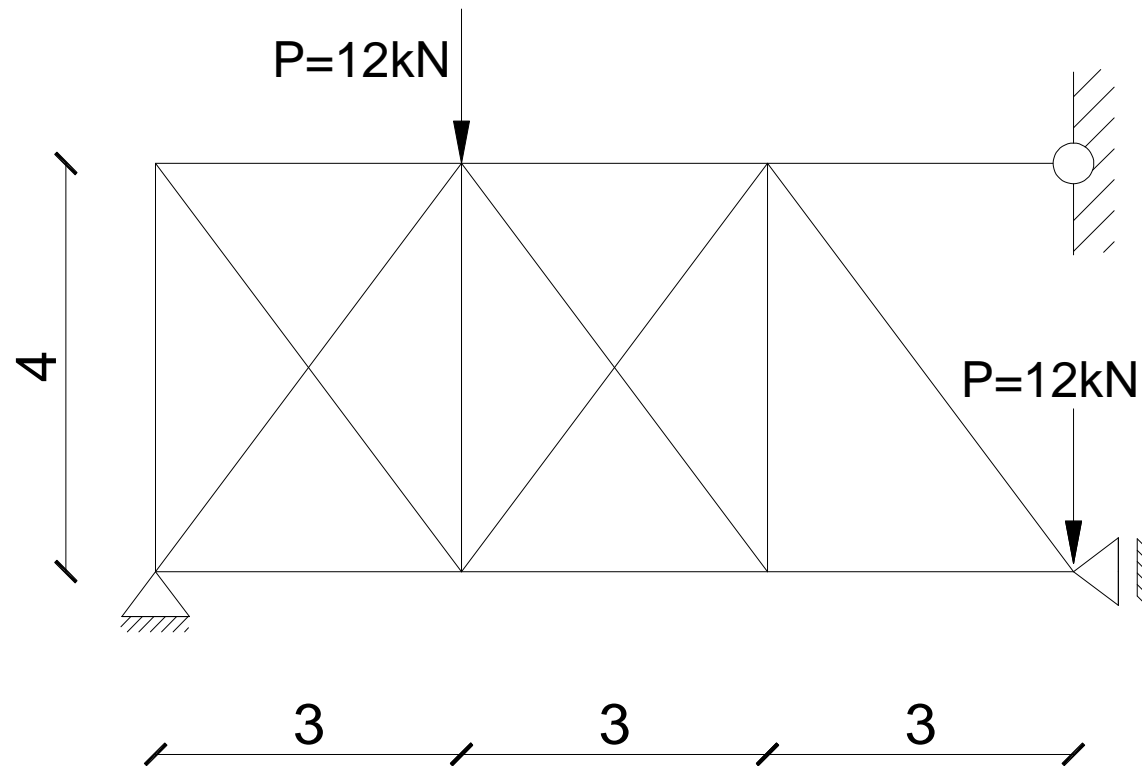
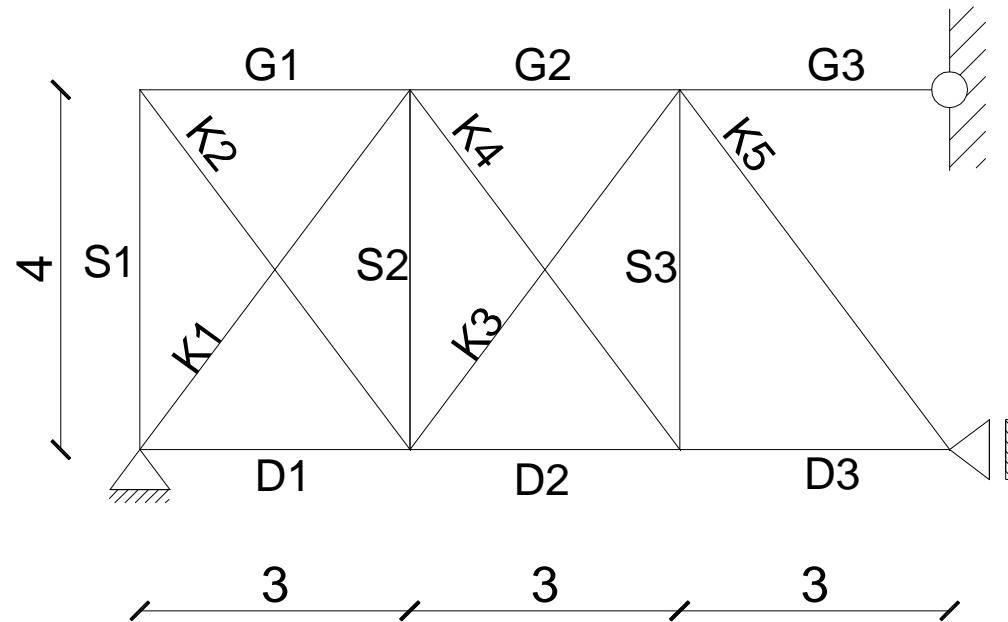


Zadanie: Wyznaczyć siły w prętach kratownicy. Zadanie rozwiązać metodą sił.



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Stopień statycznej niewyznaczalności kratownicy

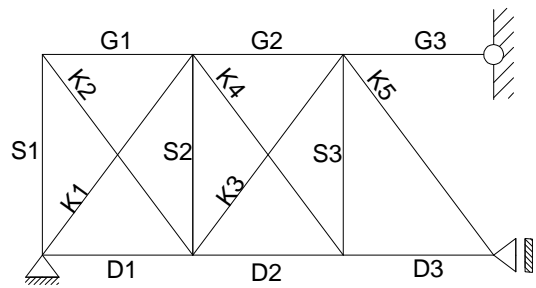


$$n_s = l_r + l_p - 2w = 5 + 14 - 2 \cdot 8 = 3$$

Układ trzykrotnie statycznie niewyznaczalny

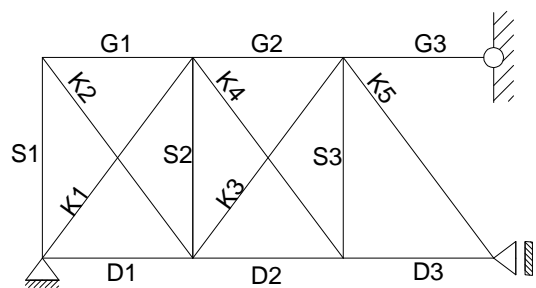
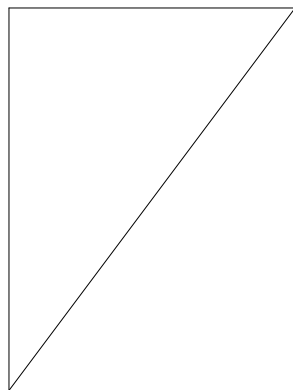
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Dobór schematu podstawowego statycznie wyznaczalnego



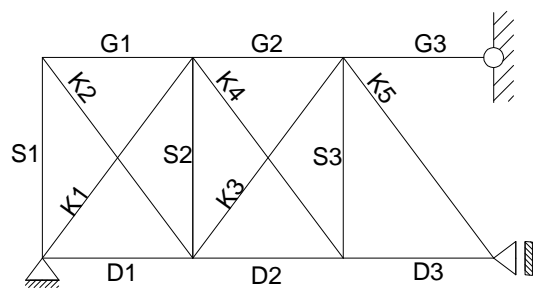
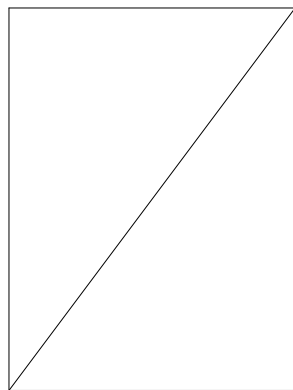
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Dobór schematu podstawowego statycznie wyznaczalnego



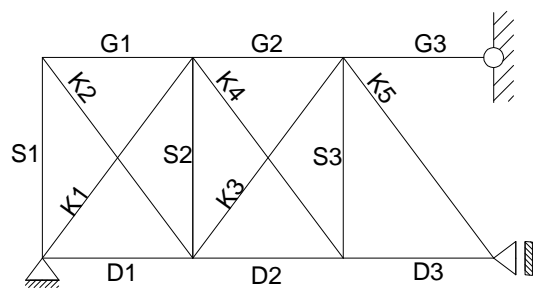
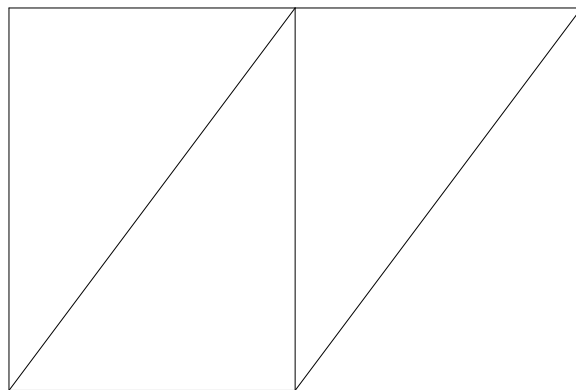
dr inż. Hanna Weber

Dobór schematu podstawowego statycznie wyznaczalnego



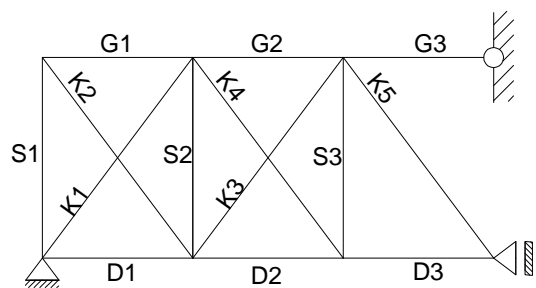
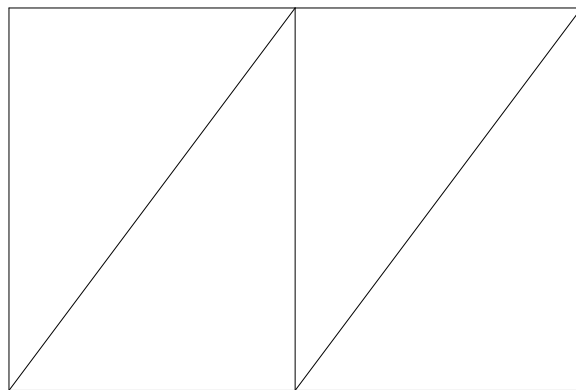
dr inż. Hanna Weber

Dobór schematu podstawowego statycznie wyznaczalnego



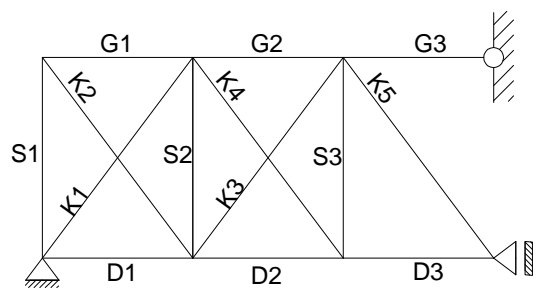
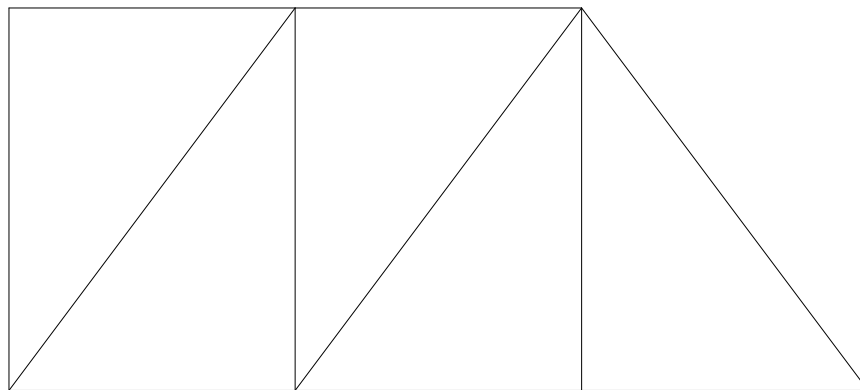
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Dobór schematu podstawowego statycznie wyznaczalnego



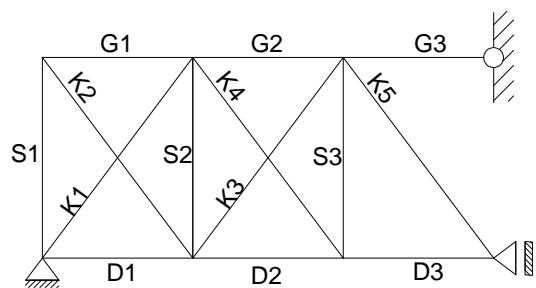
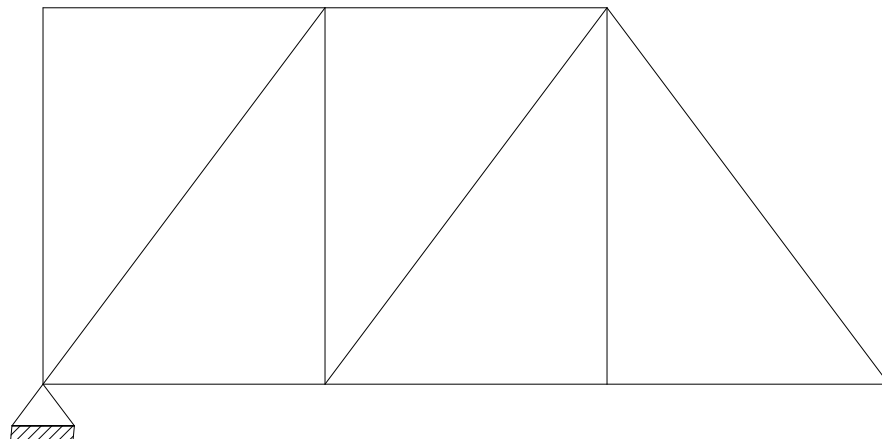
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Dobór schematu podstawowego statycznie wyznaczalnego



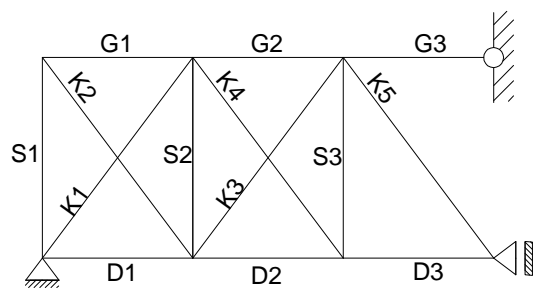
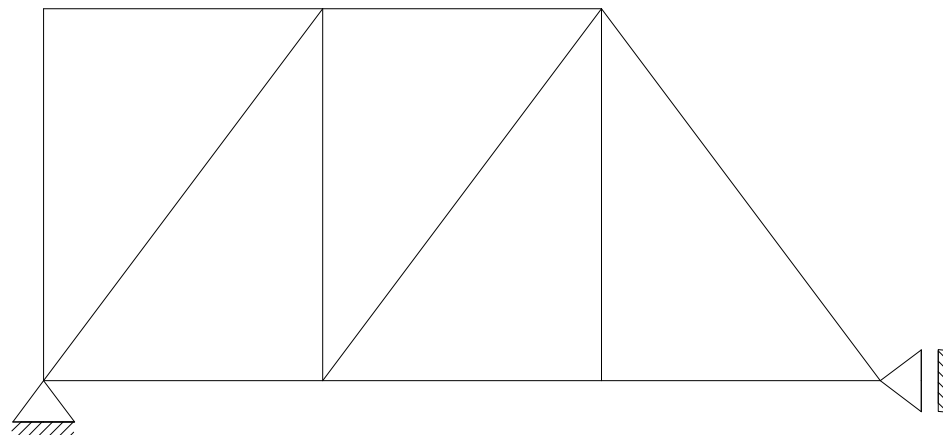
dr inż. Hanna Weber

Dobór schematu podstawowego statycznie wyznaczalnego



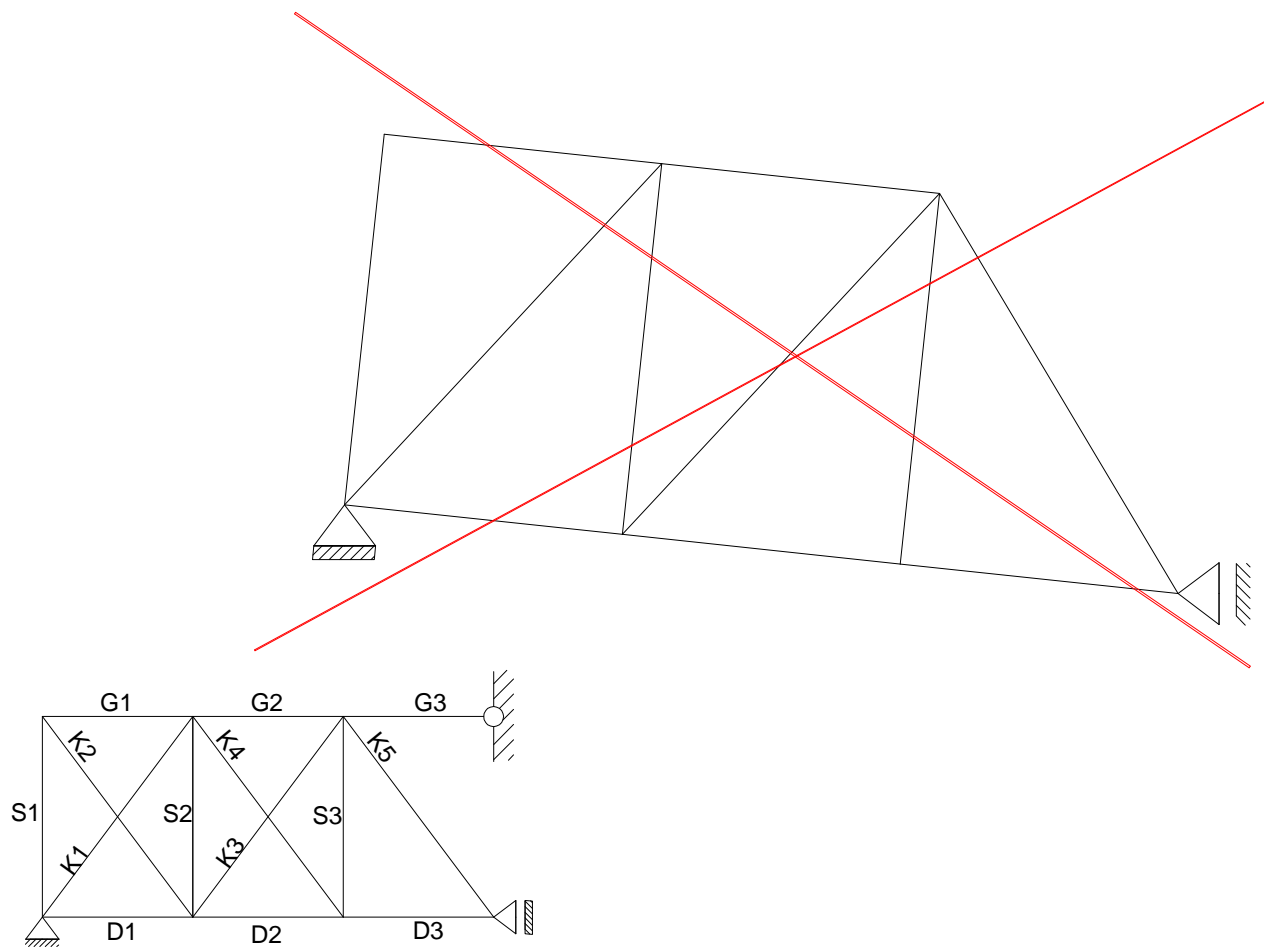
dr inż. Hanna Weber

Dobór schematu podstawowego statycznie wyznaczalnego



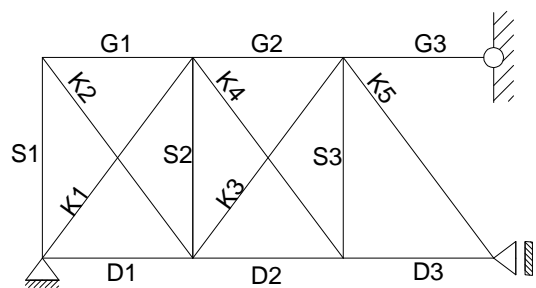
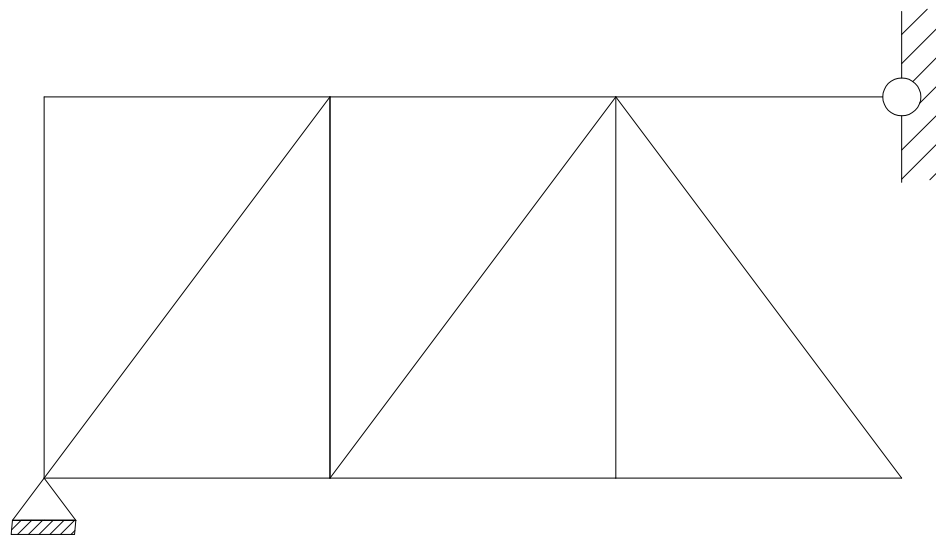
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Dobór schematu podstawowego statycznie wyznaczalnego



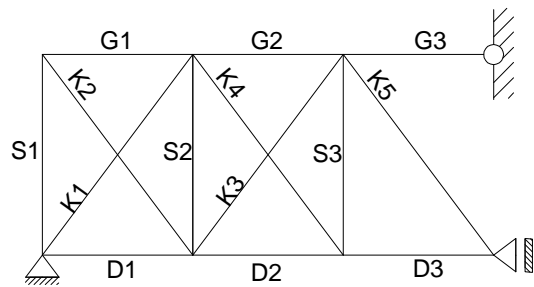
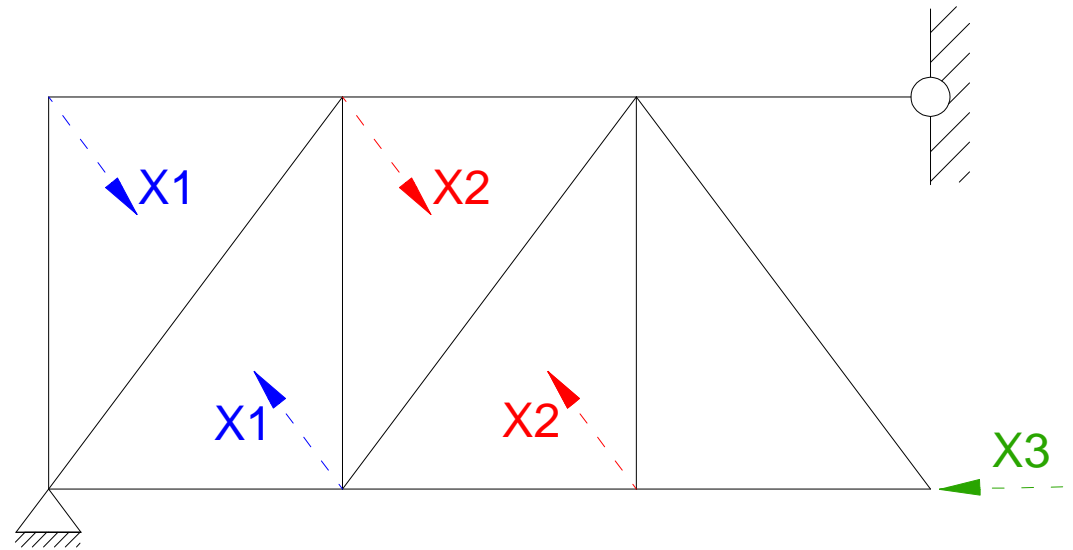
dr inż. Hanna Weber

Dobór schematu podstawowego statycznie wyznaczalnego



dr inż. Hanna Weber

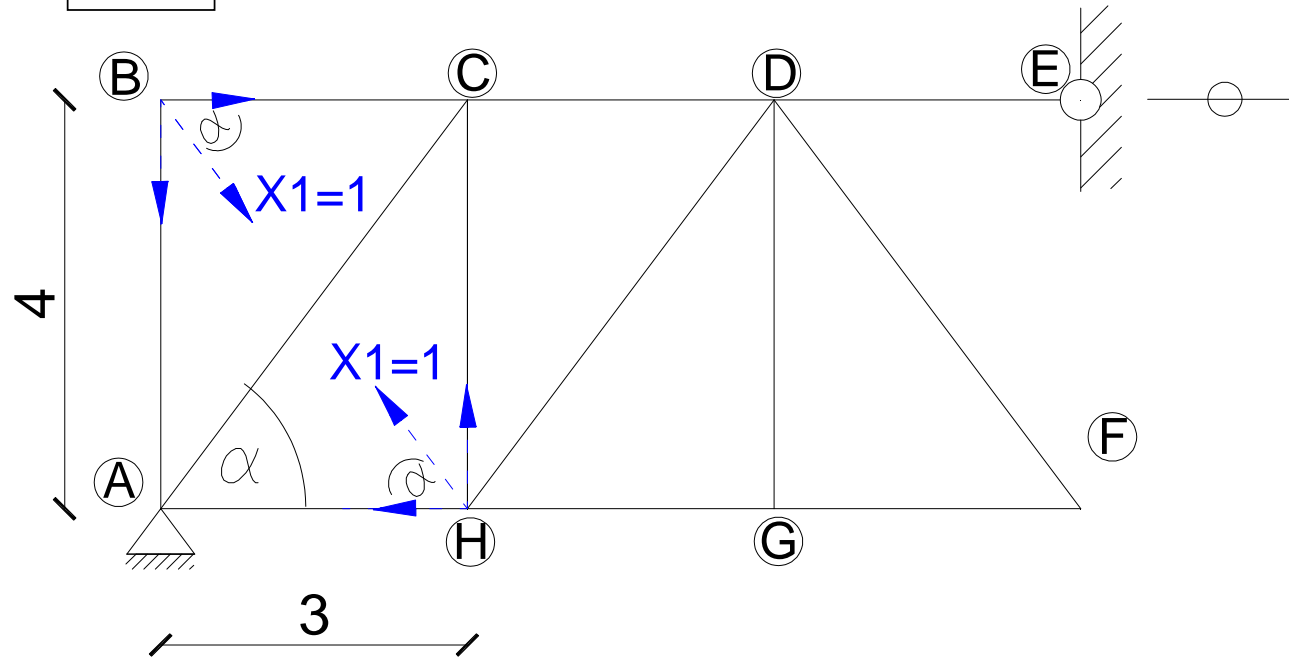
Przyjęty schemat podstawowy statycznie wyznaczalny



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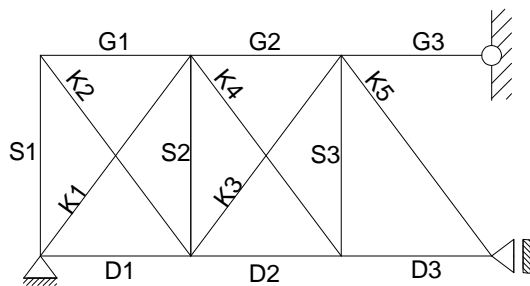
Wykresy jednostkowe

$X_1=1$



$$\sin \alpha = 4/5 = 0,8$$

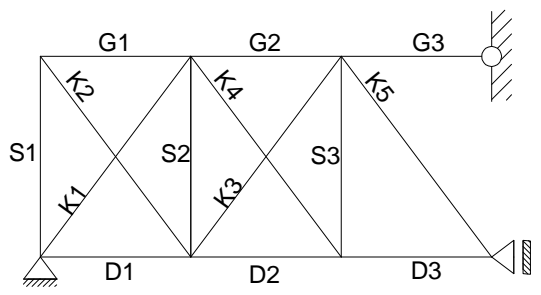
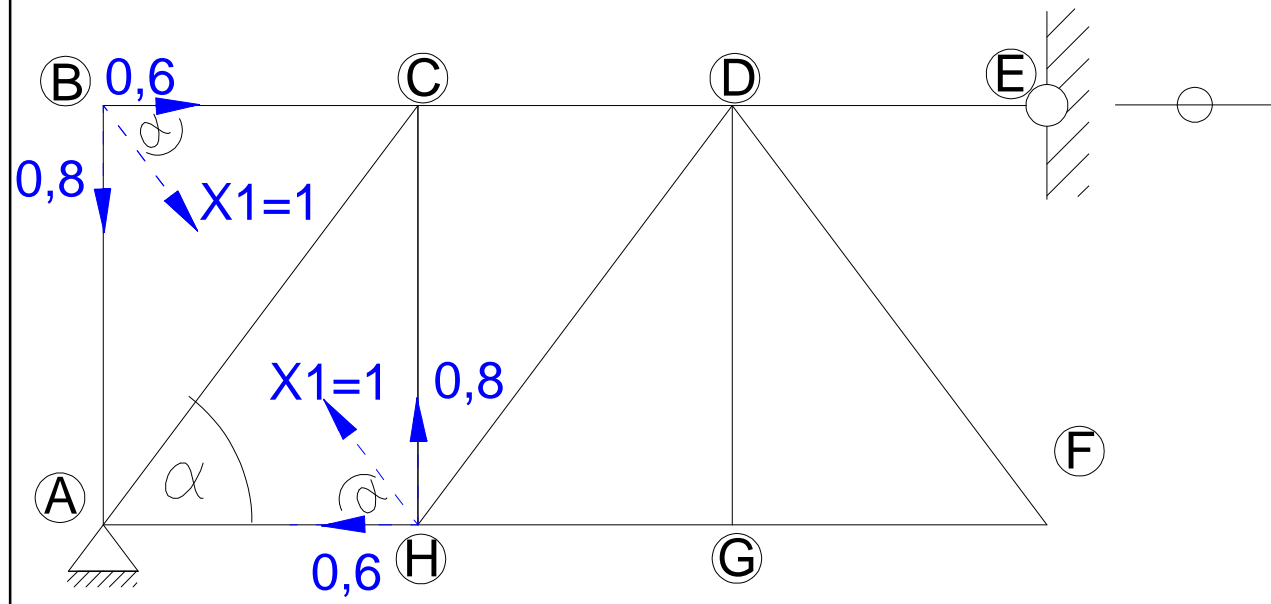
$$\cos \alpha = 3/5 = 0,6$$



dr inż. Hanna Weber

Wykresy jednostkowe

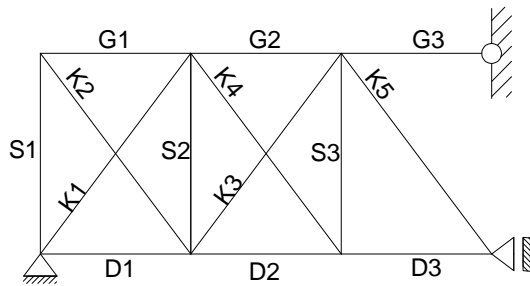
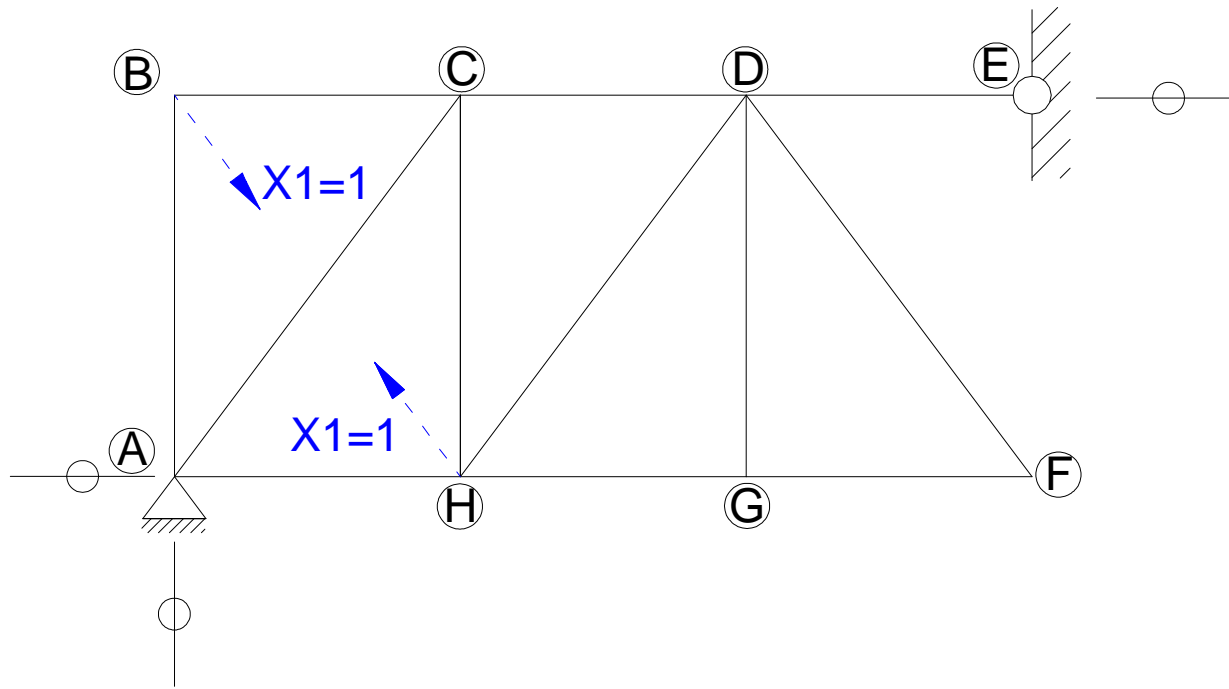
$X_1=1$



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Wykresy jednostkowe

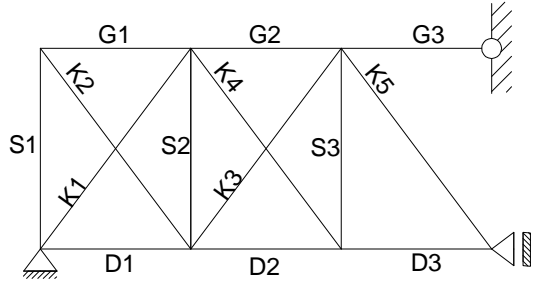
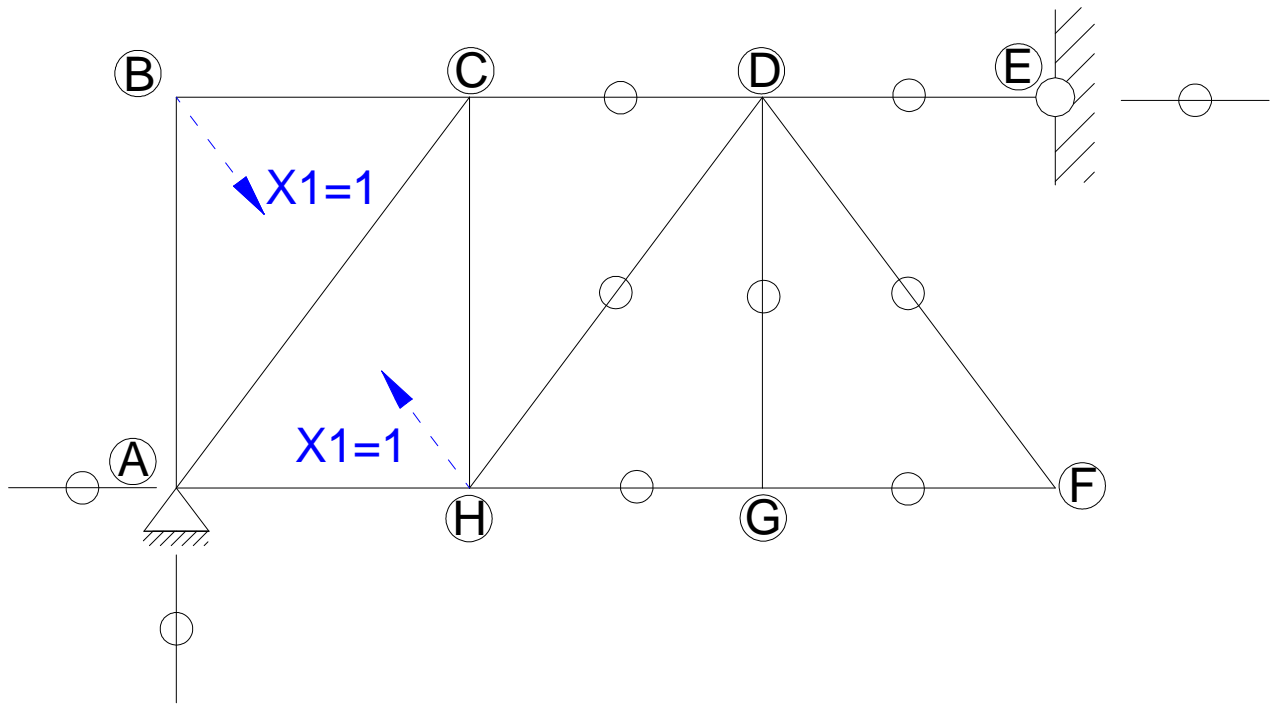
$X_1=1$



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Wykresy jednostkowe

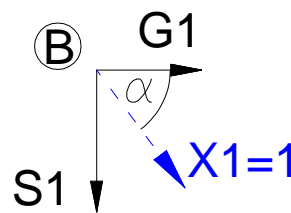
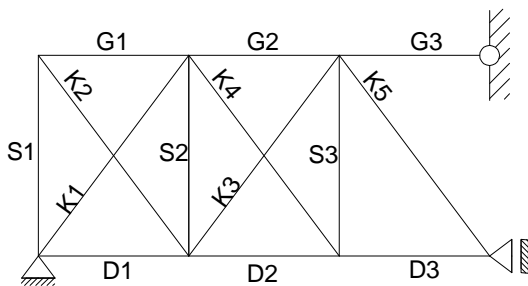
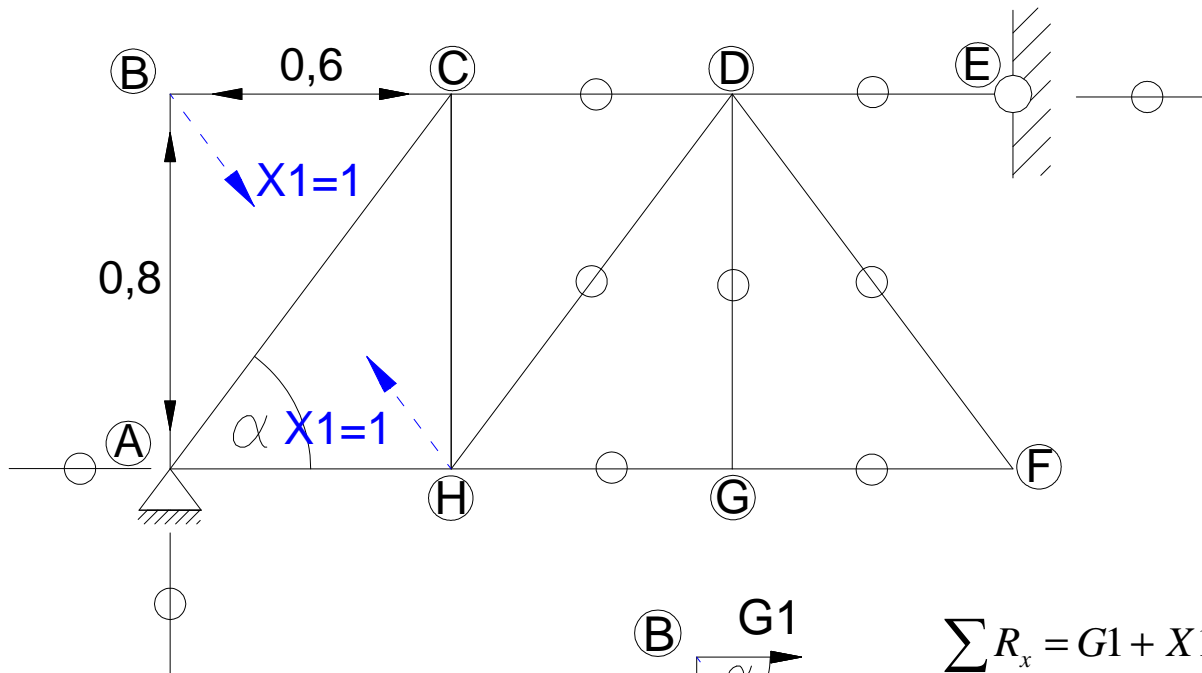
$X_1=1$



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Wykresy jednostkowe

$X1=1$



$$\sum R_x = G1 + X1 \cdot \cos \alpha = 0$$

$$G1 = -X1 \cdot \cos \alpha = -1 \cdot 0,6 = -0,6$$

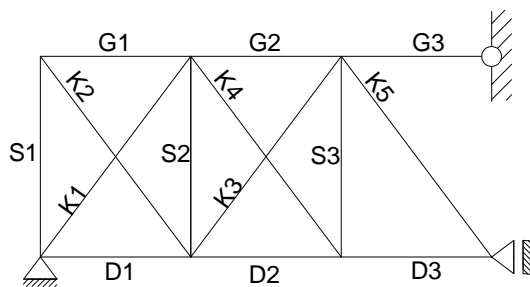
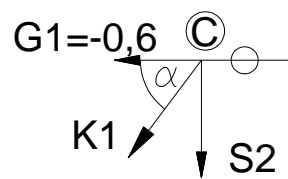
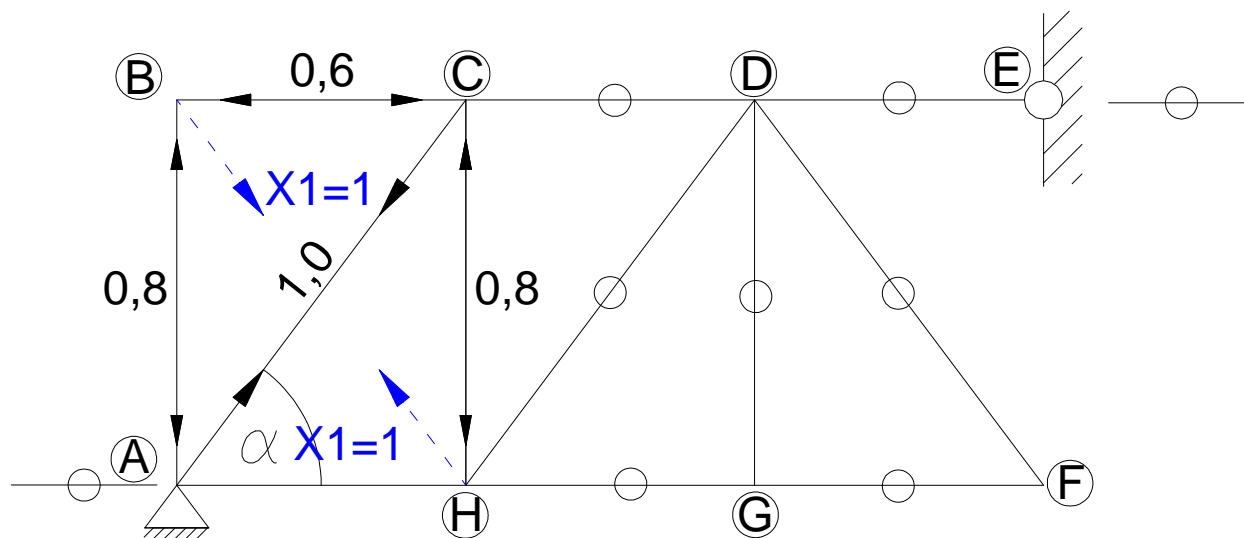
$$\sum R_y = -S1 - X1 \cdot \sin \alpha = 0$$

$$S1 = -X1 \cdot \sin \alpha = -1 \cdot 0,8 = -0,8$$

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Wykresy jednostkowe

X1=1



$$\sum R_x = -G_1 - K_1 \cdot \cos \alpha = 0$$

$$K_1 = -G_1 / \cos \alpha = -(-0.6) / 0.6 = 1$$

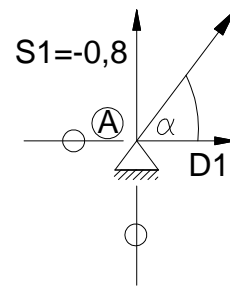
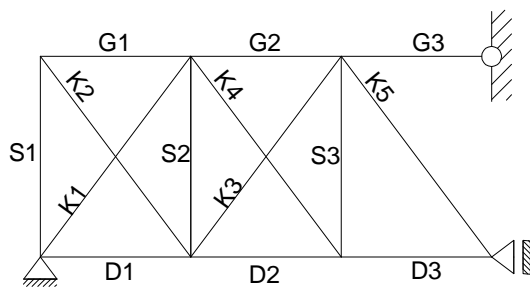
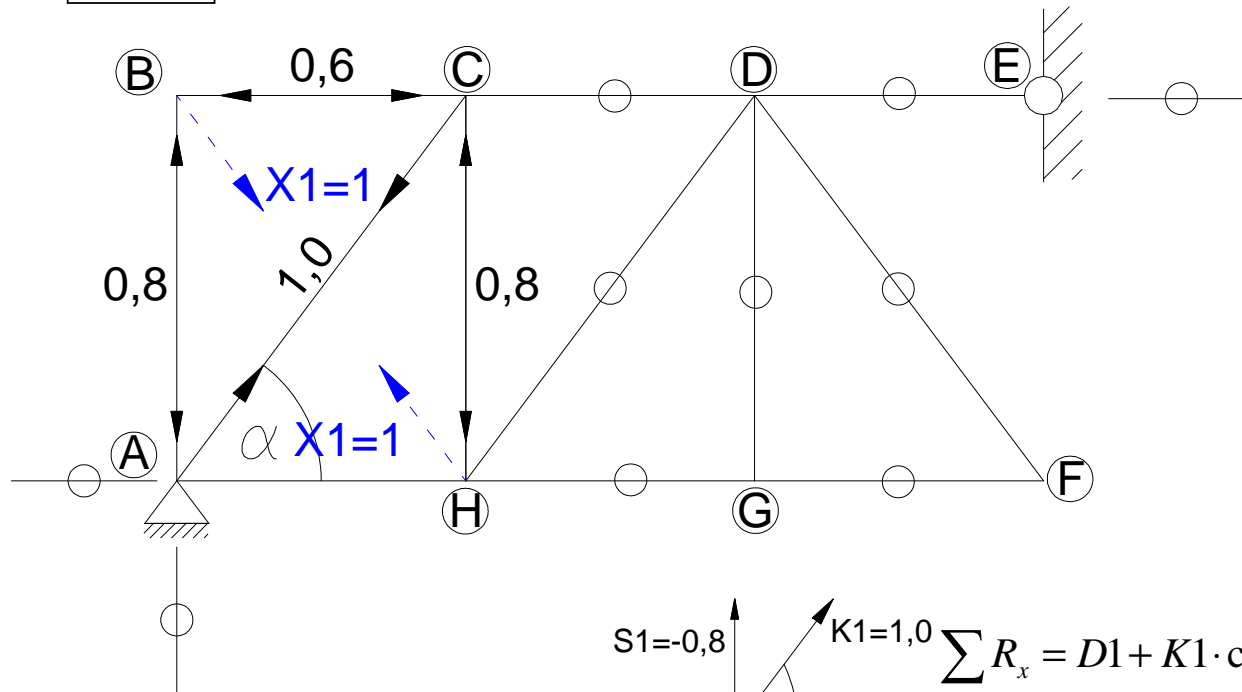
$$\sum R_y = -S_2 - K_1 \cdot \sin \alpha = 0$$

$$S_2 = -K_1 \cdot \sin \alpha = -1 \cdot 0.8 = -0.8$$

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Wykresy jednostkowe

$X_1=1$



$$\sum R_x = D1 + K1 \cdot \cos \alpha = 0$$

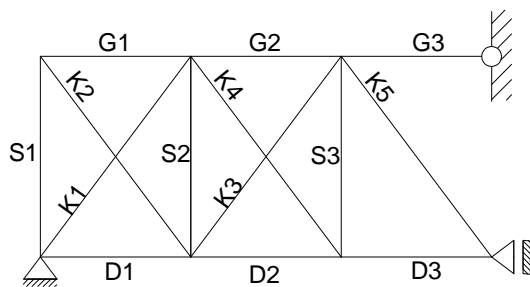
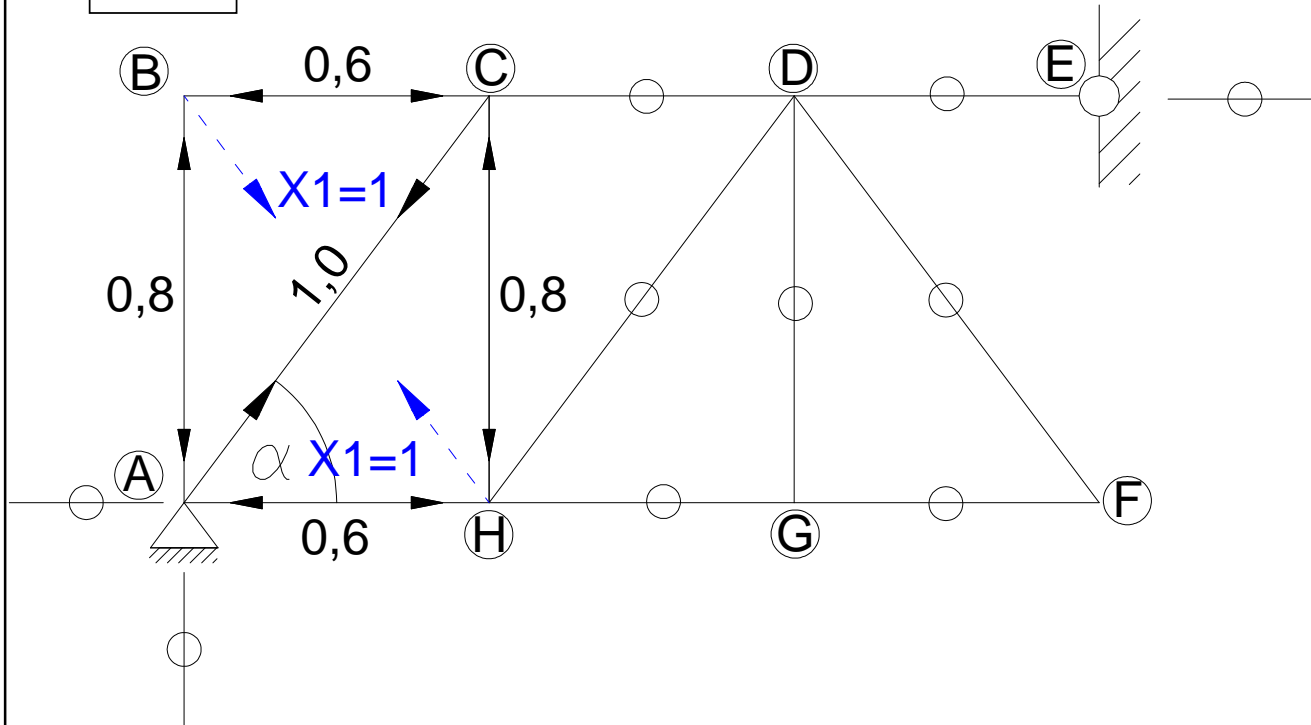
$$D1 = -K1 \cos \alpha = -0,6$$

$$\sum R_y = S1 + K1 \cdot \sin \alpha = -0,8 + 0,8 = 0$$

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Wykresy jednostkowe

$X_1=1$

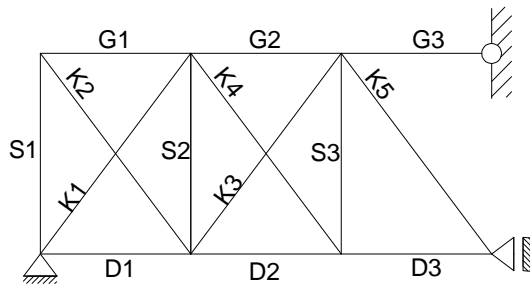
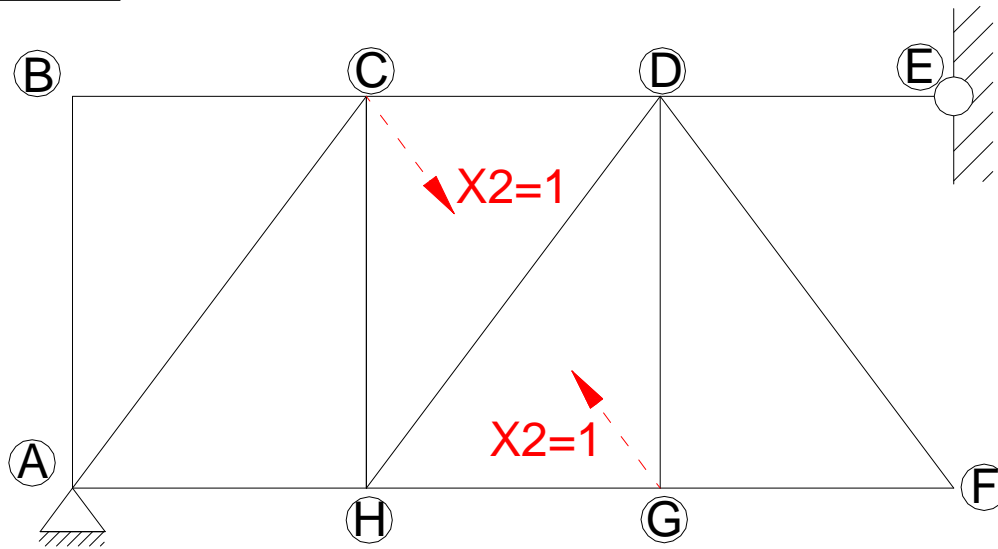


Pręt	N1
D1	-0,6
D2	0,0
D3	0,0
G1	-0,6
G2	0,0
G3	0,0
S1	-0,8
S2	-0,8
S3	0,0
K1	1,0
K2	1,0
K3	0,0
K4	0,0
K5	0,0

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Wykresy jednostkowe

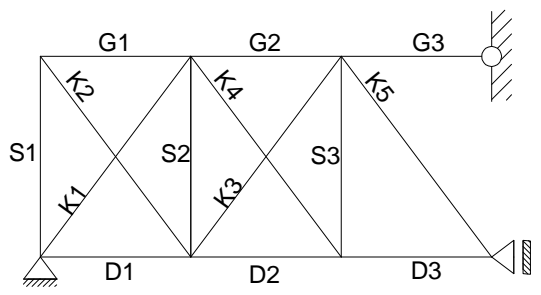
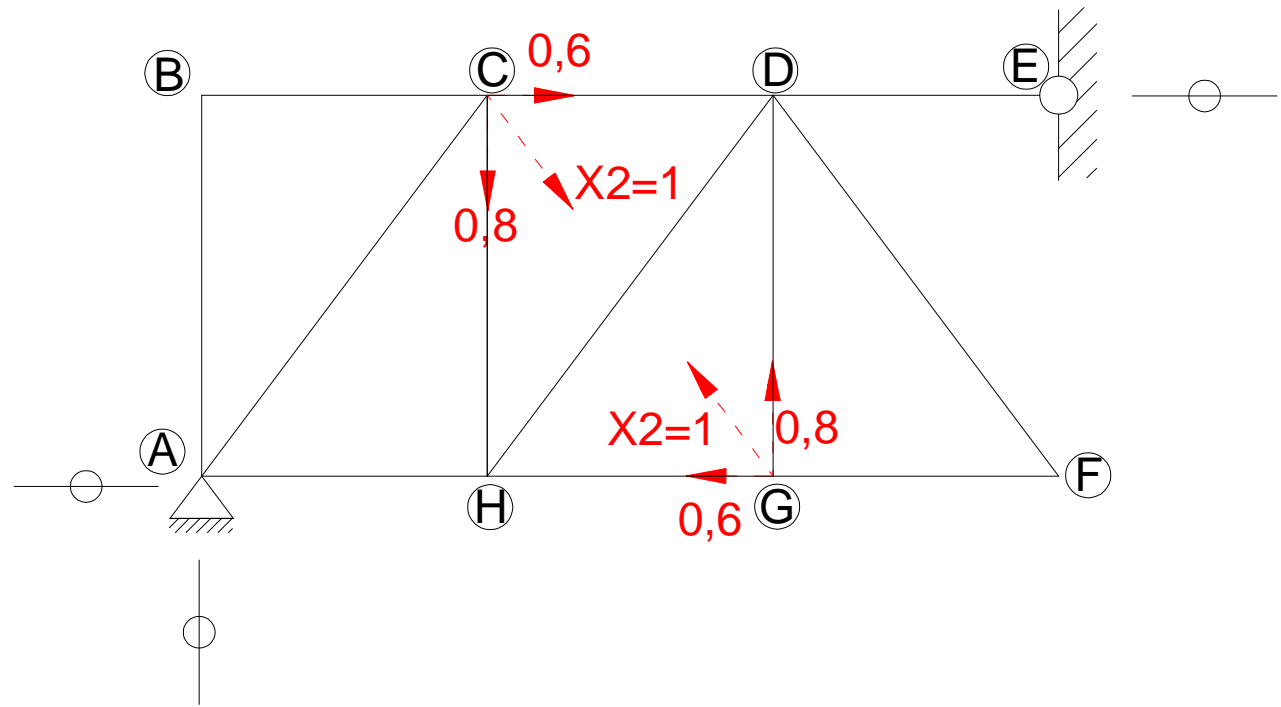
$X_2=1$



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Wykresy jednostkowe

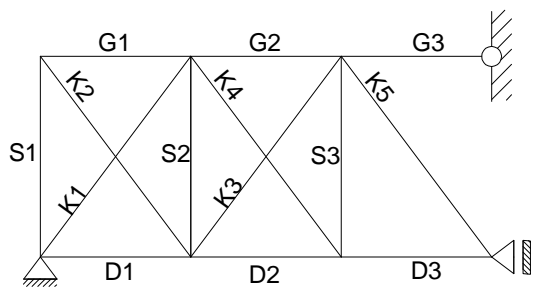
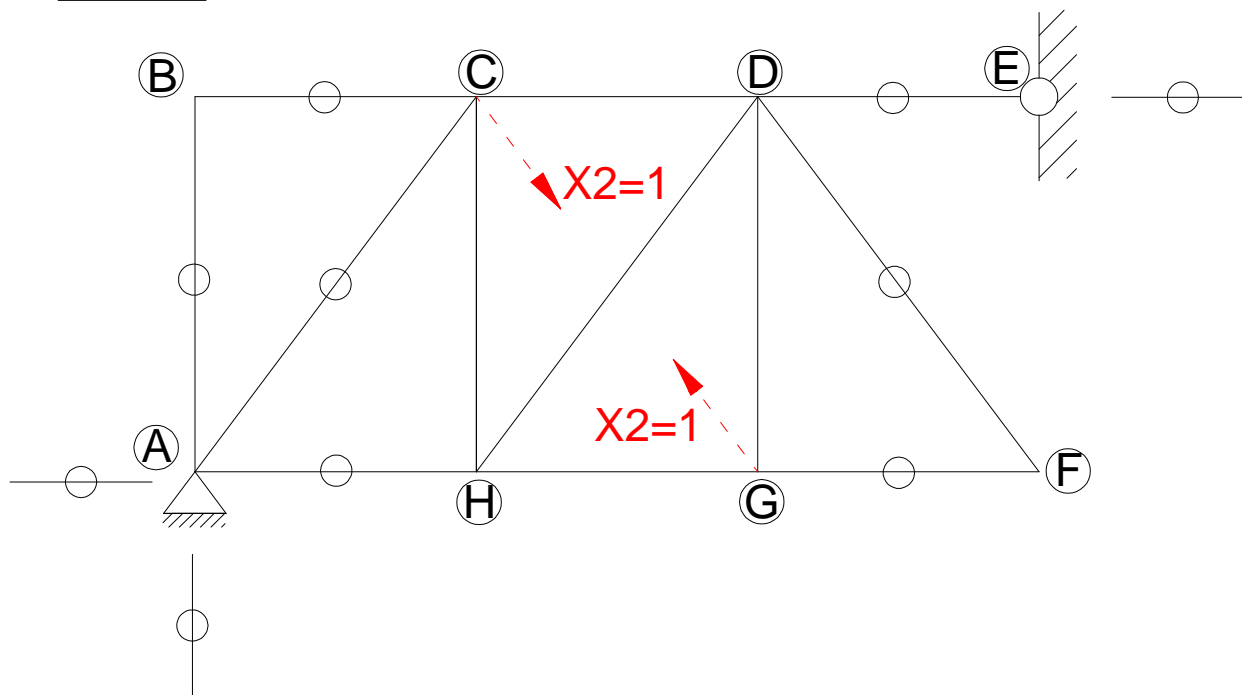
$X_2=1$



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Wykresy jednostkowe

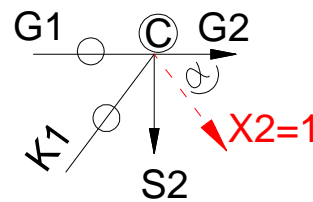
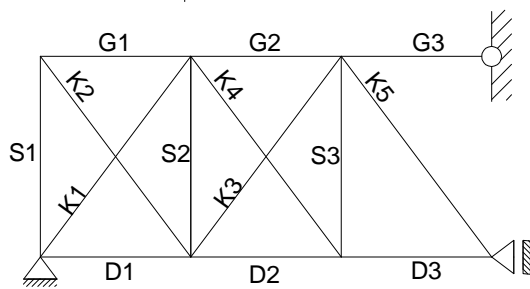
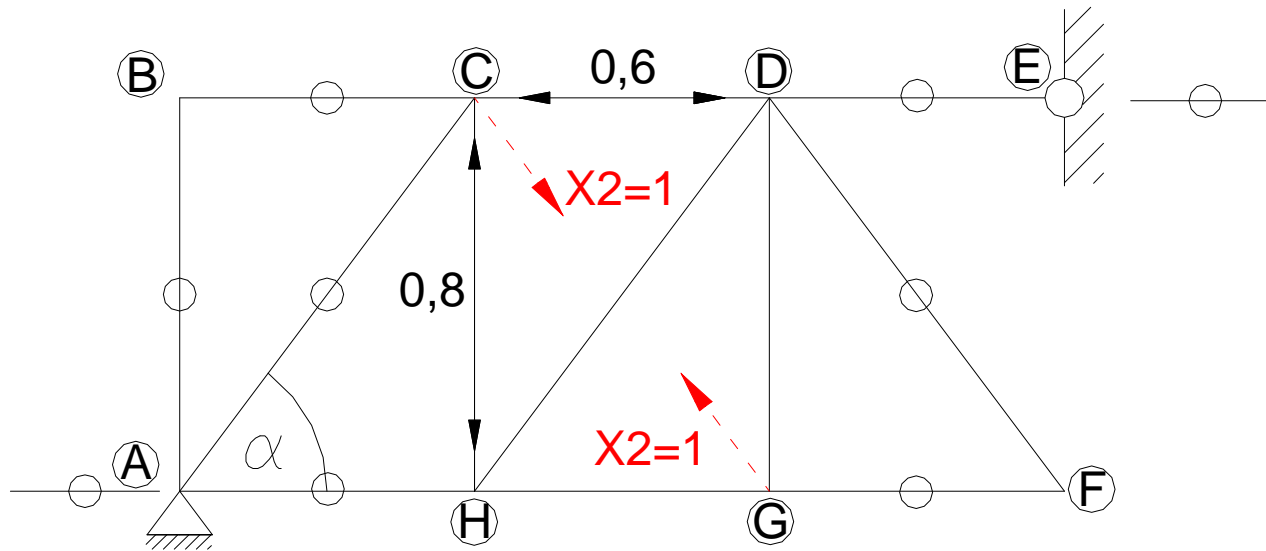
$X_2=1$



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Wykresy jednostkowe

$X_2=1$



$$\sum R_x = G_2 + X_2 \cdot \cos \alpha = 0$$

$$G_2 = -X_2 \cdot \cos \alpha = -1 \cdot 0,6 = -0,6$$

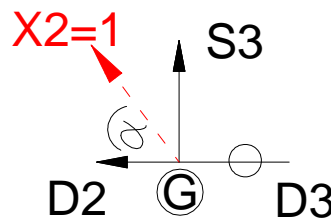
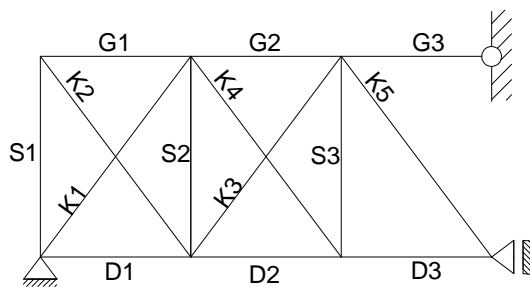
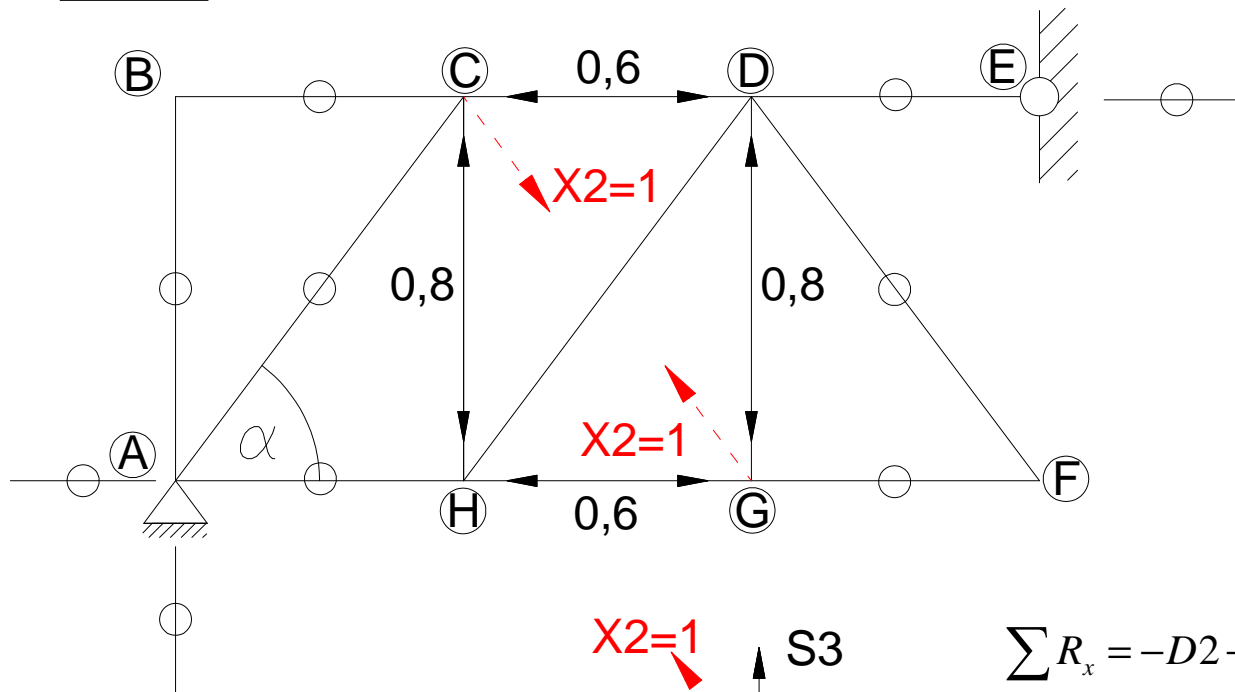
$$\sum R_y = -S_2 - X_2 \cdot \sin \alpha = 0$$

$$S_2 = -X_2 \cdot \sin \alpha = -1 \cdot 0,8 = -0,8$$

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Wykresy jednostkowe

$X_2=1$



$$\sum R_x = -D_2 - X_2 \cdot \cos \alpha = 0$$

$$D_2 = -X_2 \cos \alpha = -1 \cdot 0,6 = -0,6$$

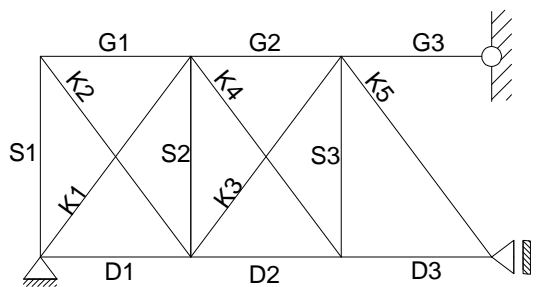
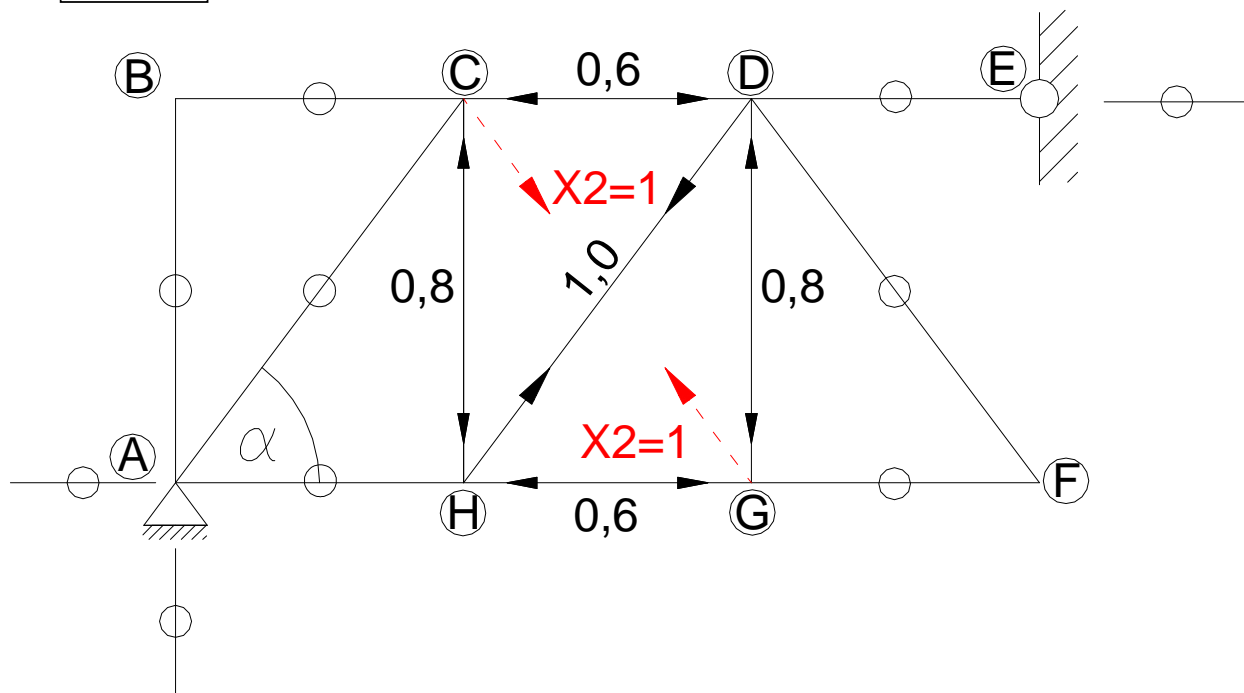
$$\sum R_y = S_3 + X_2 \cdot \sin \alpha = 0$$

$$S_3 = -X_2 \cdot \sin \alpha = -1 \cdot 0,8 = -0,8$$

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Wykresy jednostkowe

$X_2=1$

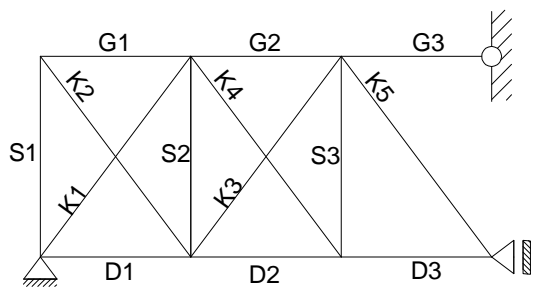
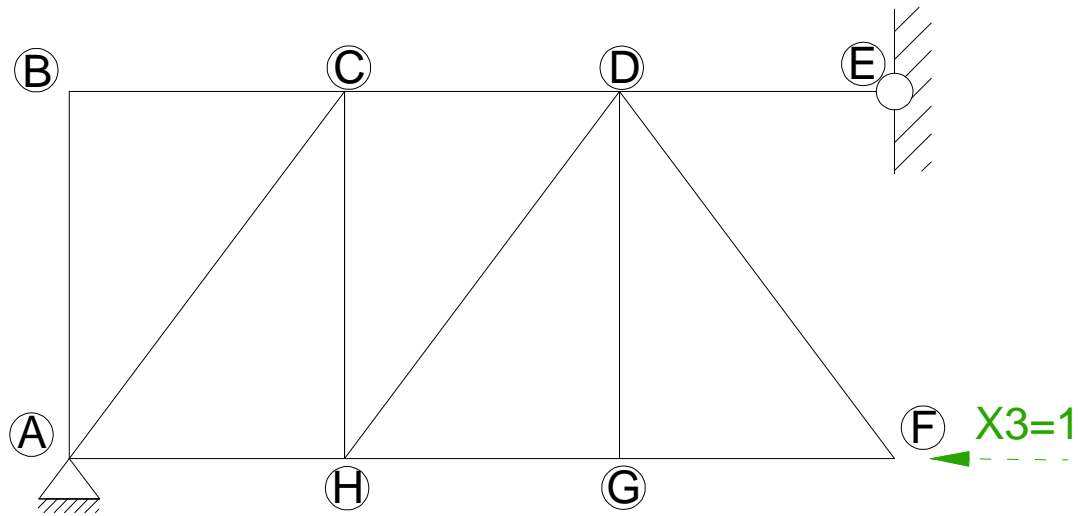


Pręt	N2
D1	0,0
D2	-0,6
D3	0,0
G1	0,0
G2	-0,6
G3	0,0
S1	0,0
S2	-0,8
S3	-0,8
K1	0,0
K2	0,0
K3	1,0
K4	1,0
K5	0,0

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Wykresy jednostkowe

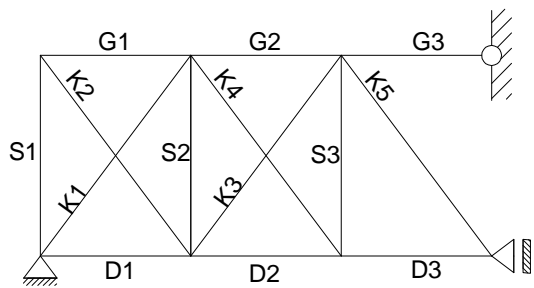
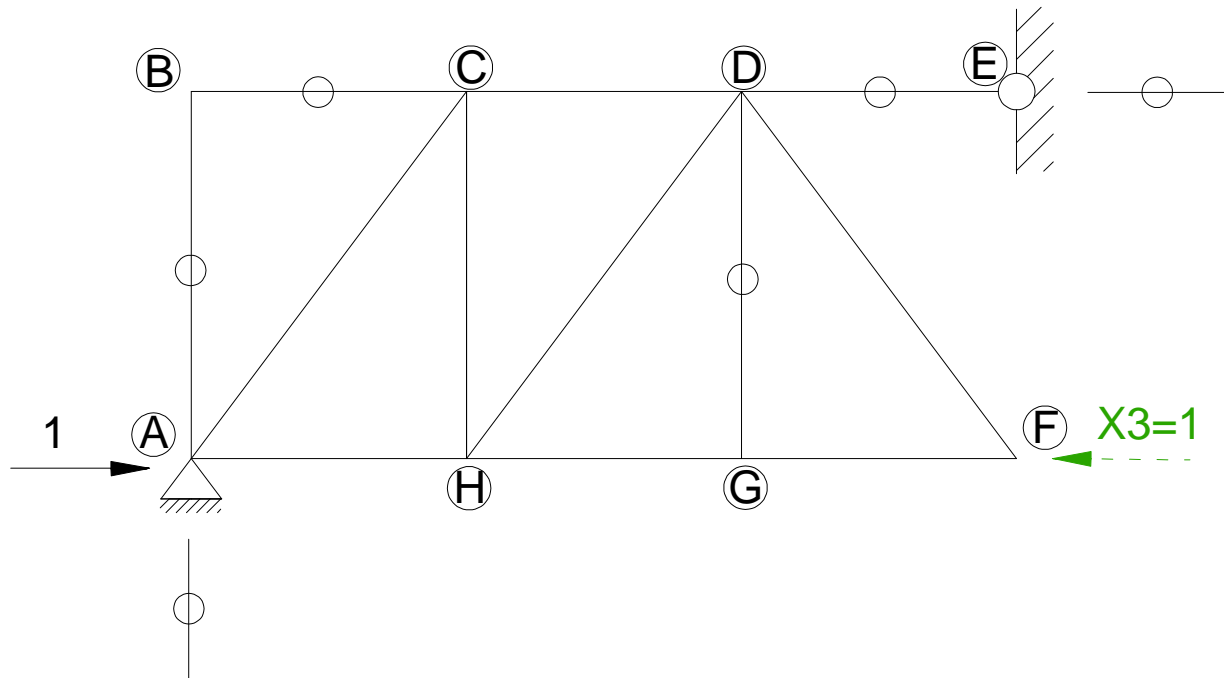
$$X_3=1$$



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Wykresy jednostkowe

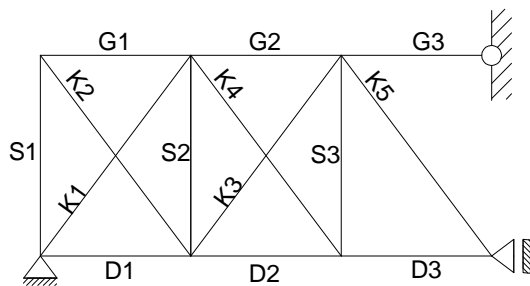
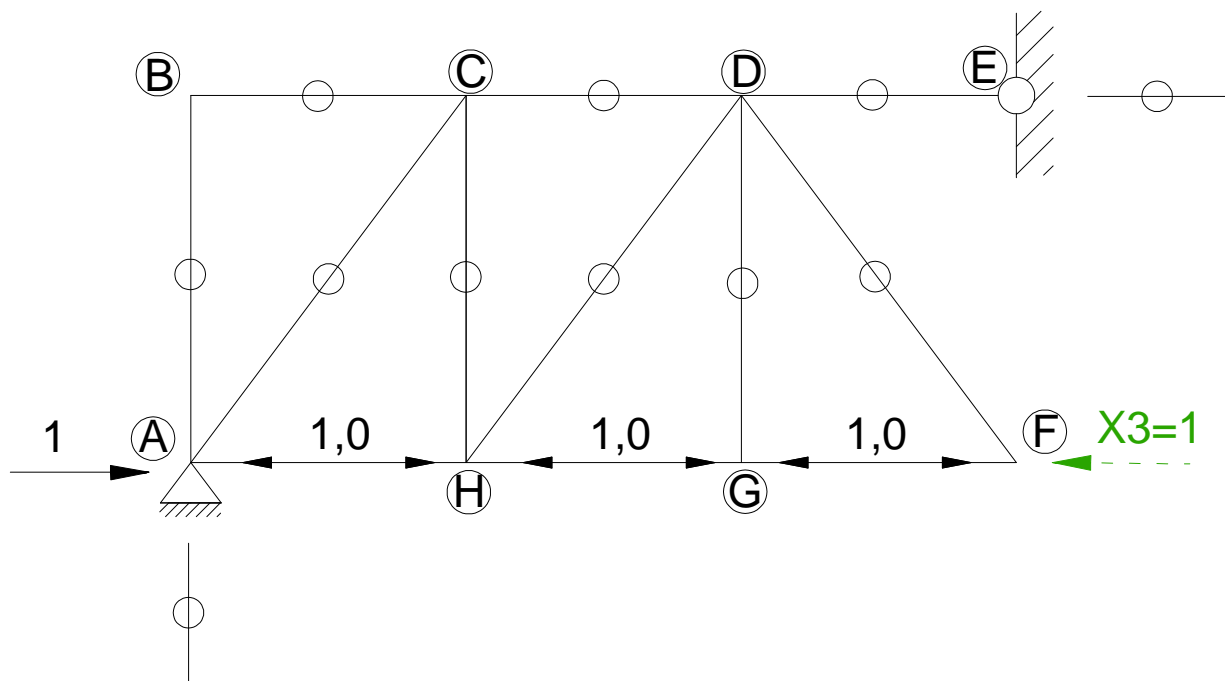
$$X_3=1$$



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Wykresy jednostkowe

X3=1

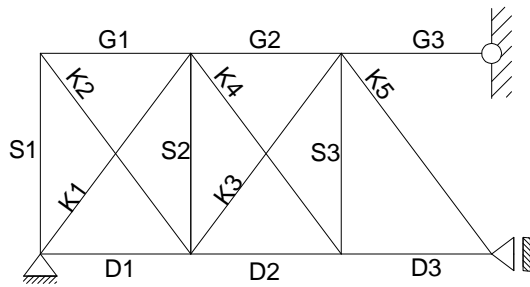
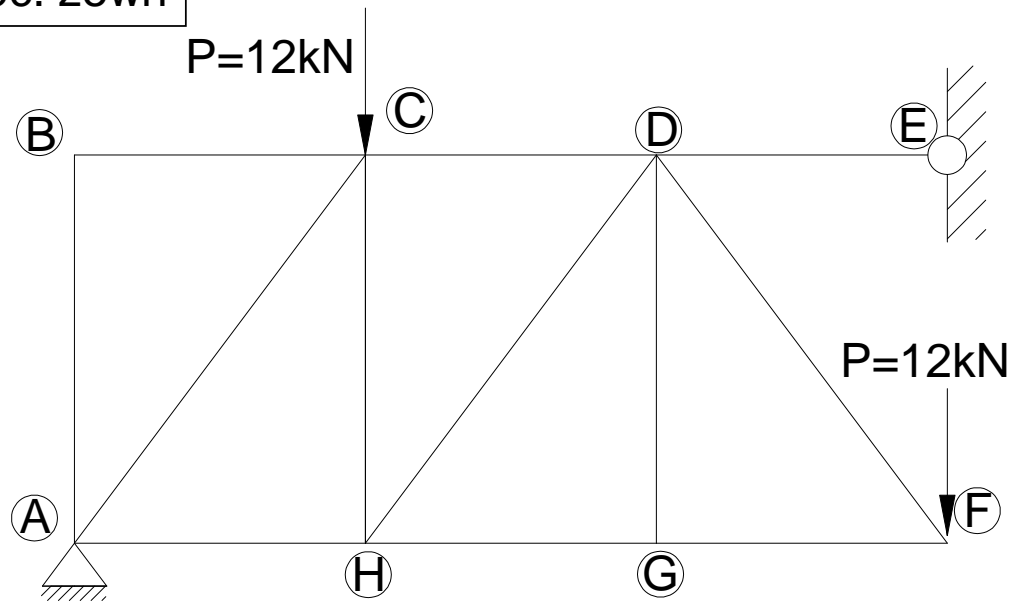


Pręt	N3
D1	-1,0
D2	-1,0
D3	-1,0
G1	0,0
G2	0,0
G3	0,0
S1	0,0
S2	0,0
S3	0,0
K1	0,0
K2	0,0
K3	0,0
K4	0,0
K5	0,0

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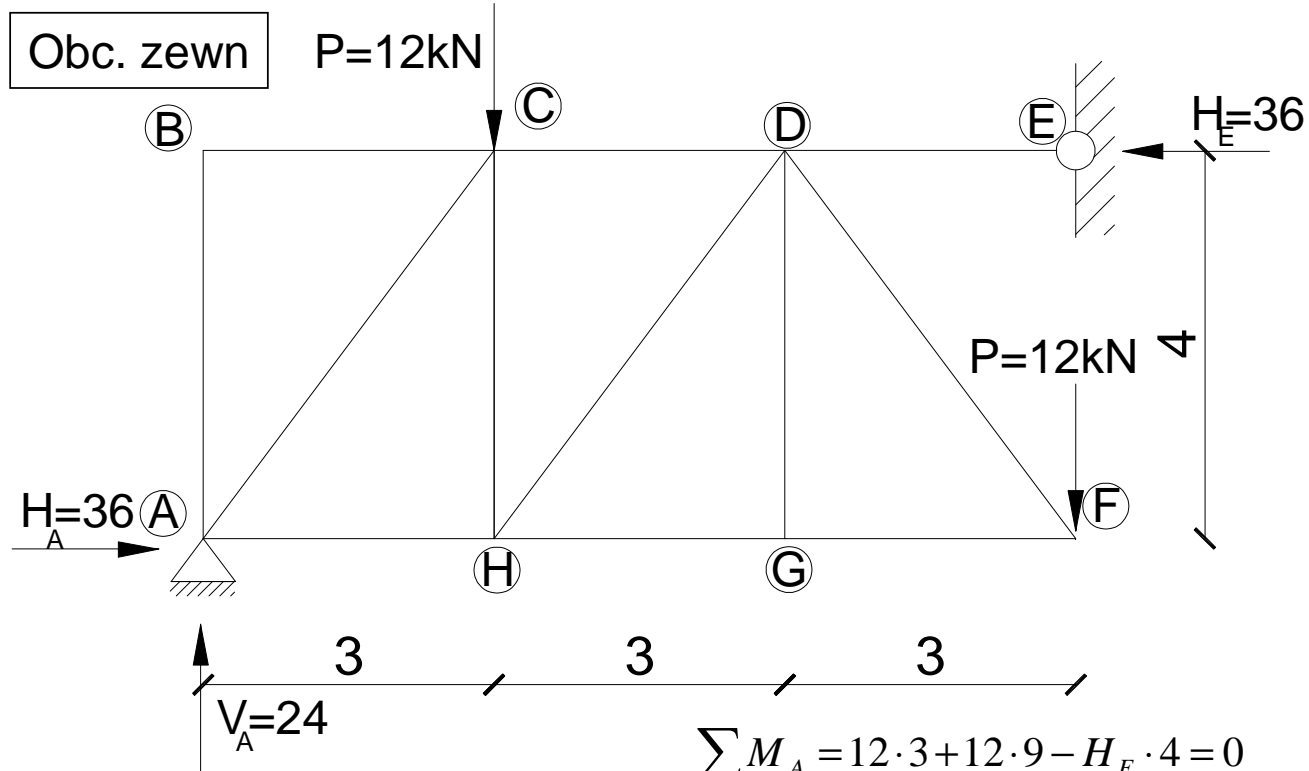
Wykresy jednostkowe

Obc. zewn



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Wykresy jednostkowe

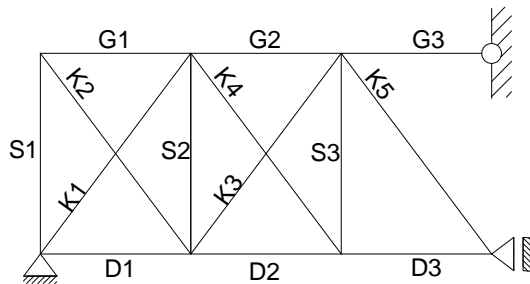


$$\sum M_A = 12 \cdot 3 + 12 \cdot 9 - H_E \cdot 4 = 0$$

$$H_E = 144 / 4 = 36 \text{ kN}$$

$$\sum R_X = H_A - H_E = 0 \rightarrow H_A = H_E = 36 \text{ kN}$$

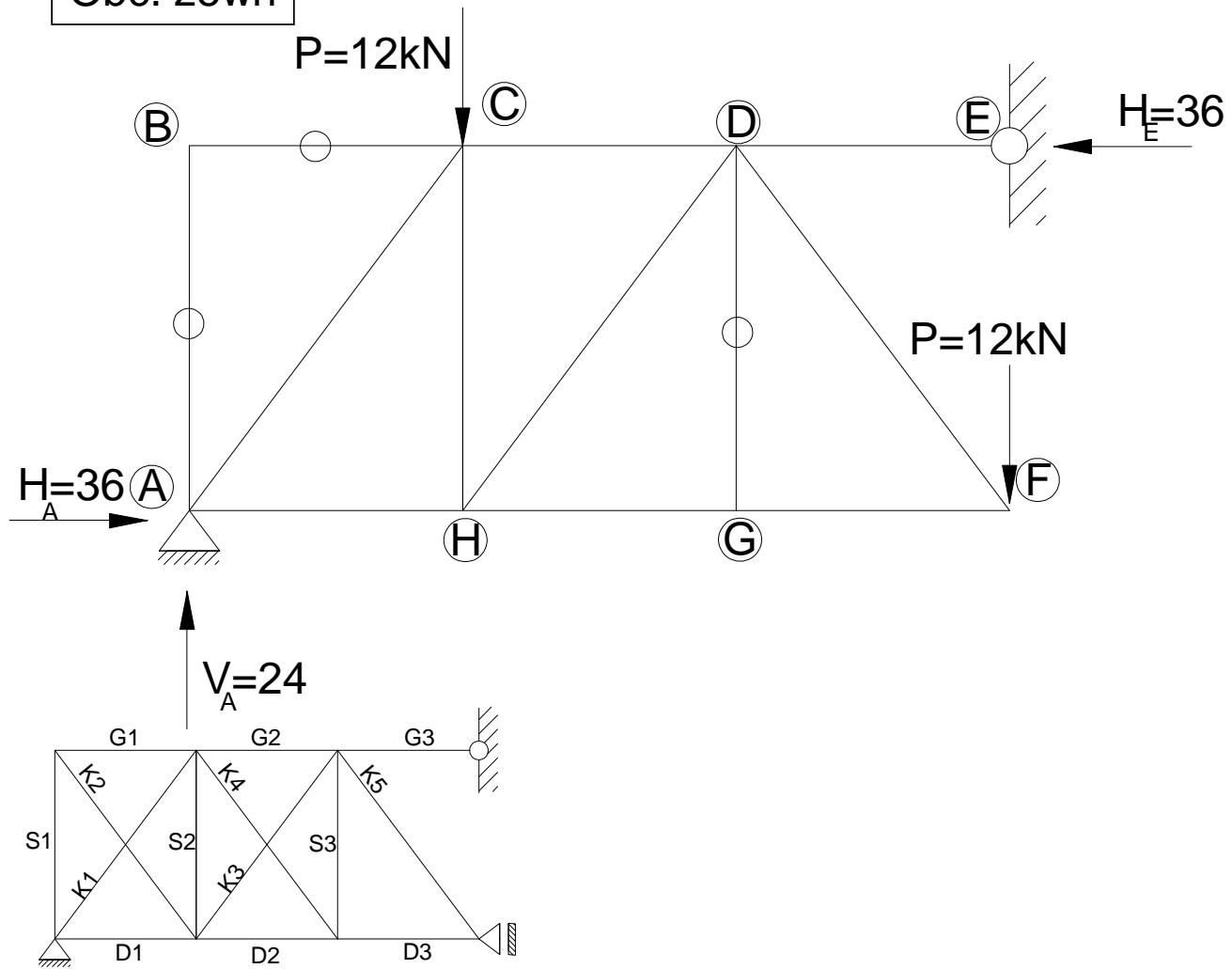
$$\sum R_Y = V_A - 12 - 12 = 0 \rightarrow V_A = 24 \text{ kN}$$



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Wykresy jednostkowe

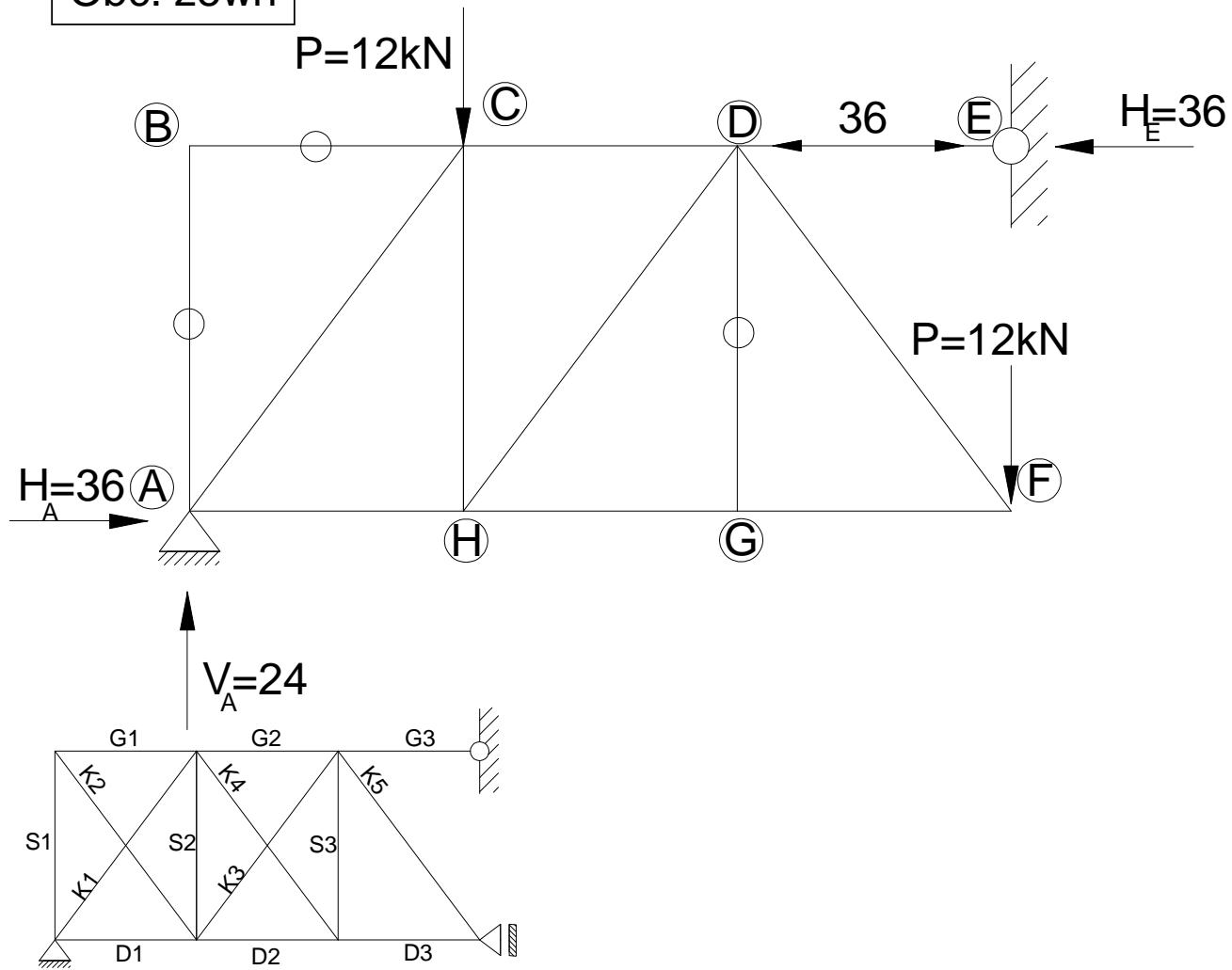
Obc. zewn



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Wykresy jednostkowe

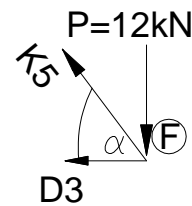
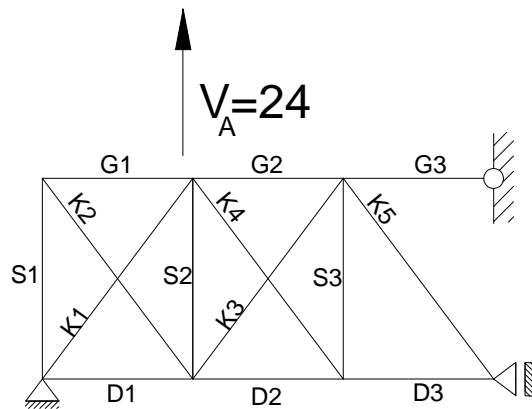
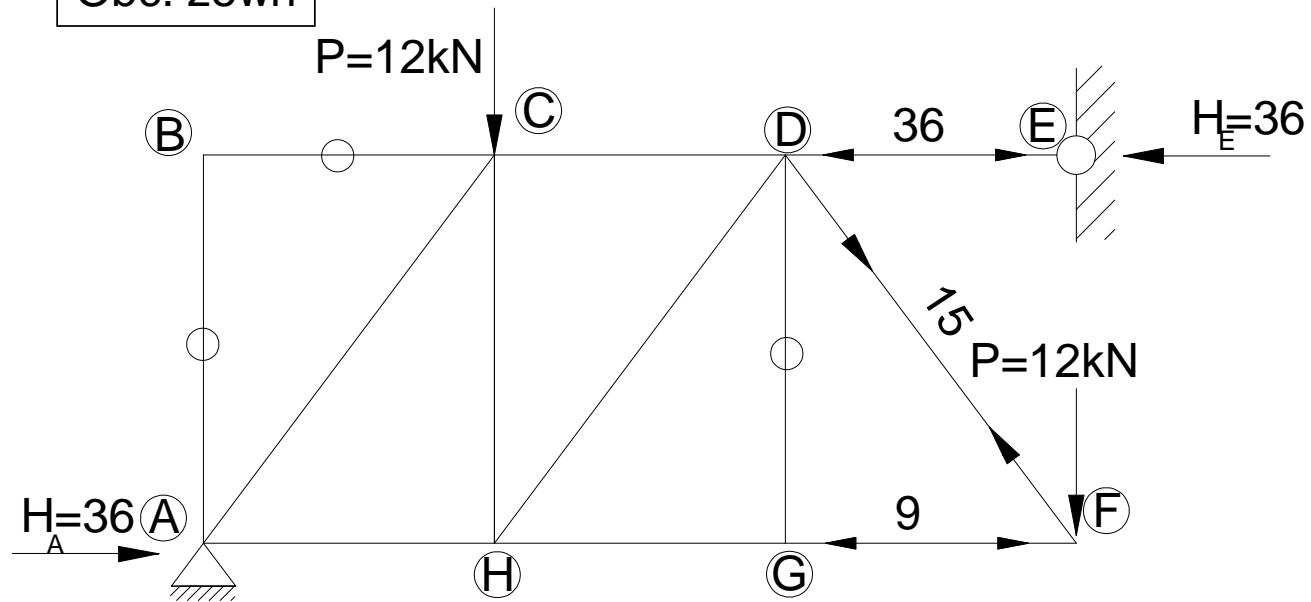
Obc. zewn



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Wykresy jednostkowe

Obc. zewn



$$\sum R_y = -12 + K5 \sin \alpha = 0$$

$$K5 = 12 / \sin \alpha = 12 / 0,8 = 15 \text{ kN}$$

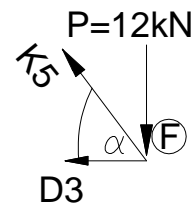
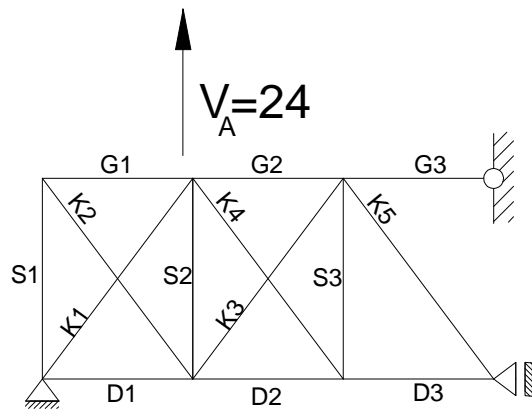
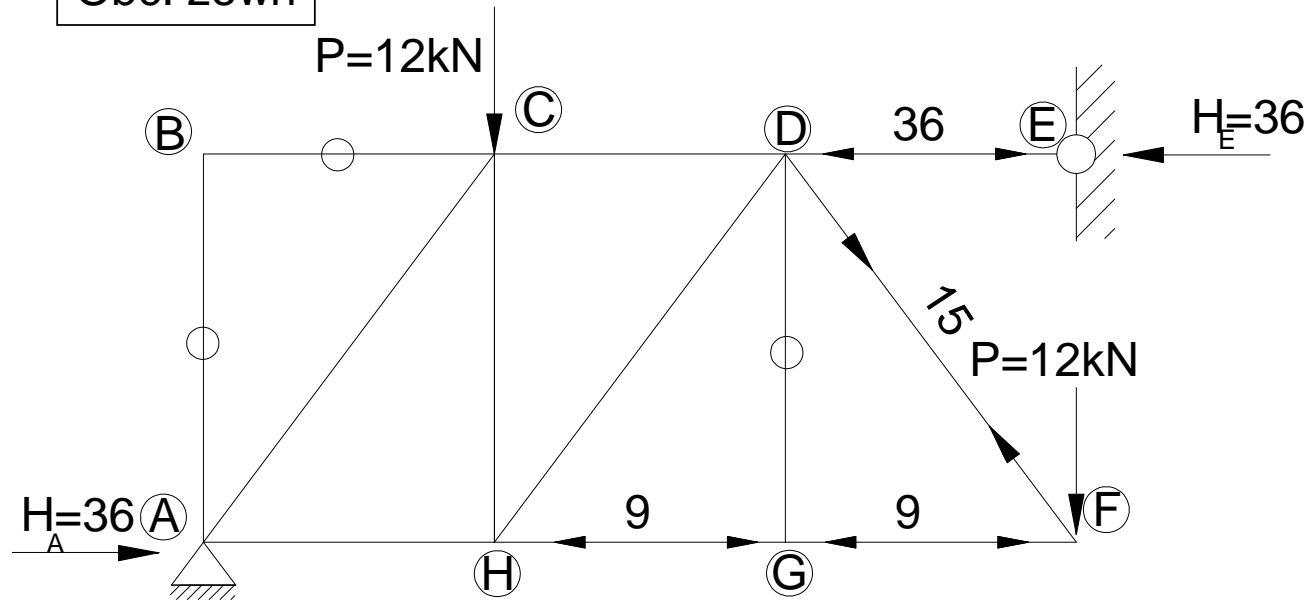
$$\sum R_x = -D3 - K5 \cos \alpha = 0$$

$$D3 = -K5 \cos \alpha = -15 \cdot 0,6 = -9$$

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Wykresy jednostkowe

Obc. zewn



$$\sum R_y = -12 + K5 \sin \alpha = 0$$

$$K5 = 12 / \sin \alpha = 12 / 0,8 = 15 \text{ kN}$$

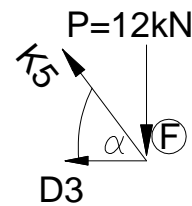
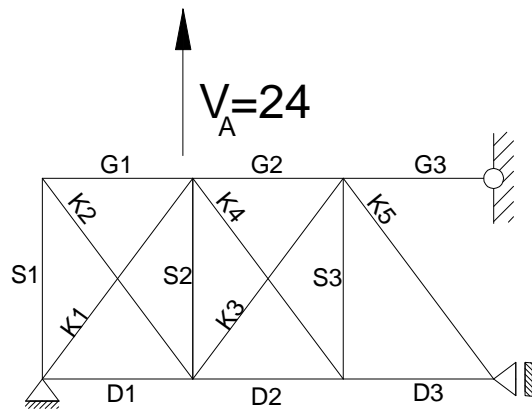
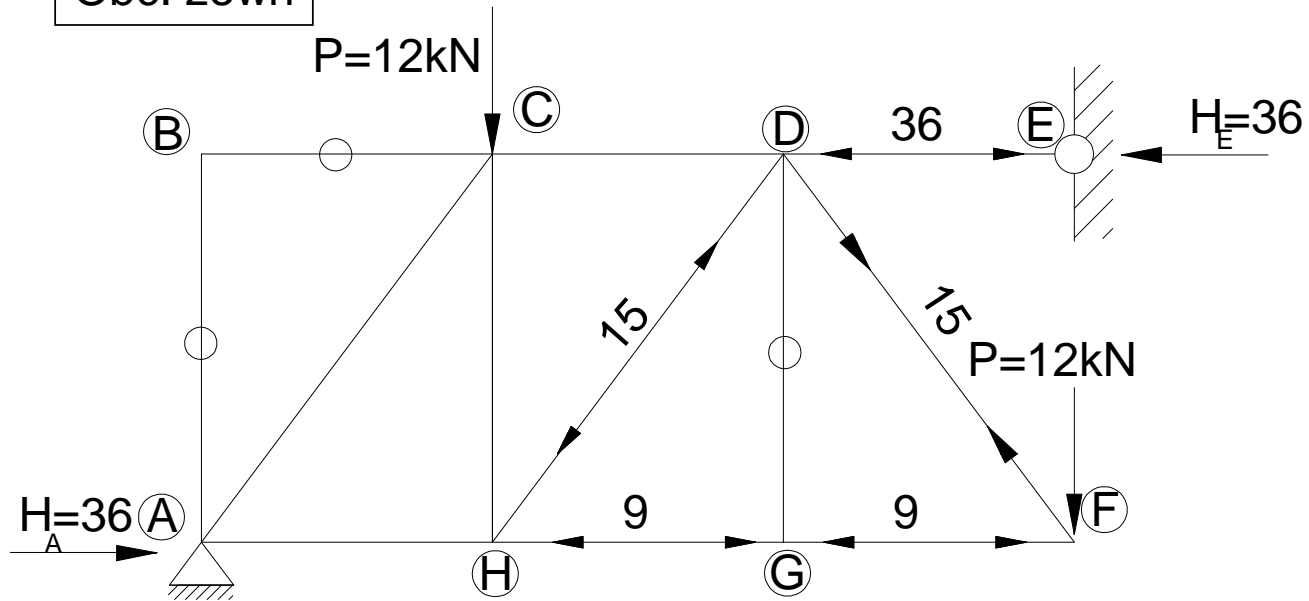
$$\sum R_x = -D3 - K5 \cos \alpha = 0$$

$$D3 = -K5 \cos \alpha = -15 \cdot 0,6 = -9$$

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Wykresy jednostkowe

Obc. zewn



$$\sum R_y = -12 + K5 \sin \alpha = 0$$

$$K5 = 12 / \sin \alpha = 12 / 0,8 = 15 \text{ kN}$$

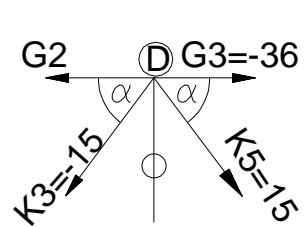
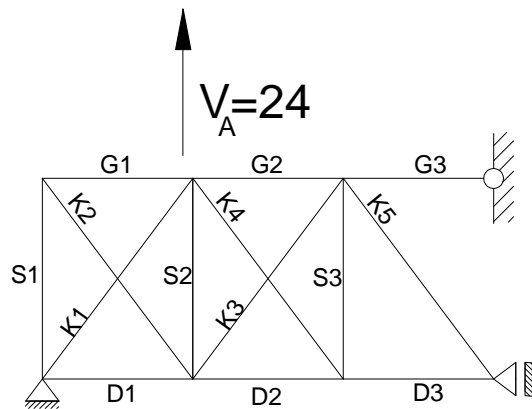
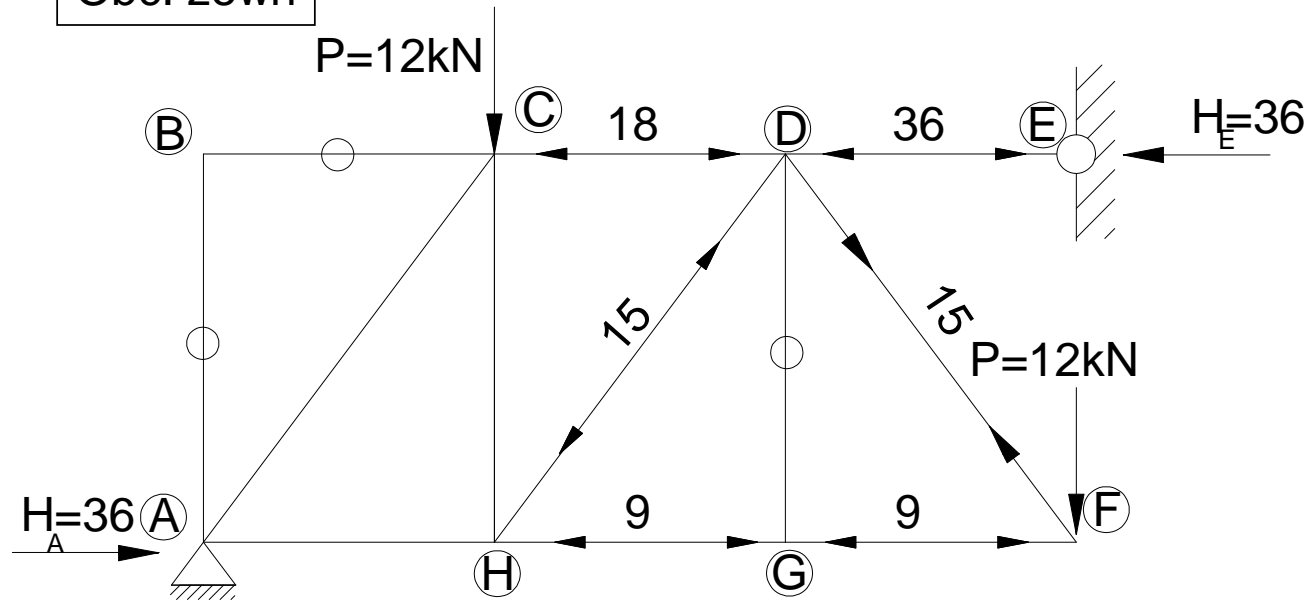
$$\sum R_x = -D3 - K5 \cos \alpha = 0$$

$$D3 = -K5 \cos \alpha = -15 \cdot 0,6 = -9$$

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Wykresy jednostkowe

Obc. zewn



$$\sum R_x = -G2 - K3 \cos \alpha + G3 + K5 \cos \alpha = 0$$

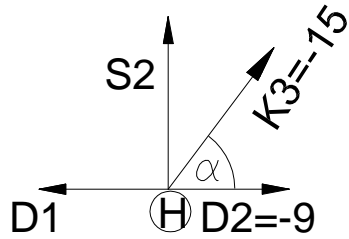
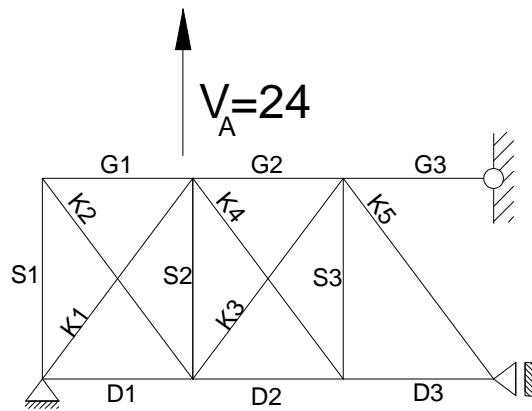
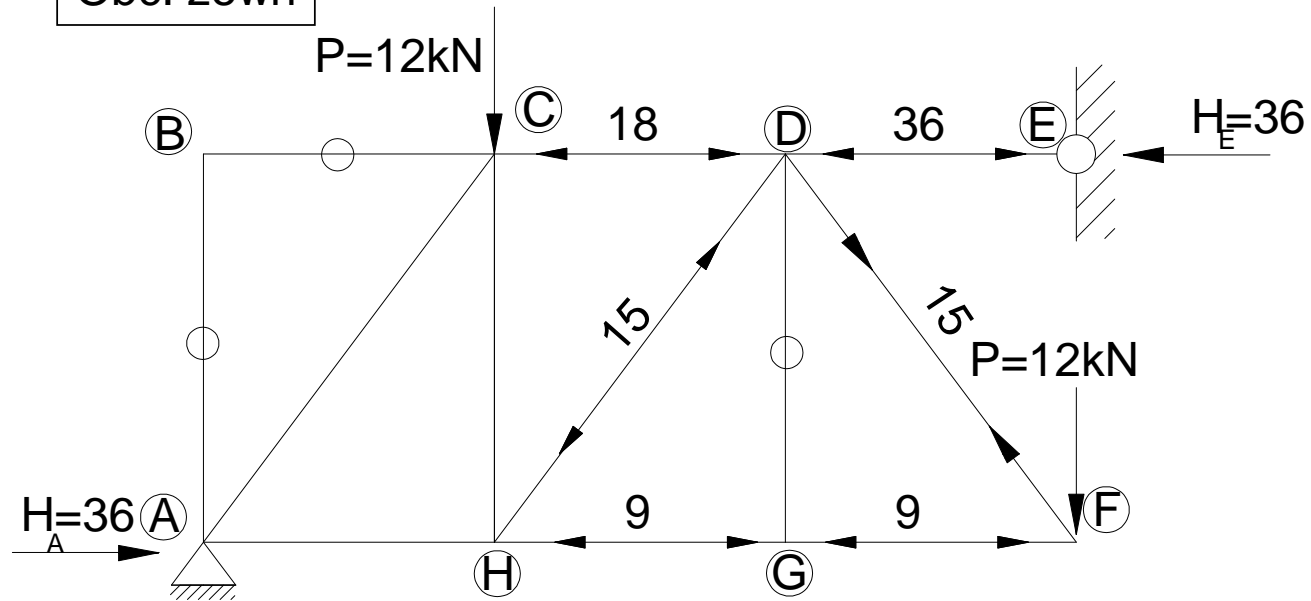
$$G2 = -K3 \cos \alpha + G3 + K5 \cos \alpha$$

$$G2 = -(-15) \cdot 0,6 - 36 + 15 \cdot 0,6 = -18$$

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Wykresy jednostkowe

Obc. zewn



$$\sum R_y = S2 + K_3 \sin \alpha = 0$$

$$S2 = -K_3 \sin \alpha = -(-15) \cdot 0,8 = 12$$

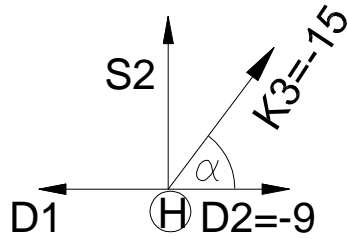
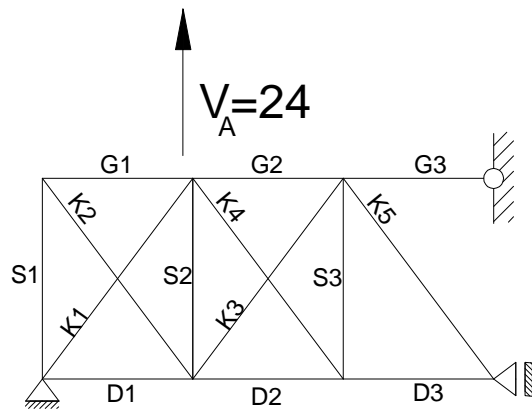
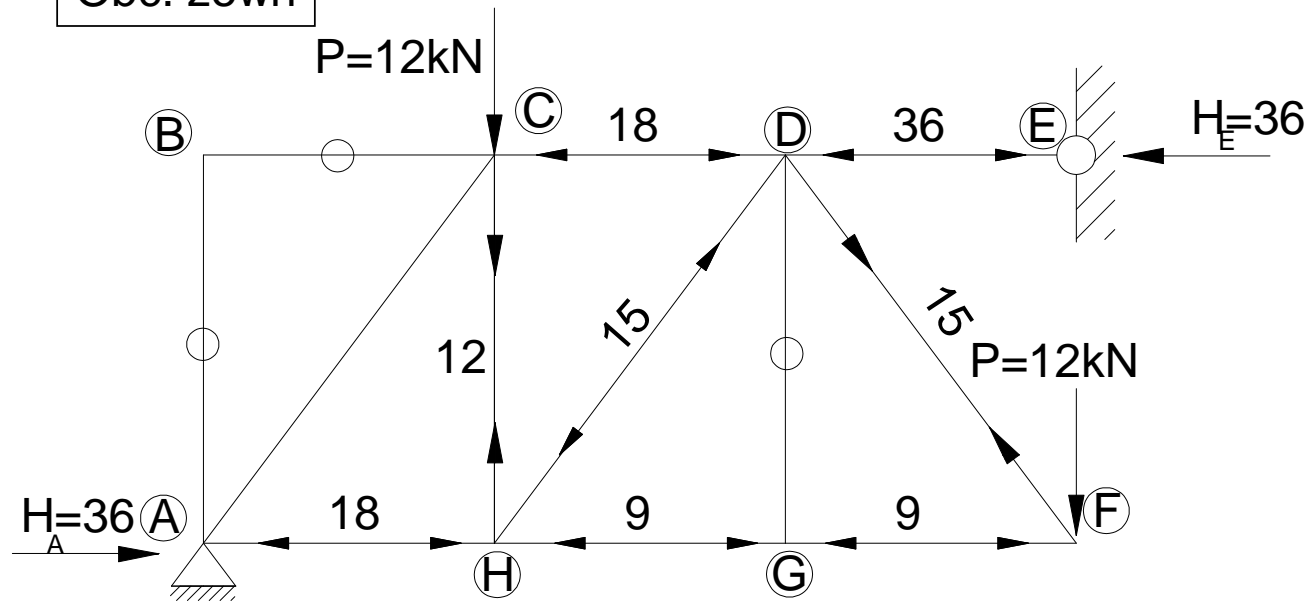
$$\sum R_x = -D1 + D2 + K_3 \cos \alpha = 0$$

$$D1 = D2 + K_3 \cos \alpha = -9 - 15 \cdot 0,6 = -18$$

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Wykresy jednostkowe

Obc. zewn



$$\sum R_y = S2 + K_3 \sin \alpha = 0$$

$$S2 = -K_3 \sin \alpha = -(-15) \cdot 0,8 = 12$$

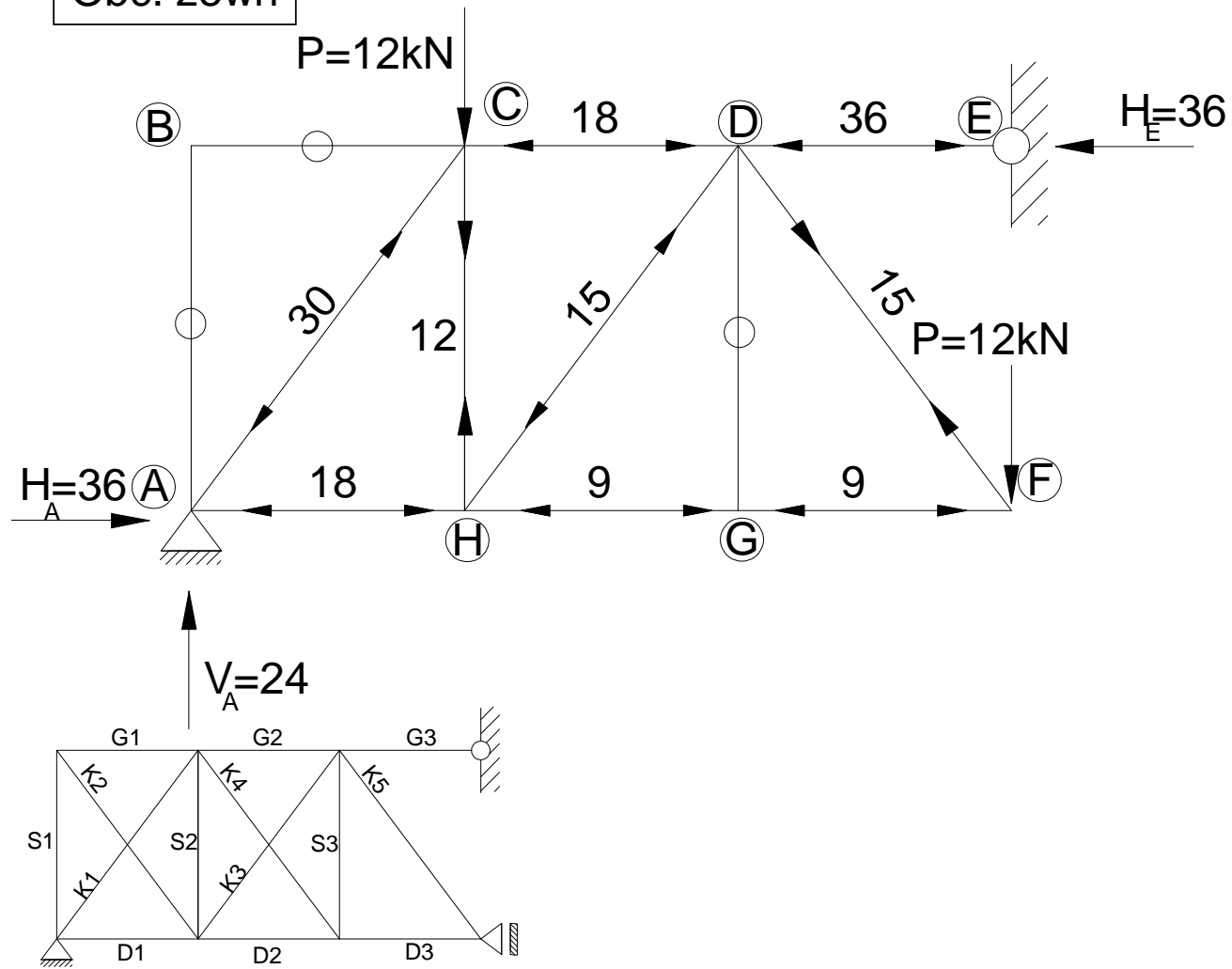
$$\sum R_x = -D1 + D2 + K_3 \cos \alpha = 0$$

$$D1 = D2 + K_3 \cos \alpha = -9 - 15 \cdot 0,6 = -18$$

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Wykresy jednostkowe

Obc. zewn



Pręt	N0
D1	-18
D2	-9
D3	-9
G1	0
G2	-18
G3	-36
S1	0
S2	12
S3	0
K1	-30
K2	0
K3	-15
K4	0
K5	15

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Obliczenia w tabeli

Pręt	L/EA	N1	N2	N3	N0	N1*N1*L/EA	N1*N2*L/EA	N1*N3*L/EA	N1*N0*L/EA
D1	3/EA	-0,6	0	-1	-18	1,08/EA	0	1,8/EA	32,4/EA
D2	3/EA	0	-0,6	-1	-9	0	0	0	0
D3	3/EA	0	0	-1	-9	0	0	0	0
G1	3/EA	-0,6	0	0	0	1,08/EA	0	0	0
G2	3/EA	0	-0,6	0	-18	0	0	0	0
G3	3/EA	0	0	0	-36	0	0	0	0
S1	4/EA	-0,8	0	0	0	2,56/EA	0	0	0
S2	4/EA	-0,8	-0,8	0	12	2,56/EA	2,56/EA	0	-38,4/EA
S3	4/EA	0	-0,8	0	0	0	0	0	0
K1	5/EA	1	0	0	-30	5/EA	0	0	-150/EA
K2	5/EA	1	0	0	0	5/EA	0	0	0
K3	5/EA	0	1	0	-15	0	0	0	0
K4	5/EA	0	1	0	0	0	0	0	0
K5	5/EA	0	0	0	15	0	0	0	0
						$\delta_{11}=17,28/EA$	$\delta_{12}=2,56/EA$	$\delta_{13}=1,8/EA$	$\delta_{10}=-156/EA$

Obliczenia w tabeli

Pręt	L/EA	N1	N2	N3	N0	N2*N2*L/EA	N2*N3*L/EA	N2*N0*L/EA	N3*N3*L/EA	N3*N0*L/EA
D1	3/EA	-0,6	0	-1	-18	0	0	0	3/EA	54/EA
D2	3/EA	0	-0,6	-1	-9	1,08/EA	1,8/EA	16,2/EA	3/EA	27/EA
D3	3/EA	0	0	-1	-9	0	0	0	3/EA	27/EA
G1	3/EA	-0,6	0	0	0	0	0	0	0	0
G2	3/EA	0	-0,6	0	-18	1,08/EA	0	32,4/EA	0	0
G3	3/EA	0	0	0	-36	0	0	0	0	0
S1	4/EA	-0,8	0	0	0	0	0	0	0	0
S2	4/EA	-0,8	-0,8	0	12	2,56/EA	0	-38,4/EA	0	0
S3	4/EA	0	-0,8	0	0	2,56/EA	0	0	0	0
K1	5/EA	1	0	0	-30	0	0	0	0	0
K2	5/EA	1	0	0	0	0	0	0	0	0
K3	5/EA	0	1	0	-15	5/EA	0	-75/EA	0	0
K4	5/EA	0	1	0	0	5/EA	0	0	0	0
K5	5/EA	0	0	0	15	0	0	0	0	0
						$\delta_{22} = 17,28/EA$	$\delta_{23} = 1,8/EA$	$\delta_{20} = -64,8/EA$	$\delta_{33} = 9/EA$	$\delta_{30} = 108/EA$

Wyznaczenie nadliczbowych

Układ równań metody sił:

$$\delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 + \delta_{10} = 0$$

$$\delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 + \delta_{20} = 0$$

$$\delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 + \delta_{30} = 0$$

Podstawiając wyliczone wcześniej wartości otrzymujemy:

$$\frac{17,28}{EA} \cdot X_1 + \frac{2,56}{EA} \cdot X_2 + \frac{1,8}{EA} \cdot X_3 - \frac{156}{EA} = 0$$

$$\frac{2,56}{EA} \cdot X_1 + \frac{17,28}{EA} \cdot X_2 + \frac{1,8}{EA} \cdot X_3 + \frac{-64,8}{EA} = 0$$

$$\frac{1,8}{EA} \cdot X_1 + \frac{1,8}{EA} \cdot X_2 + \frac{9}{EA} \cdot X_3 + \frac{108}{EA} = 0$$

Rozwiązanie układu równań:

$$X_1 = 10,00kN$$

$$X_2 = 3,81kN$$

$$X_3 = -14,76kN$$

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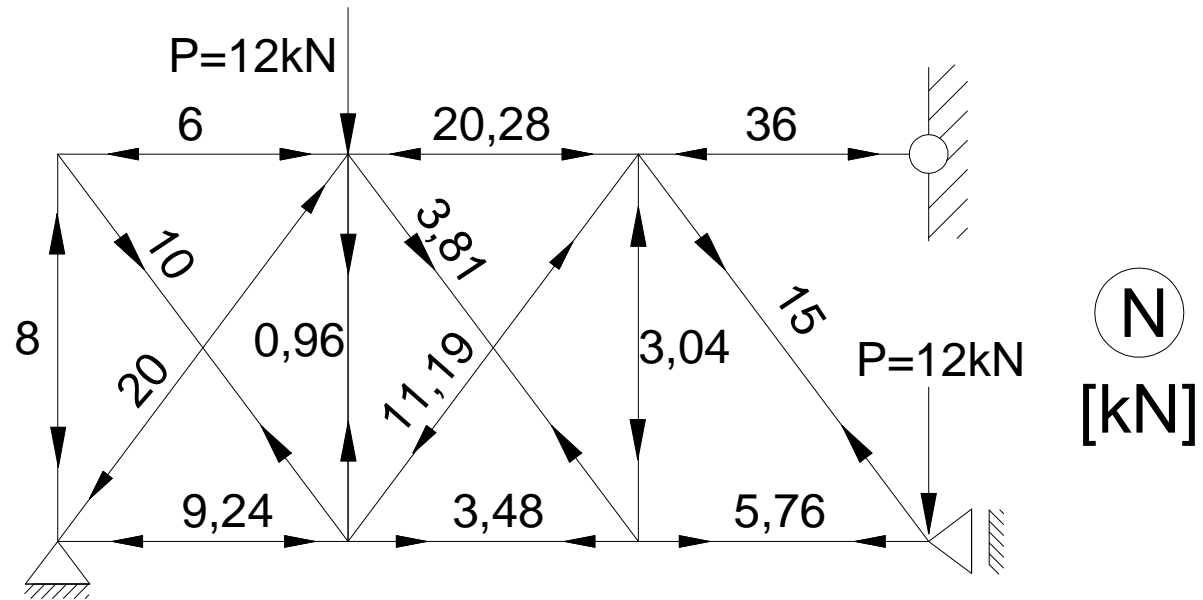
Wyznaczenie ostatecznych wartości sił normalnych dla układu statycznie niewyznaczalnego

Pręt	L/EA	N1	N2	N3	N0	X1*N1	X2*N2	X3*N3	$N=N1*X1+N2*X2+N3*X3+N0$
D1	3/EA	-0,6	0	-1	-18	-6	0	14,76	-9,24
D2	3/EA	0	-0,6	-1	-9	0	-2,28	14,76	3,48
D3	3/EA	0	0	-1	-9	0	0	14,76	5,76
G1	3/EA	-0,6	0	0	0	-6	0	0	-6
G2	3/EA	0	-0,6	0	-18	0	-2,28	0	-20,28
G3	3/EA	0	0	0	-36	0	0	0	-36
S1	4/EA	-0,8	0	0	0	-8	0	0	-8
S2	4/EA	-0,8	-0,8	0	12	-8	-3,04	0	0,96
S3	4/EA	0	-0,8	0	0	0	-3,04	0	-3,04
K1	5/EA	1	0	0	-30	10	0	0	-20
K2	5/EA	1	0	0	0	10	0	0	10
K3	5/EA	0	1	0	-15	0	3,81	0	-11,19
K4	5/EA	0	1	0	0	0	3,81	0	3,81
K5	5/EA	0	0	0	15	0	0	0	15

$X1=10\text{kN}$; $X2=3,81\text{kN}$; $X3=-14,76\text{kN}$

Wyznaczenie ostatecznych wartości sił normalnych dla układu statycznie niewyznaczalnego

Pręt	N [kN]
D1	-9,24
D2	3,48
D3	5,76
G1	-6
G2	-20,28
G3	-36
S1	-8
S2	0,96
S3	-3,04
K1	-20
K2	10
K3	-11,19
K4	3,81
K5	15



$X_1=10\text{kN}; X_2=3,81\text{kN}; X_3=-14,76\text{kN}$

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