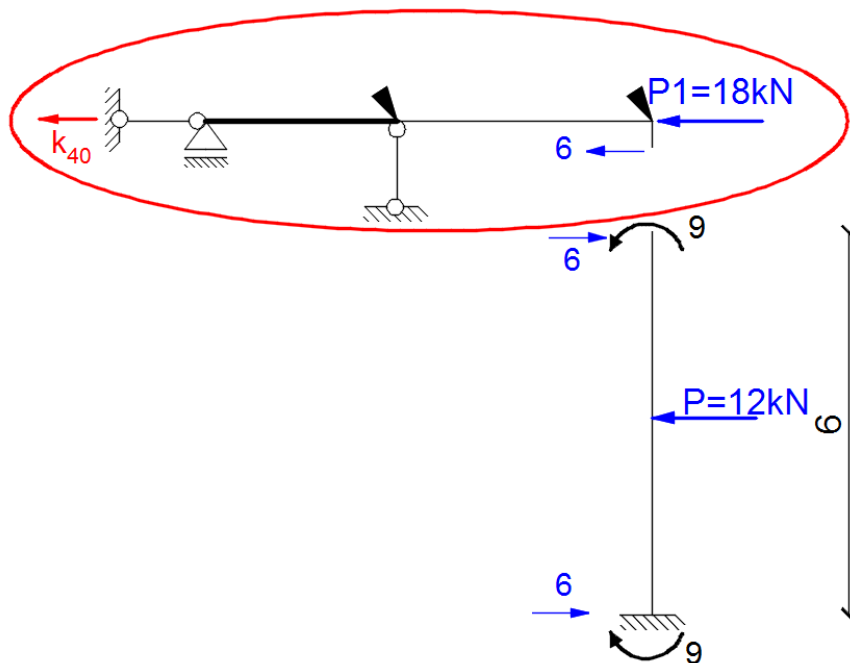
$$\frac{Pl}{8} = \frac{12 \cdot 6}{8} = 9$$

Obciążenie zewnętrzne



$$k_{10} = 4,5 - 5,33 = -0,83 \text{ kNm}$$

$$k_{20} = 5,33 - 9 = -3,67 \text{ kNm}$$



$$\sum R_x = -18 - 6 - k_{40} = 0$$

↓

$$k_{40} = -24 \text{ kN}$$

Układ równań metody przemieszczeń:

$$k_{11} \cdot \varphi_1 + k_{12} \cdot \varphi_2 + k_{13} \cdot \Delta_3 + k_{14} \cdot \Delta_4 + k_{10} = 0$$

$$k_{21} \cdot \varphi_1 + k_{22} \cdot \varphi_2 + k_{23} \cdot \Delta_3 + k_{24} \cdot \Delta_4 + k_{20} = 0$$

$$k_{31} \cdot \varphi_1 + k_{32} \cdot \varphi_2 + k_{33} \cdot \Delta_3 + k_{34} \cdot \Delta_4 + k_{30} = 0$$

$$k_{41} \cdot \varphi_1 + k_{42} \cdot \varphi_2 + k_{43} \cdot \Delta_3 + k_{44} \cdot \Delta_4 + k_{40} = 0$$

Podstawiając wyliczone wcześniej wartości otrzymujemy:

$$3EI \cdot \varphi_1 + \frac{EI}{2} \cdot \varphi_2 - \frac{7EI}{24} \cdot \Delta_3 + 0 \cdot \Delta_4 - 0,83 = 0$$

$$\frac{EI}{2} \cdot \varphi_1 + \frac{5EI}{3} \cdot \varphi_2 + \frac{3EI}{8} \cdot \Delta_3 + \frac{EI}{6} \cdot \Delta_4 - 3,67 = 0$$

$$-\frac{7EI}{24} \cdot \varphi_1 + \frac{3EI}{8} \cdot \varphi_2 + \frac{59EI}{144} \cdot \Delta_3 + 0 \cdot \Delta_4 - 27,5 = 0$$

$$0 \cdot \varphi_1 + \frac{EI}{6} \cdot \varphi_2 + 0 \cdot \Delta_3 + \frac{EI}{18} \cdot \Delta_4 - 24 = 0$$