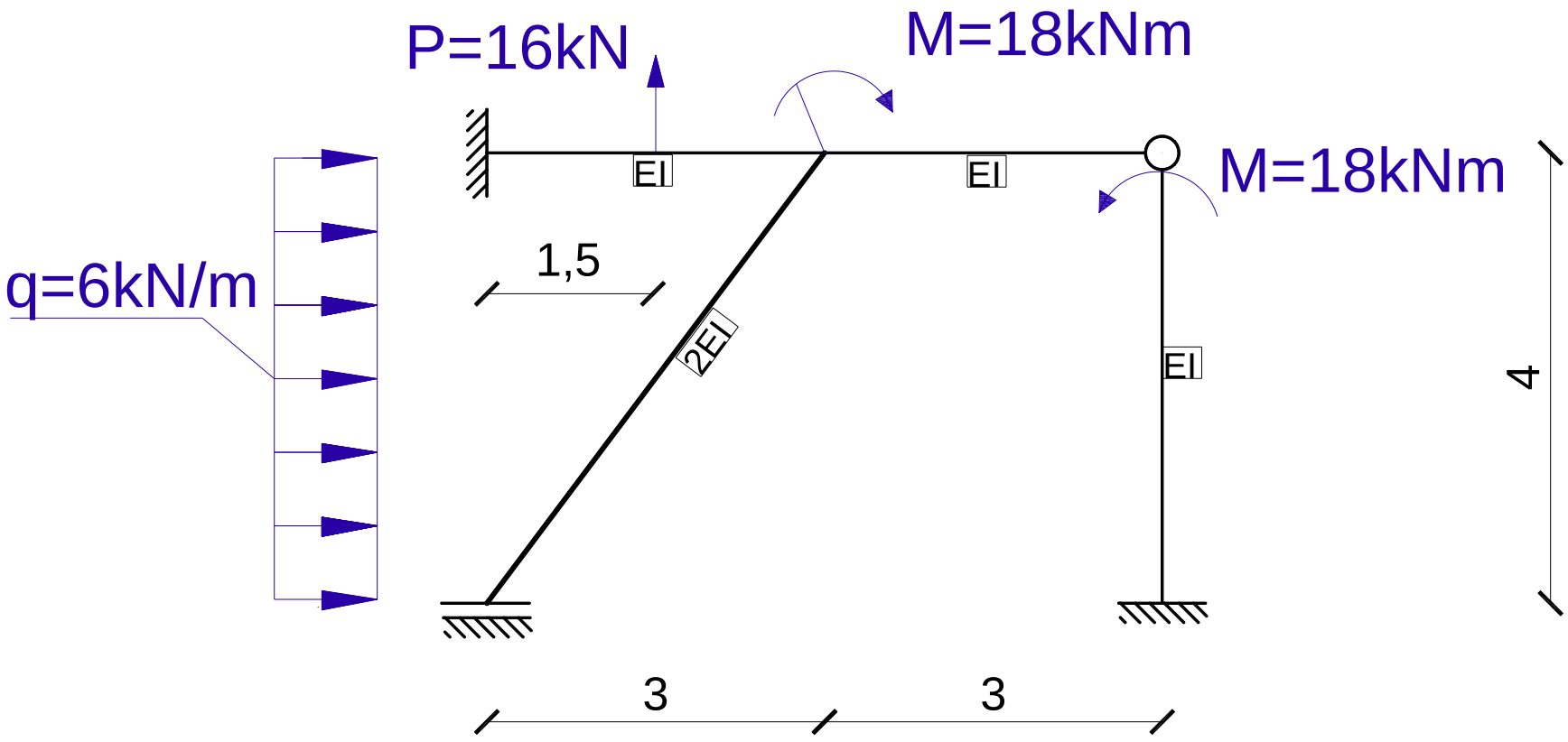
Mechanika Budowli – sem. IV

Wykład:

Obliczanie ramy przesuwnej z prętem ukośnym metodą przemieszczeń

dr inż. Hanna Weber

Zadanie 1: Wyznaczyć współczynniki układu równań metody przemieszczeń. Zadanie rozwiązać w minimalnej bazie niewiadomych.



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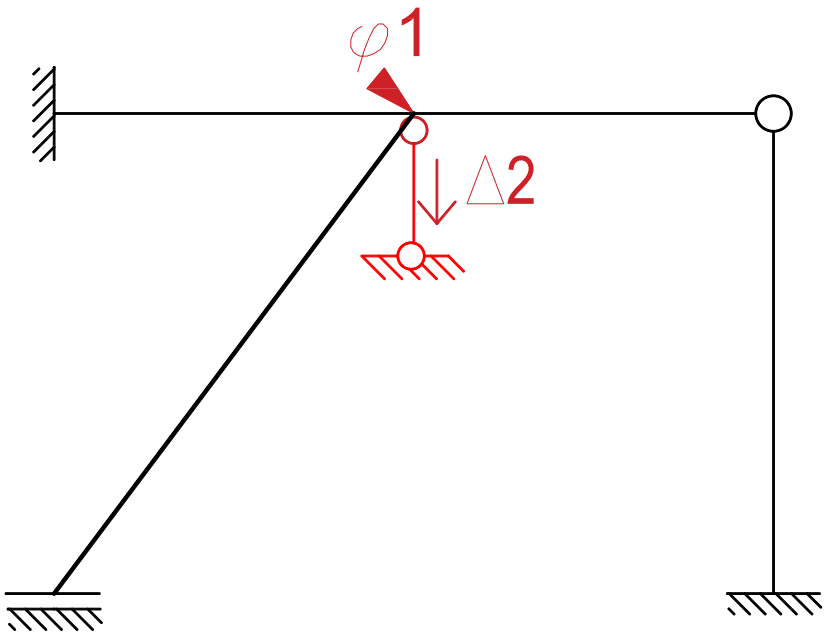
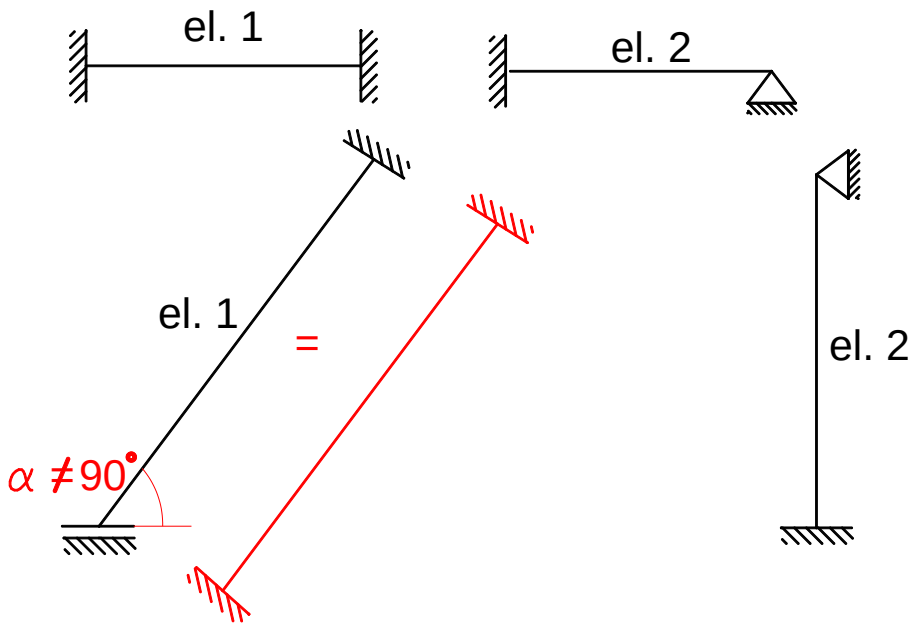
Schemat podstawowy geometrycznie wyznaczalny



Podział na elementy:



Układ dwukrotnie geometrycznie niewyznaczalny

$n_g = 2(\varphi 1, \triangle 2)$

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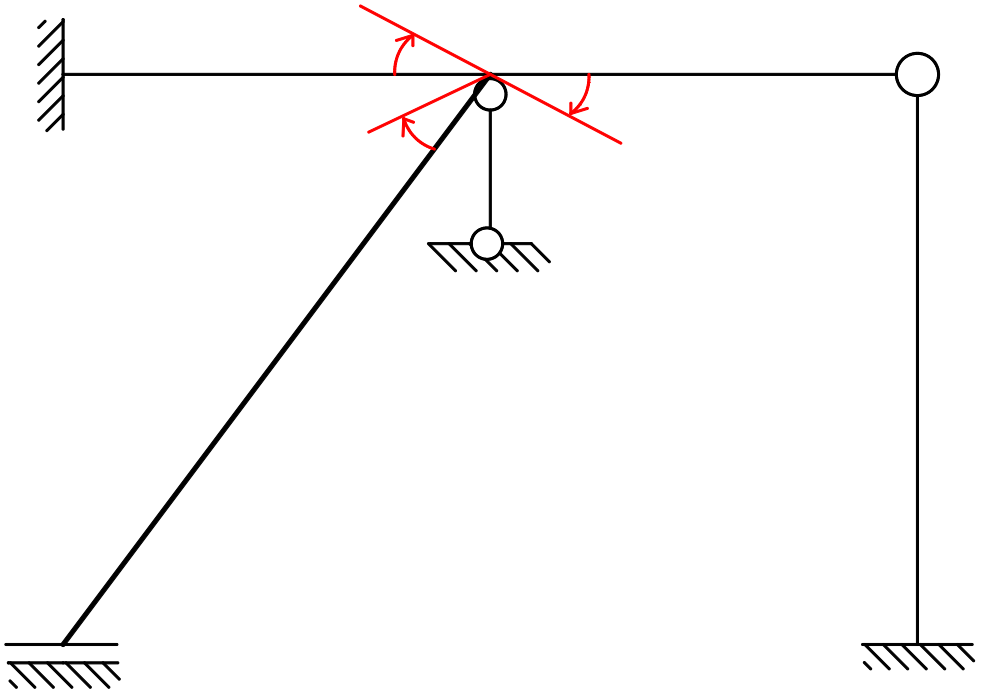
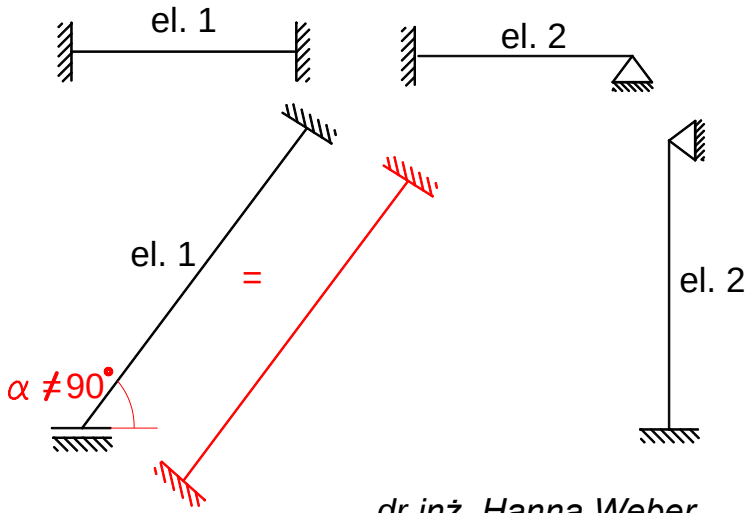
Stan $\varphi_1=1$





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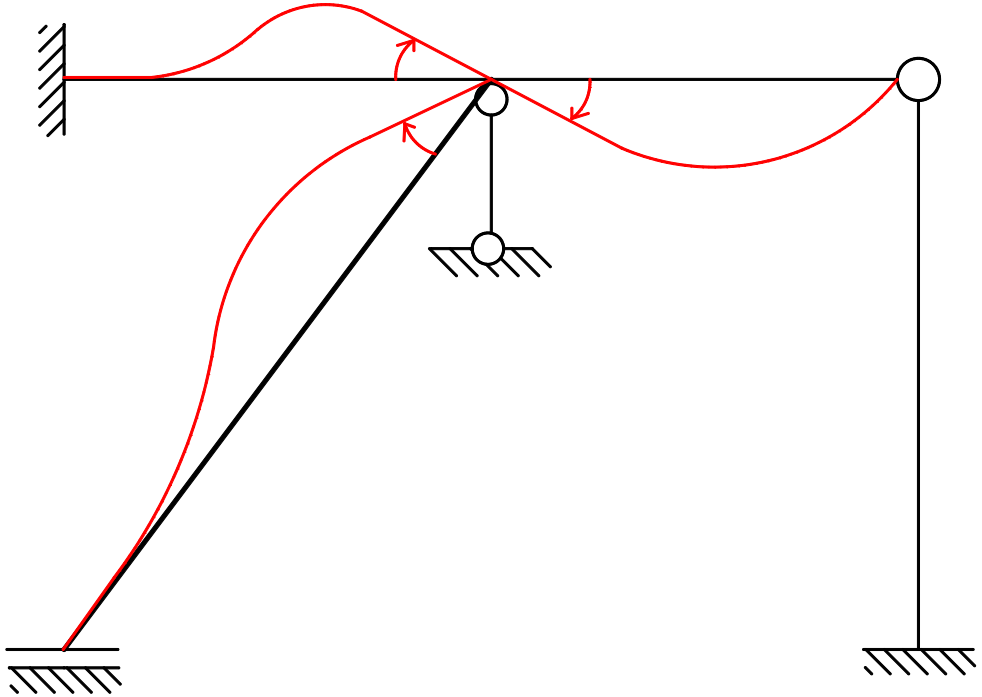

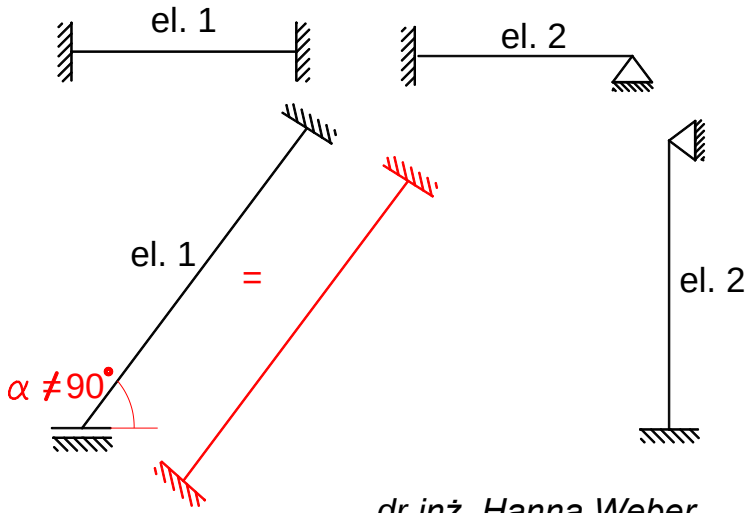

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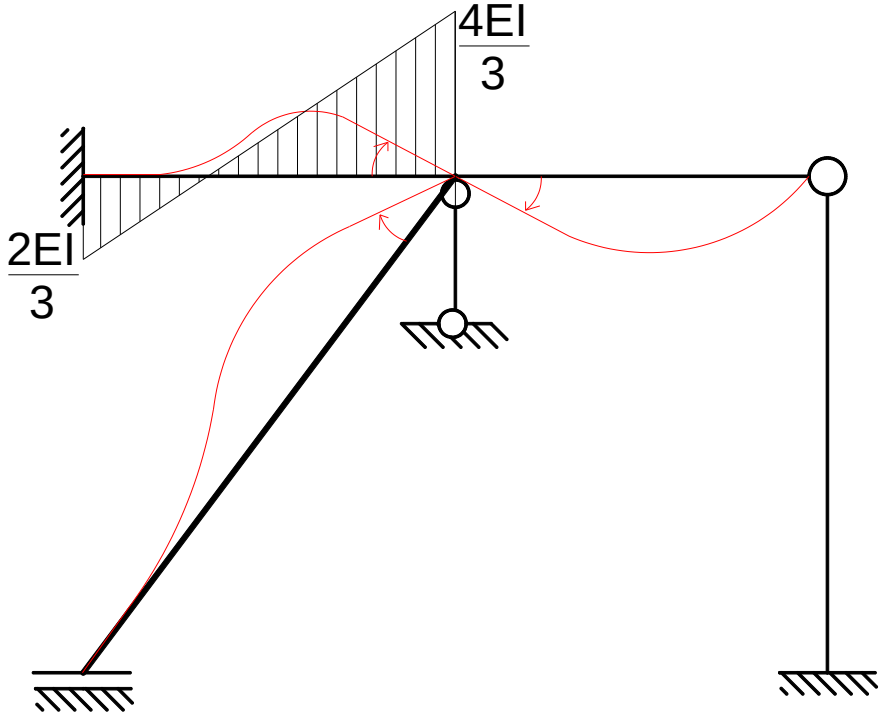

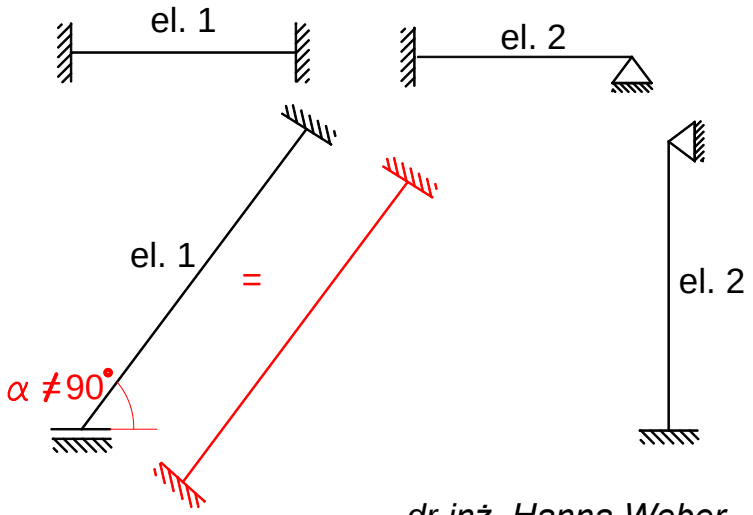

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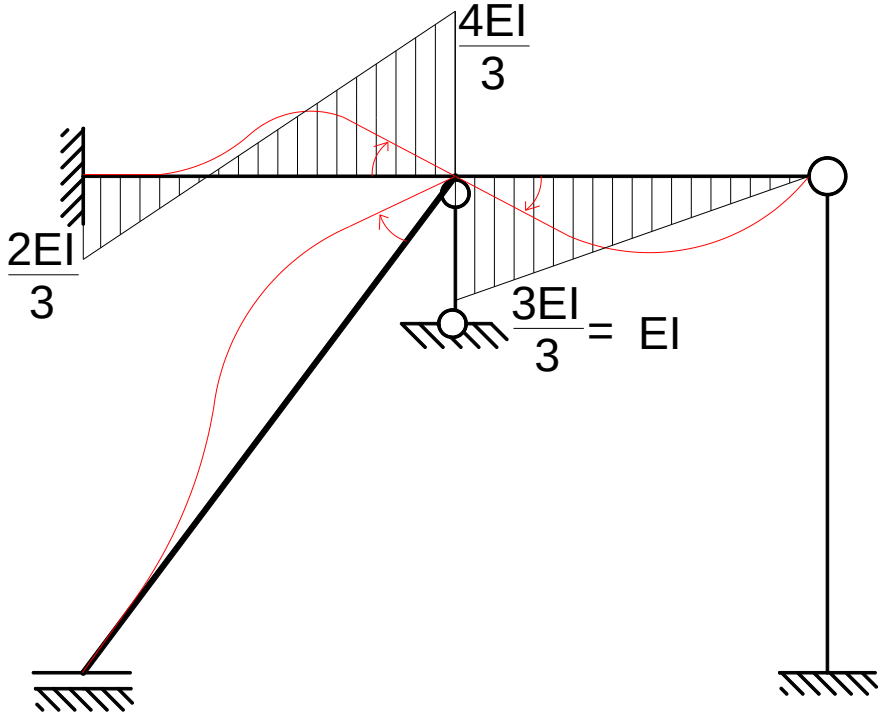

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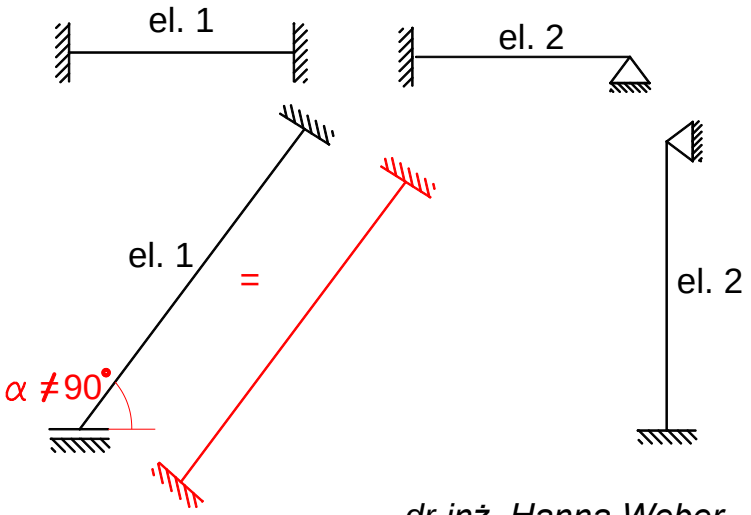

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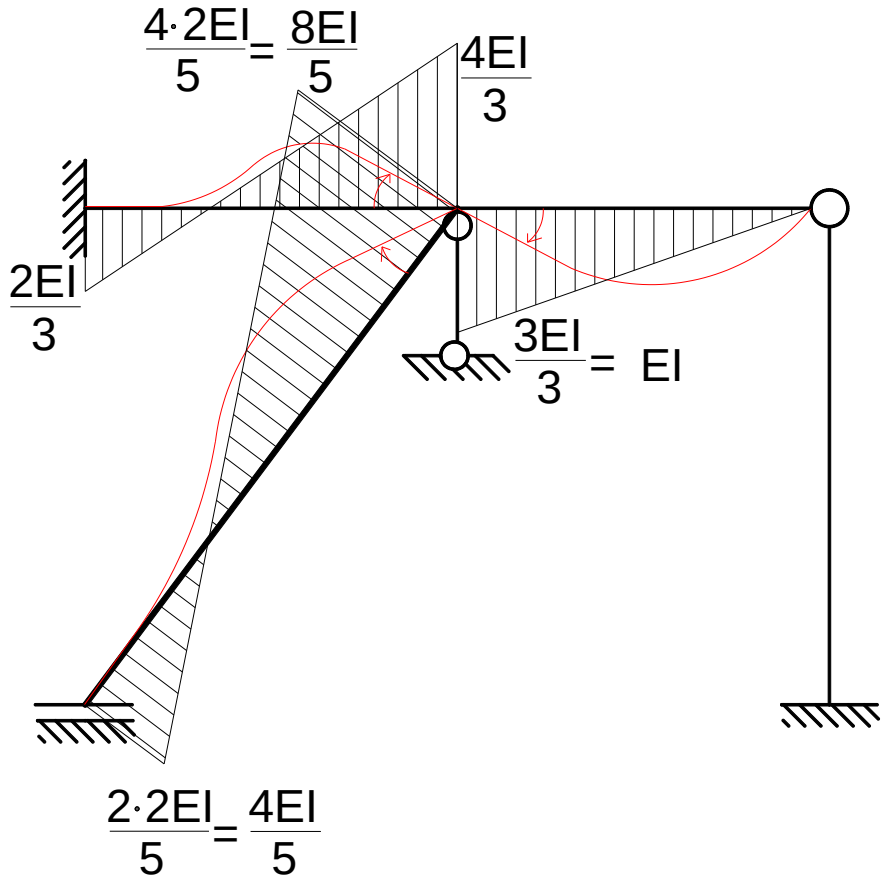

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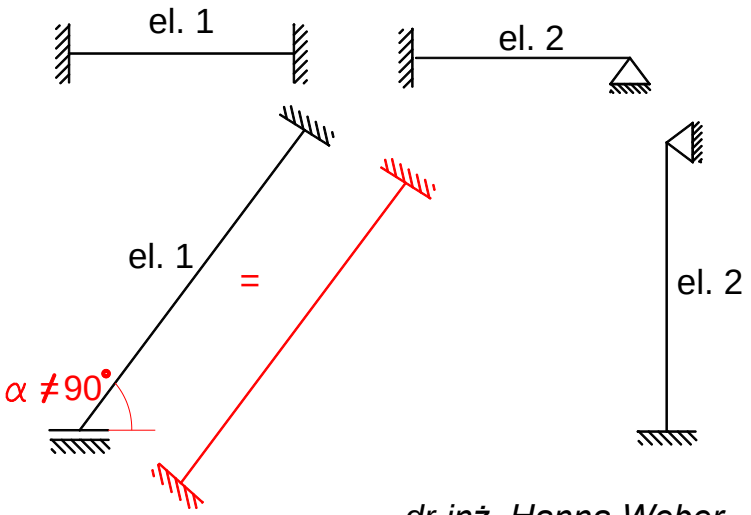

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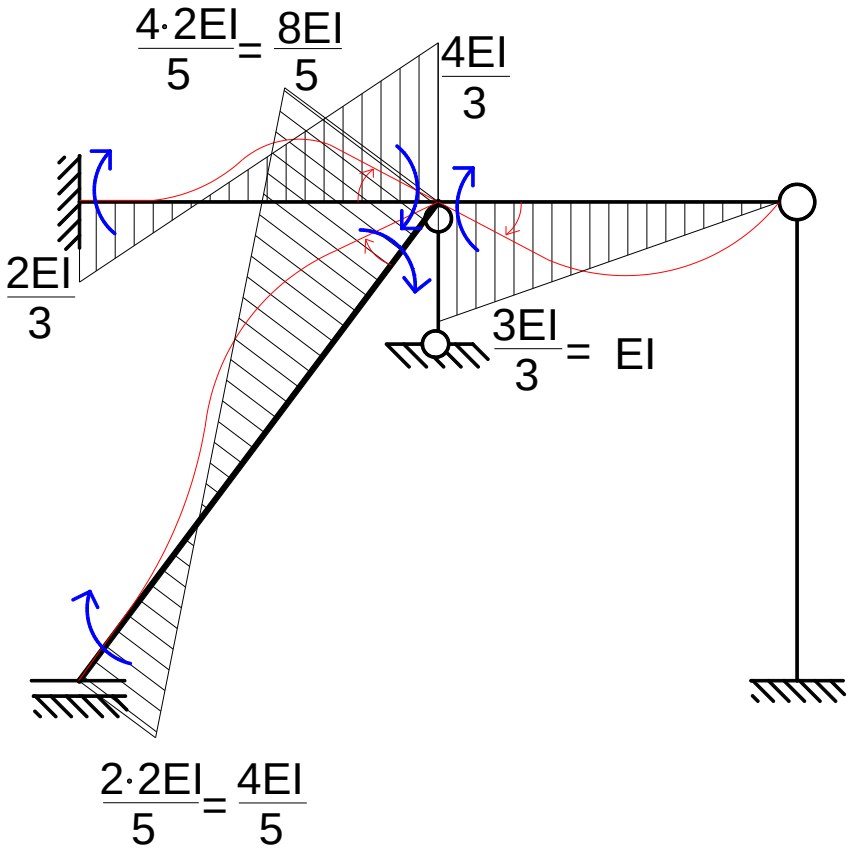

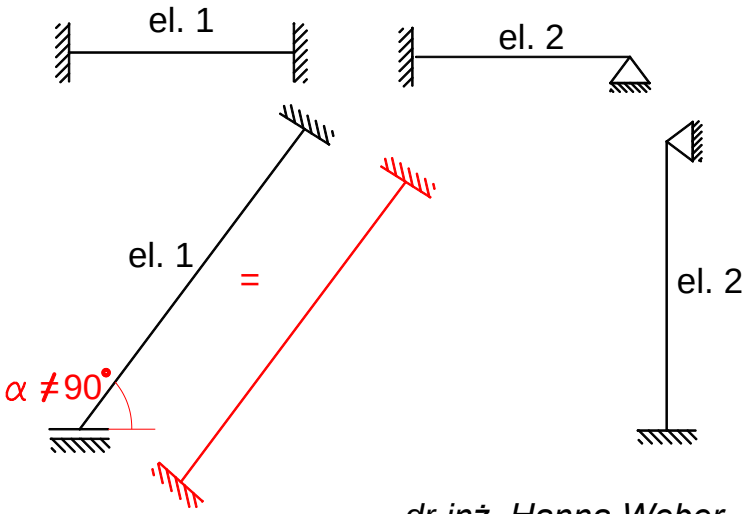

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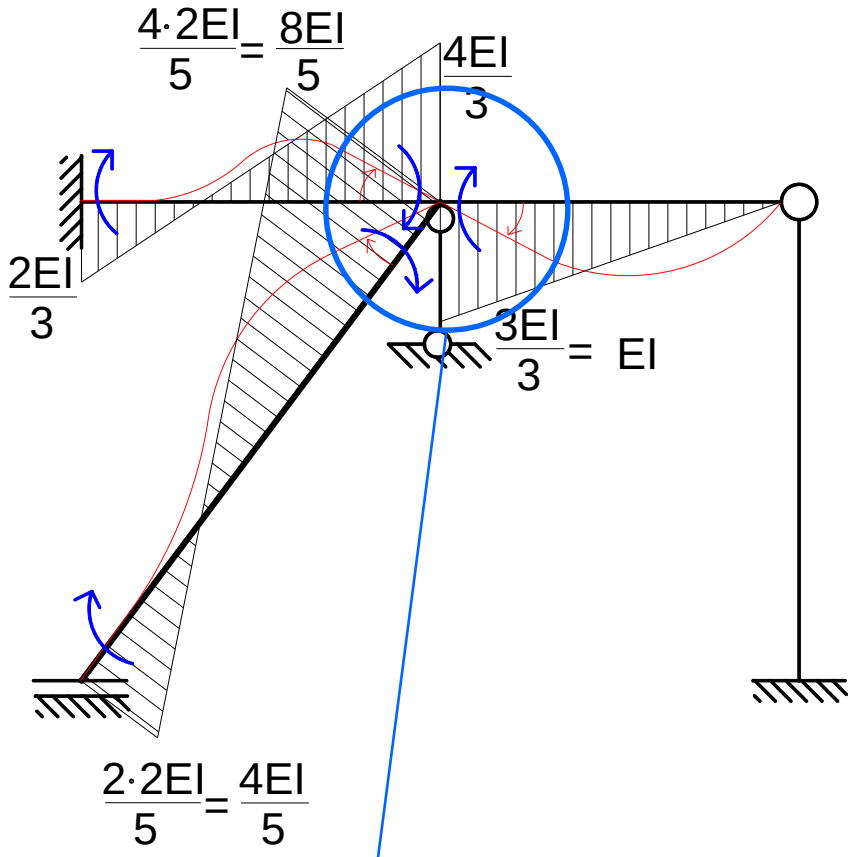
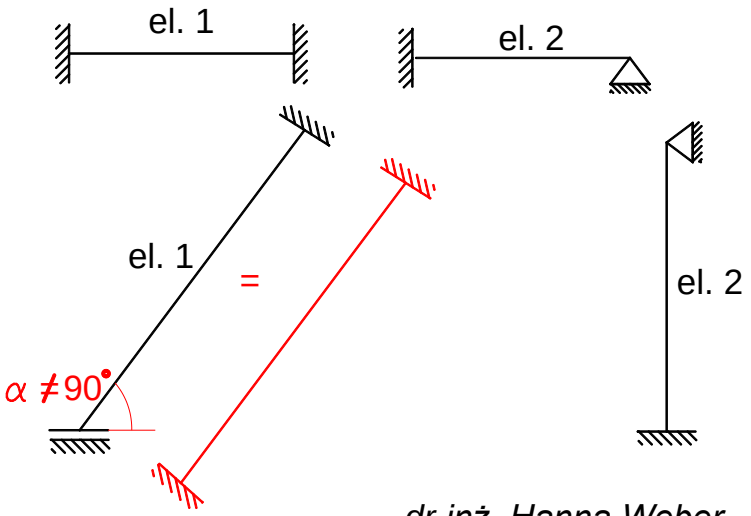


$$k_{11} = \frac{4EI}{3} + \frac{8EI}{5} + EI = \frac{59EI}{15}$$



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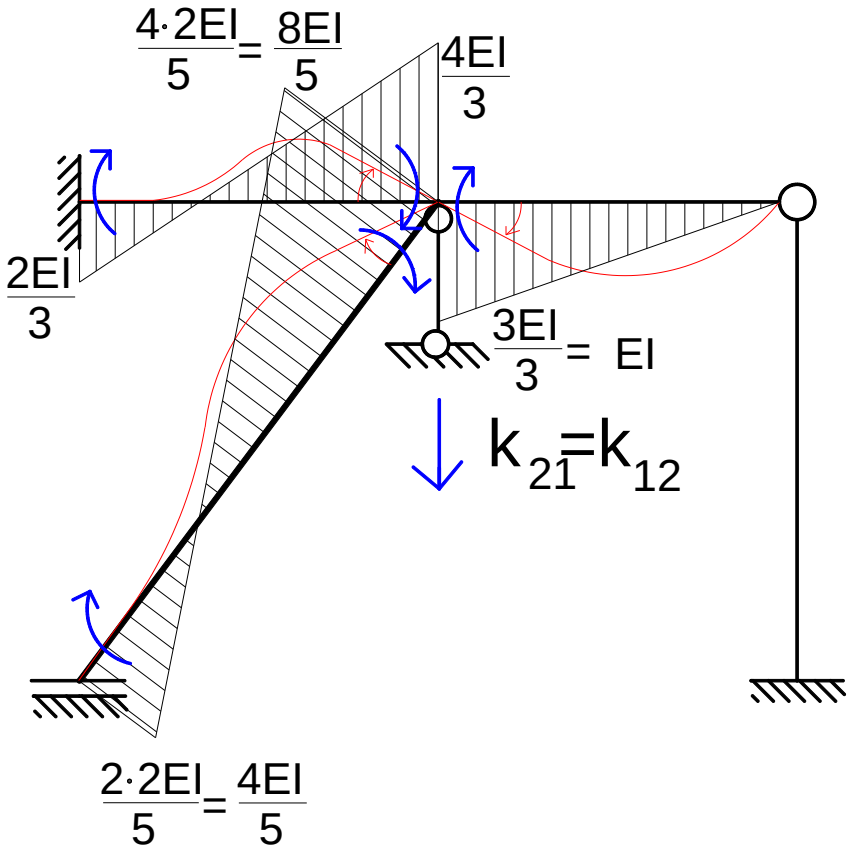

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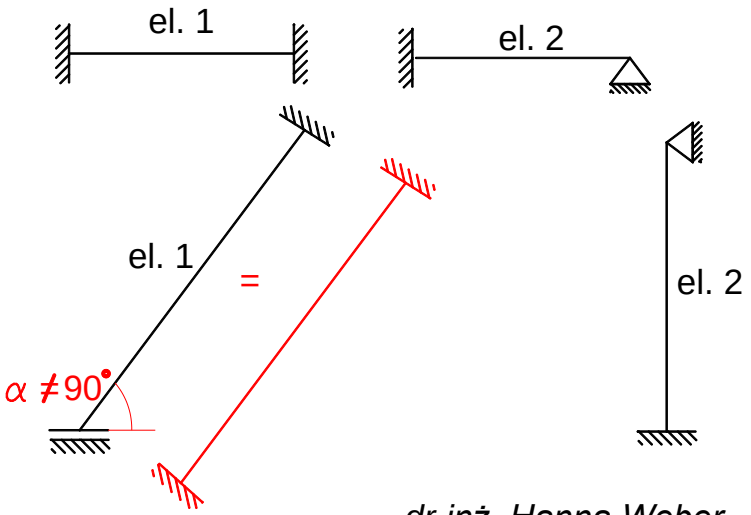

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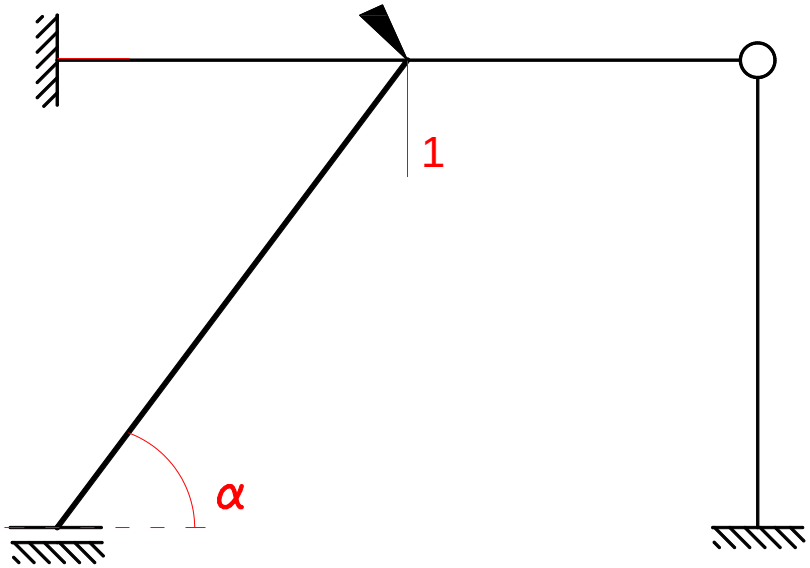

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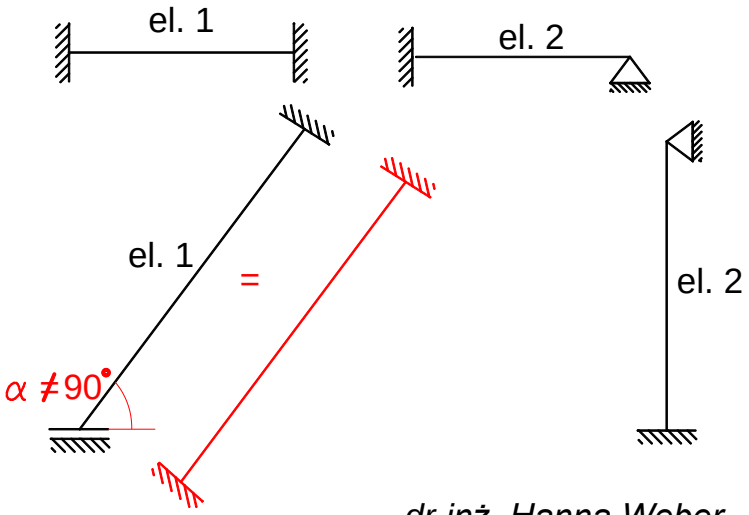

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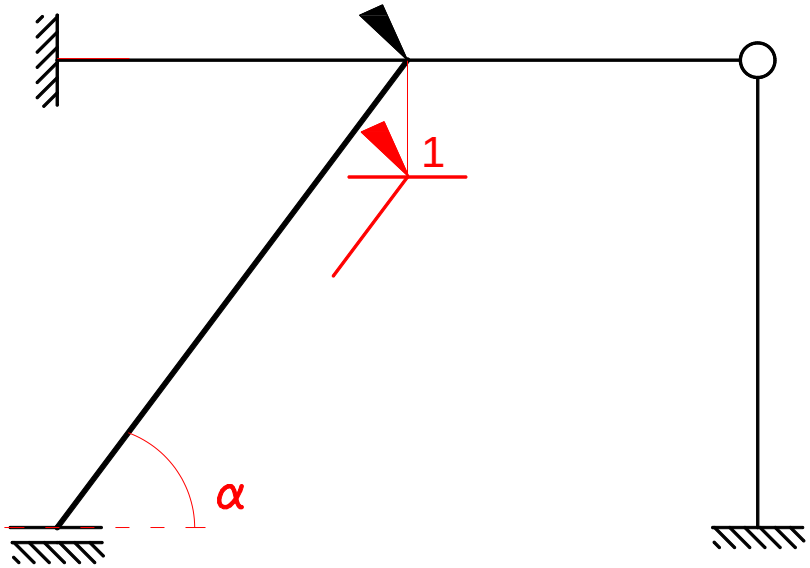

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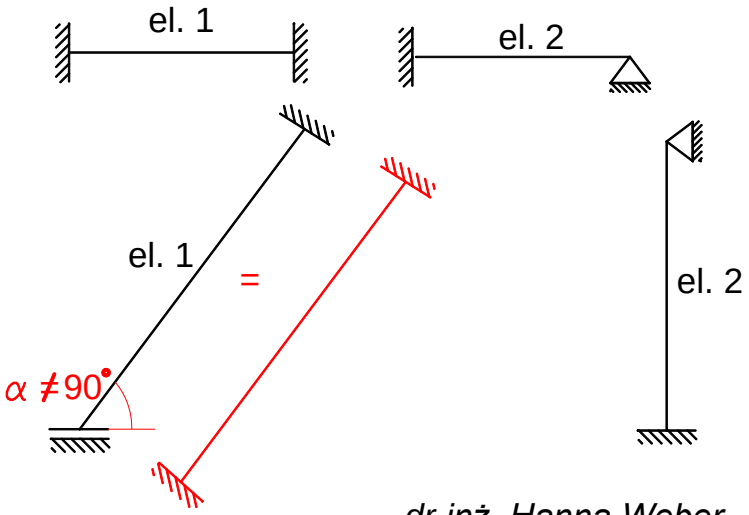

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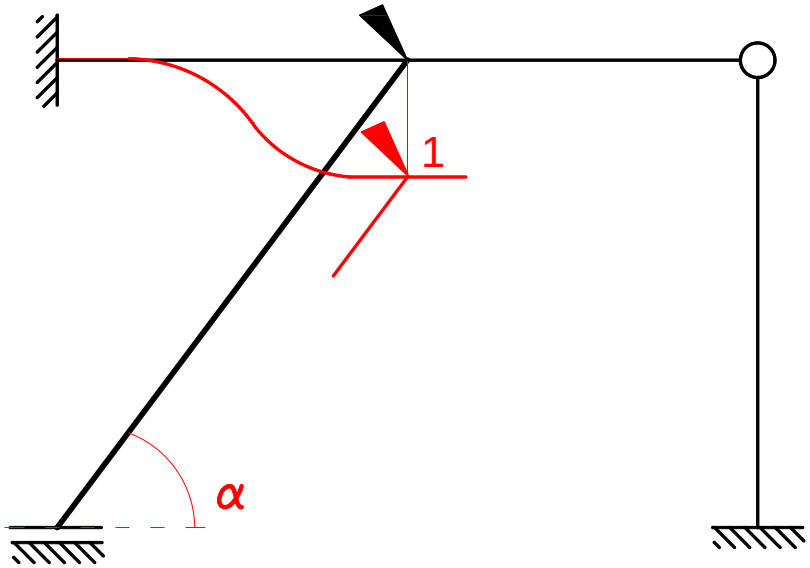

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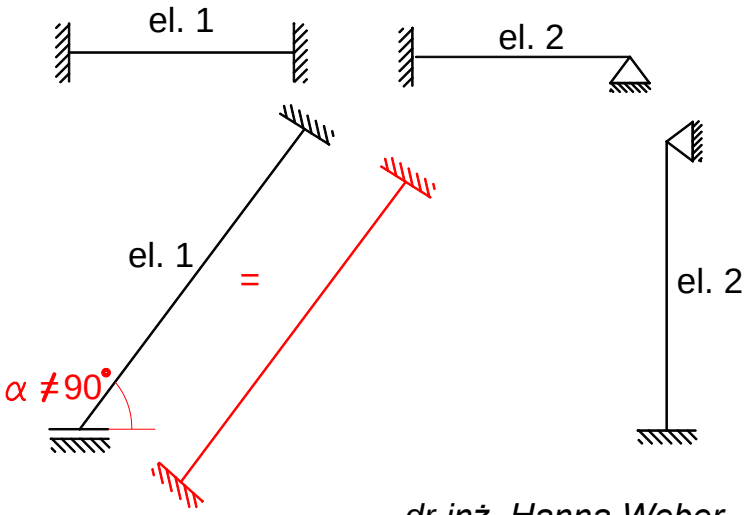

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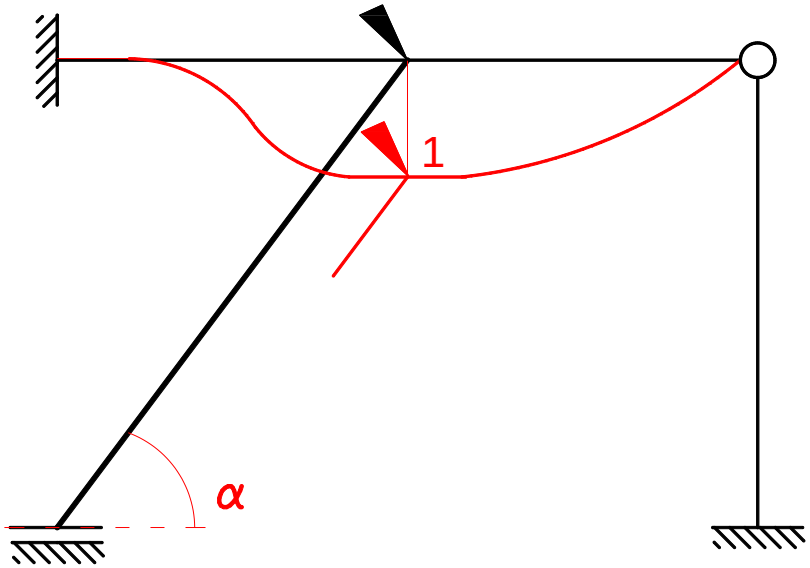

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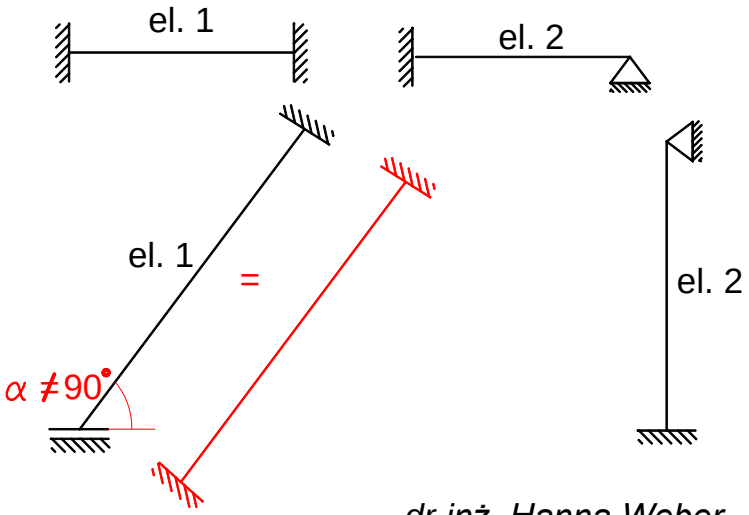

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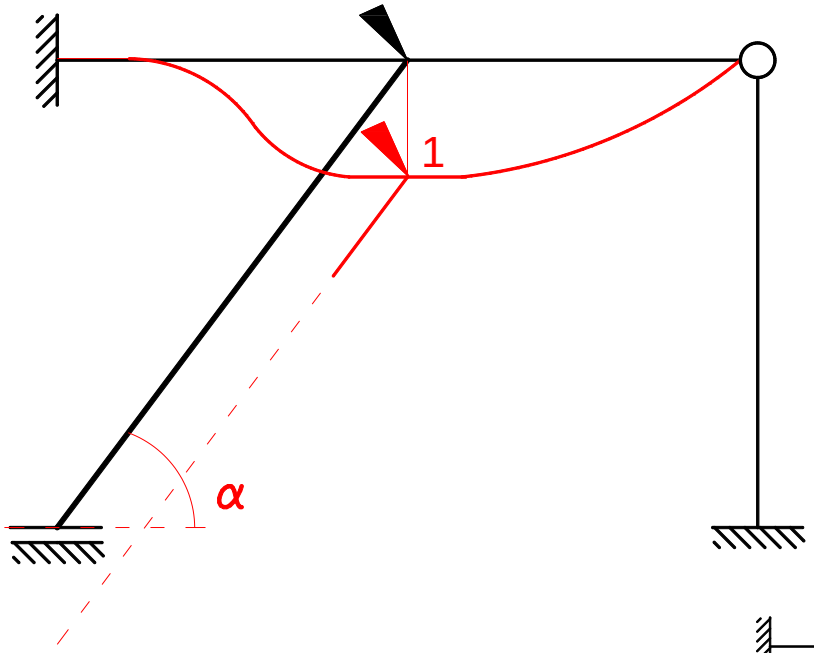

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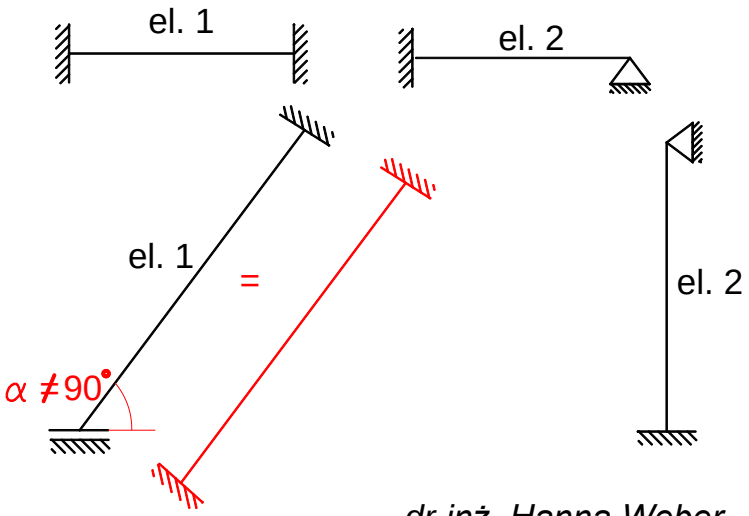

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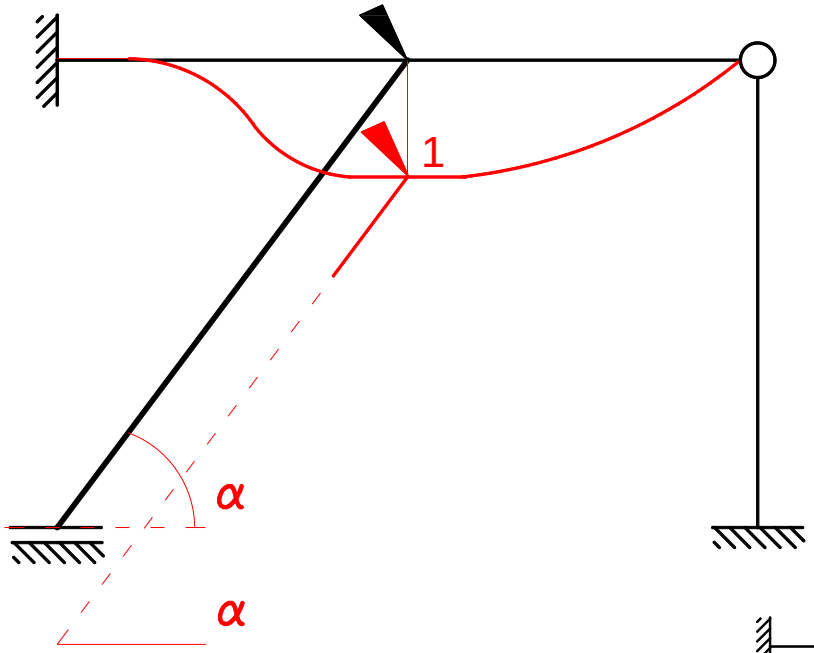

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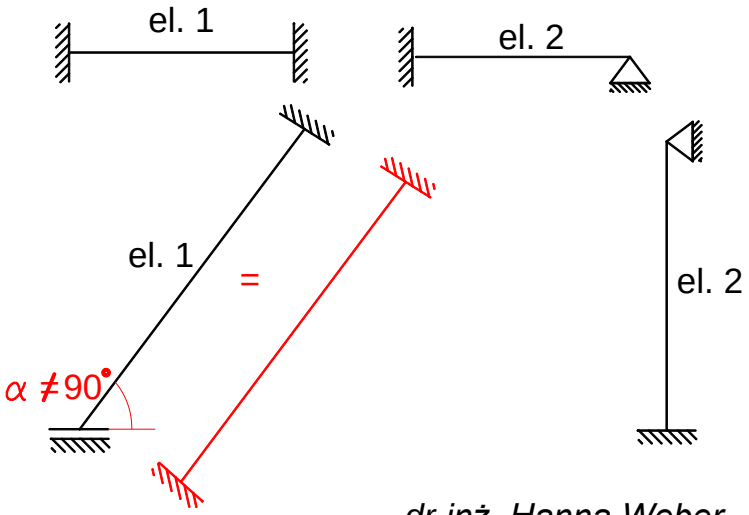

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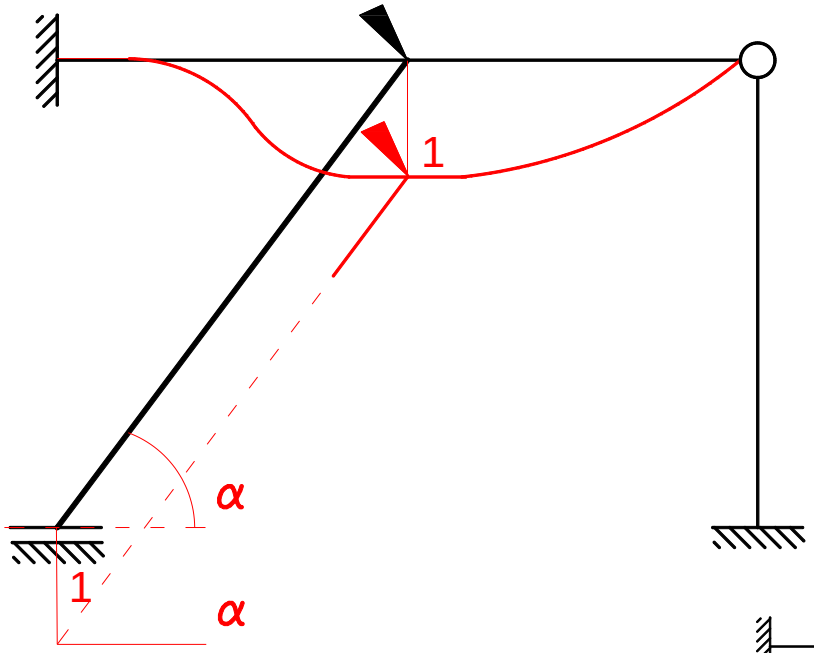

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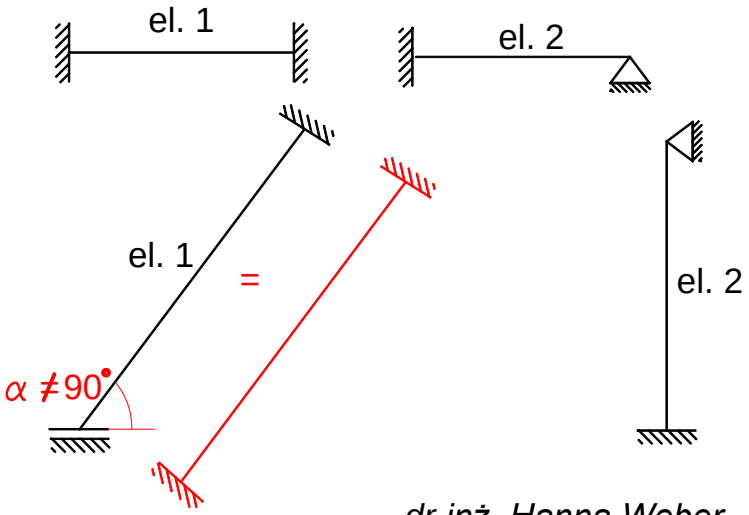

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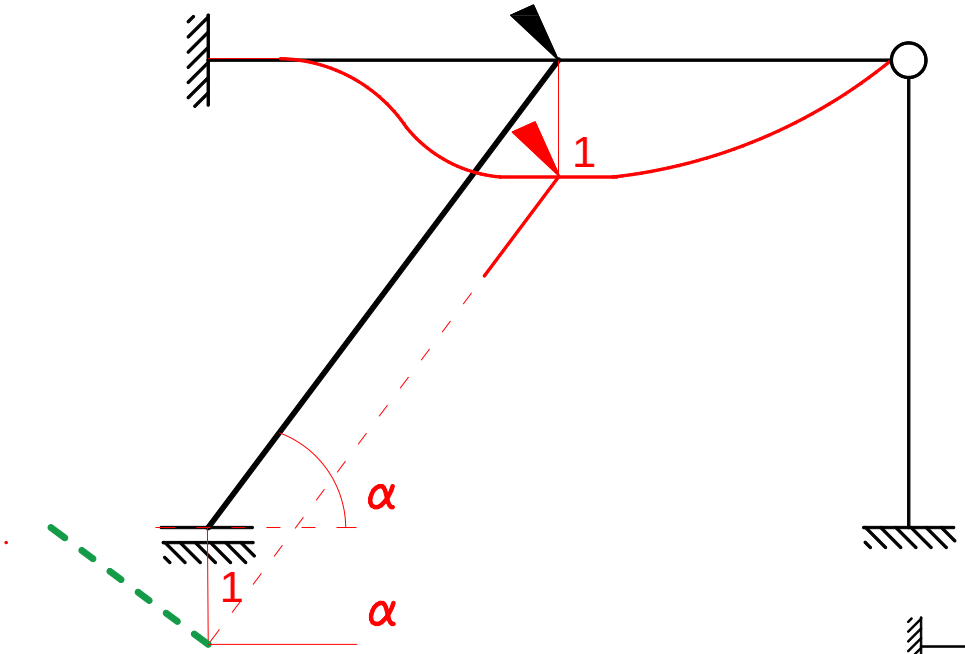

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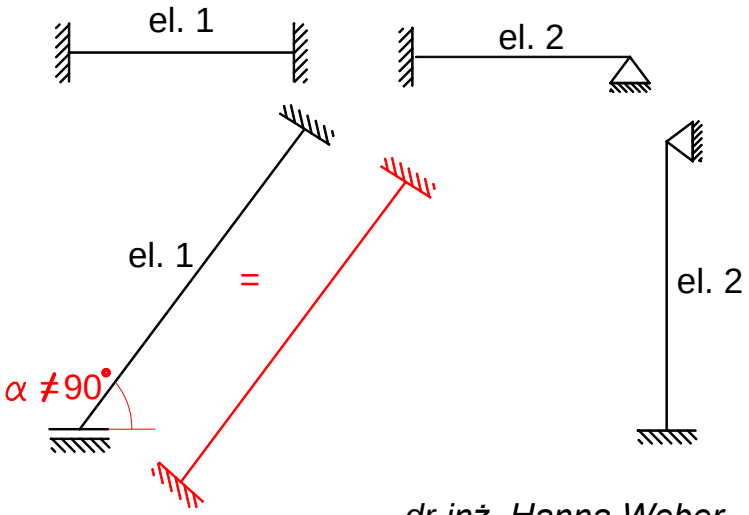

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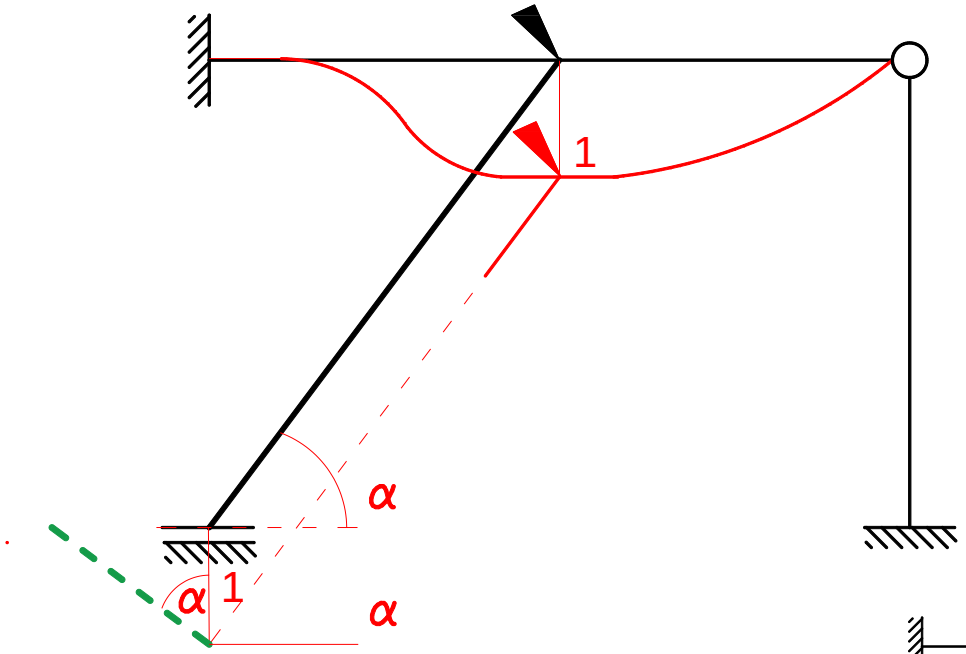

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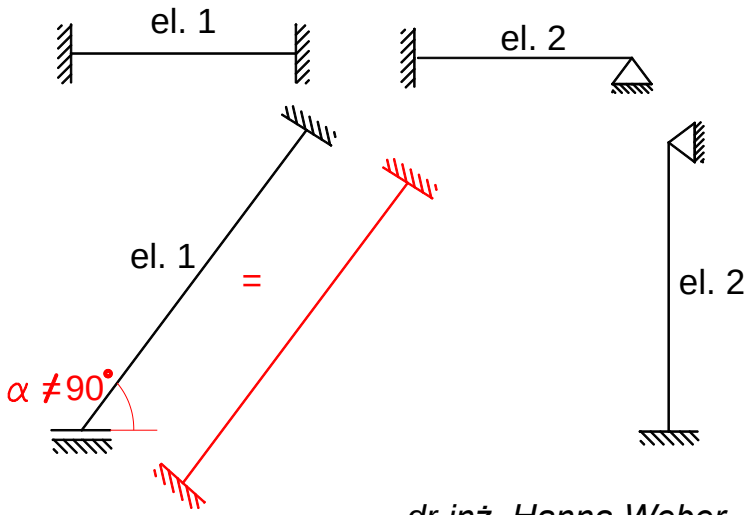

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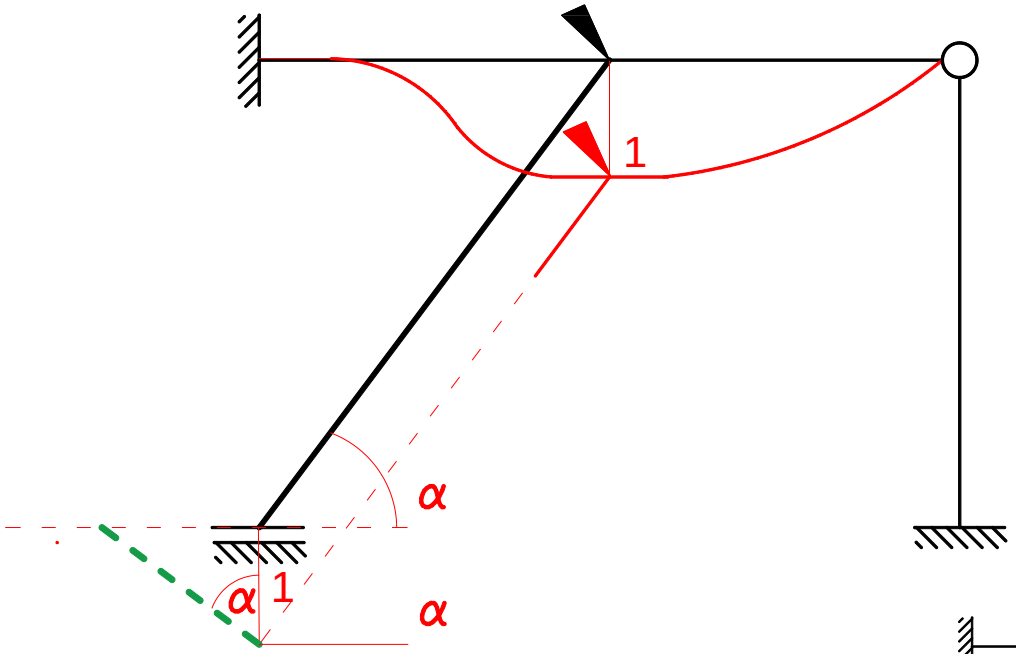

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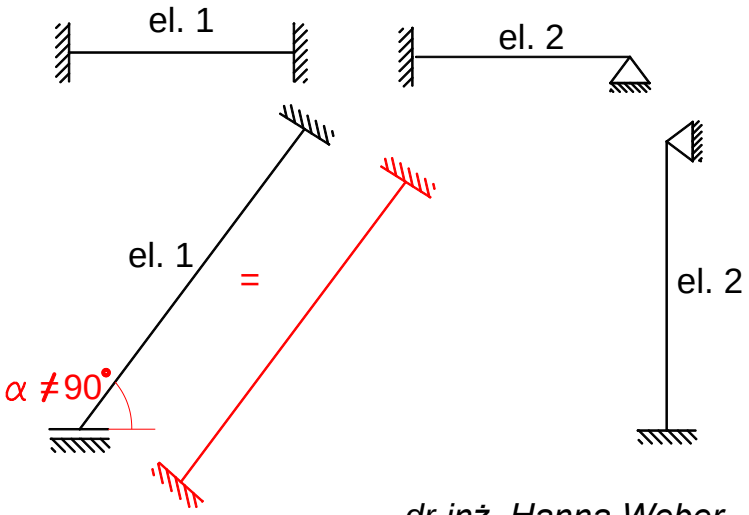

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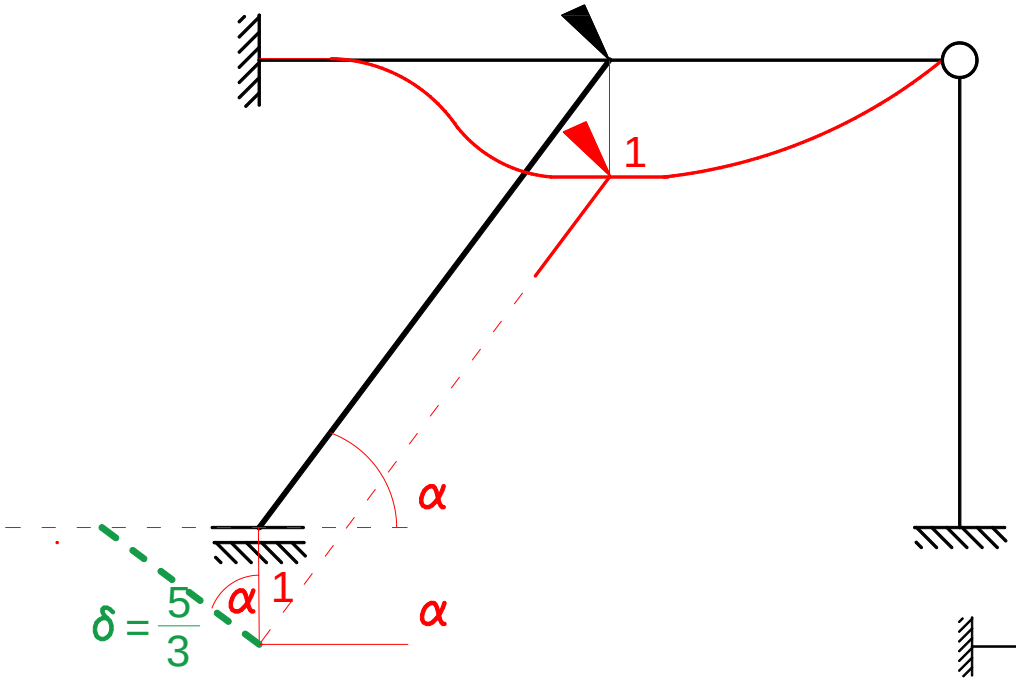

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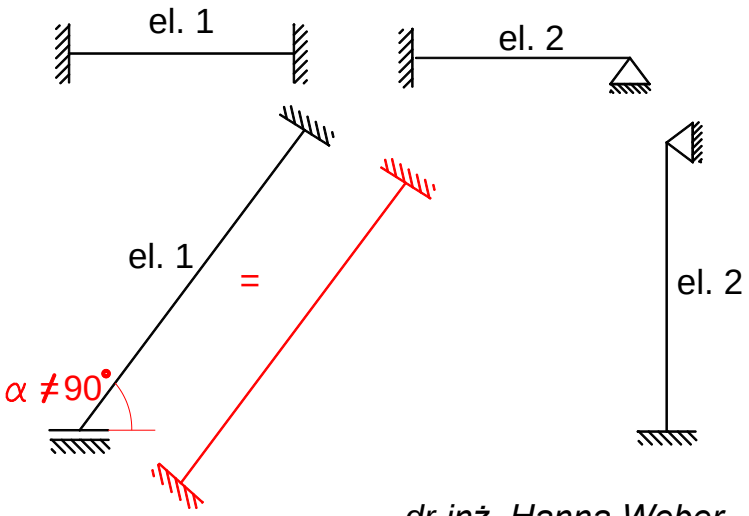

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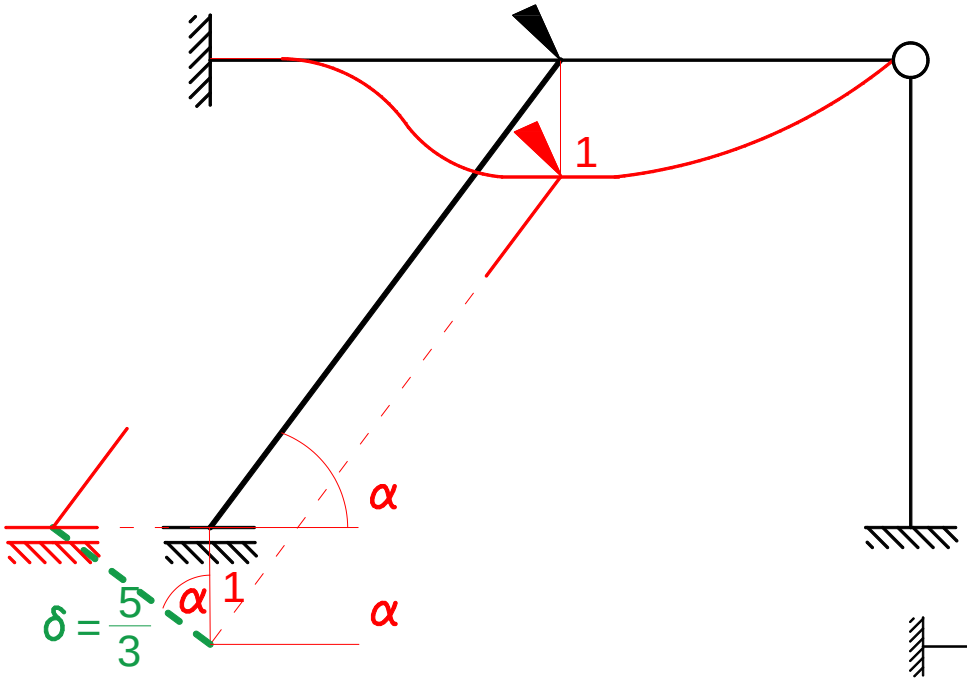

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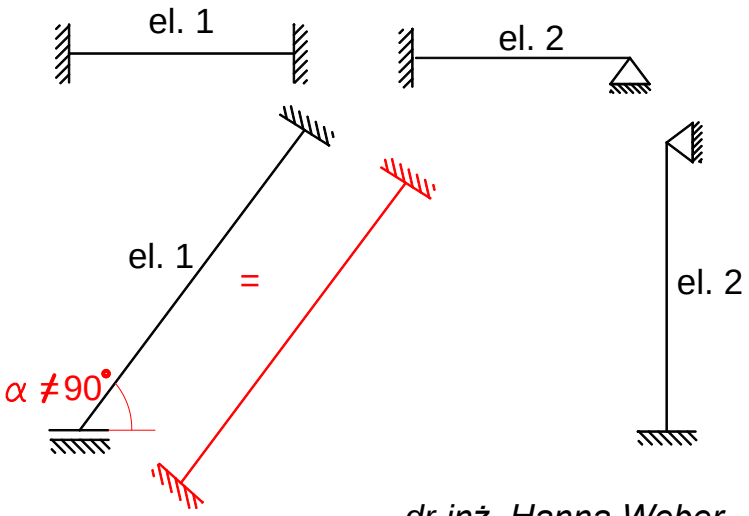

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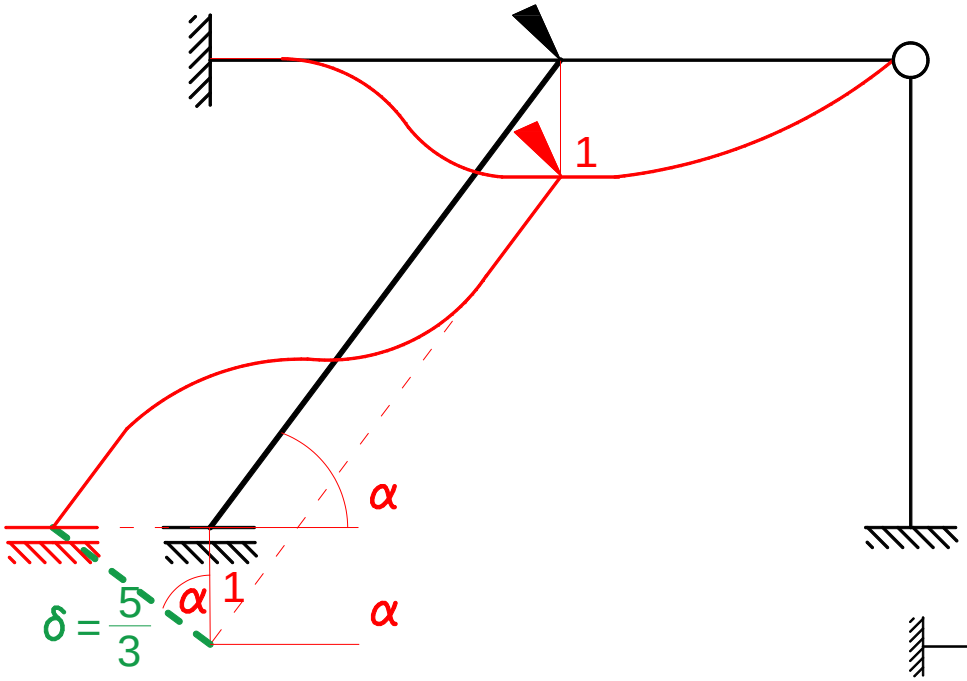

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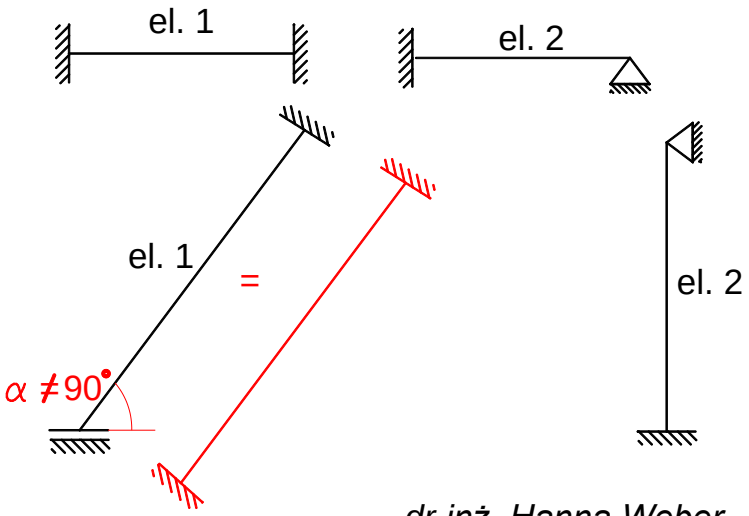

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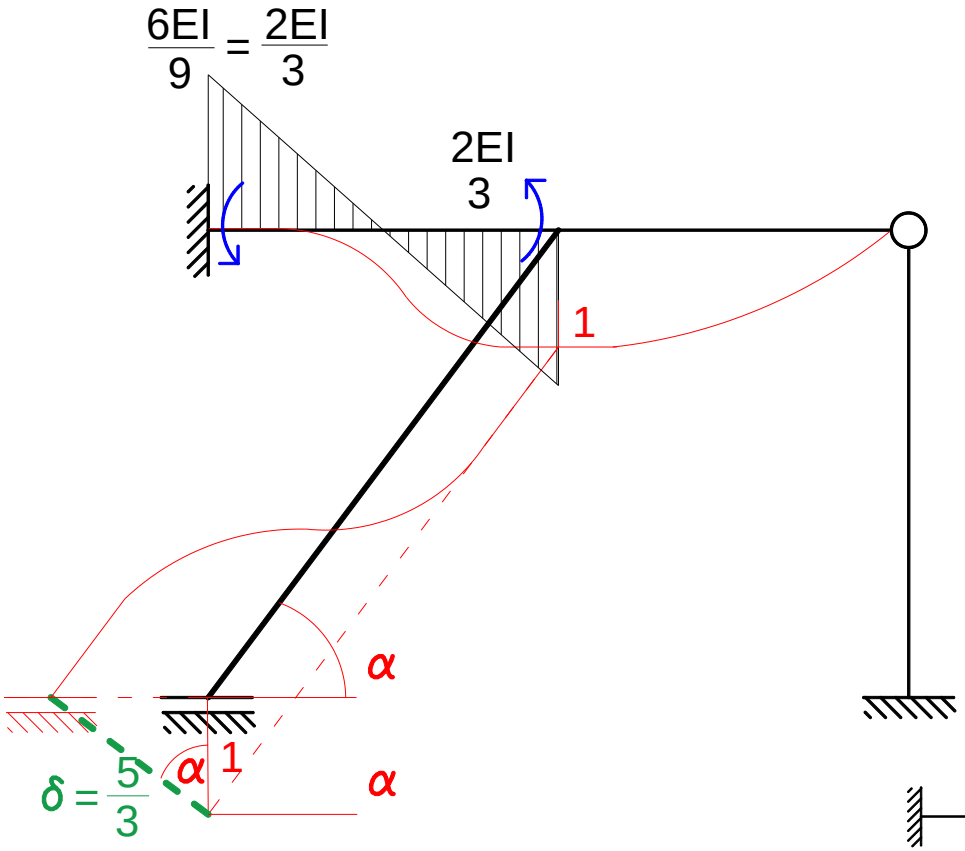

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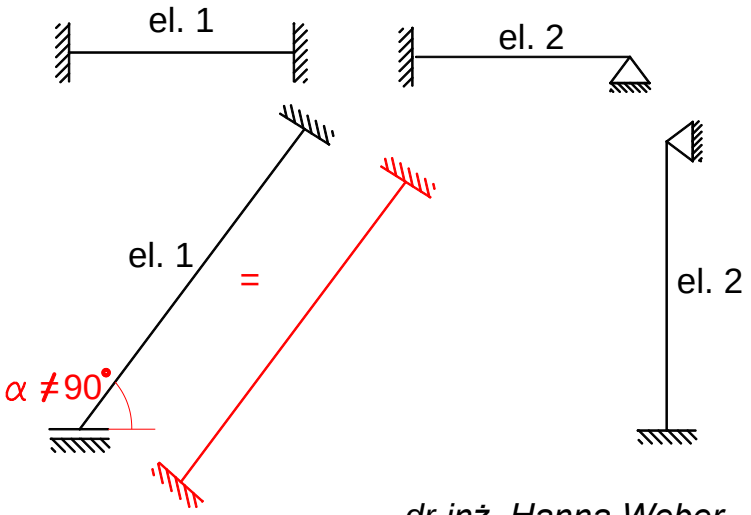

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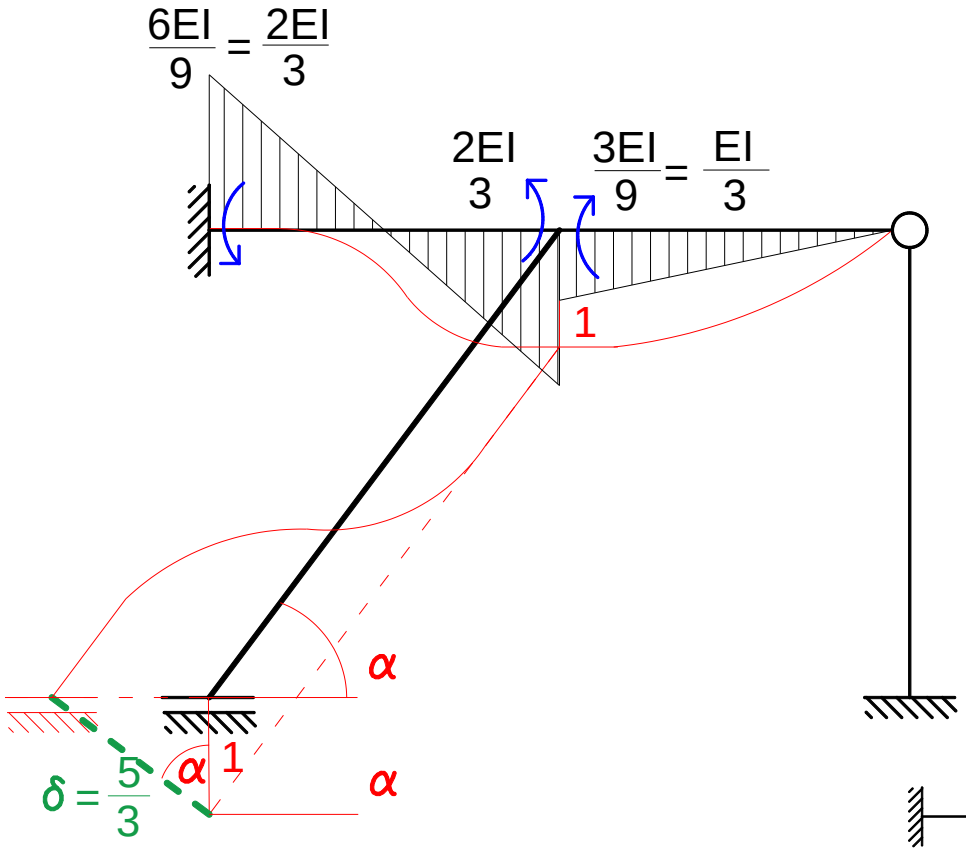

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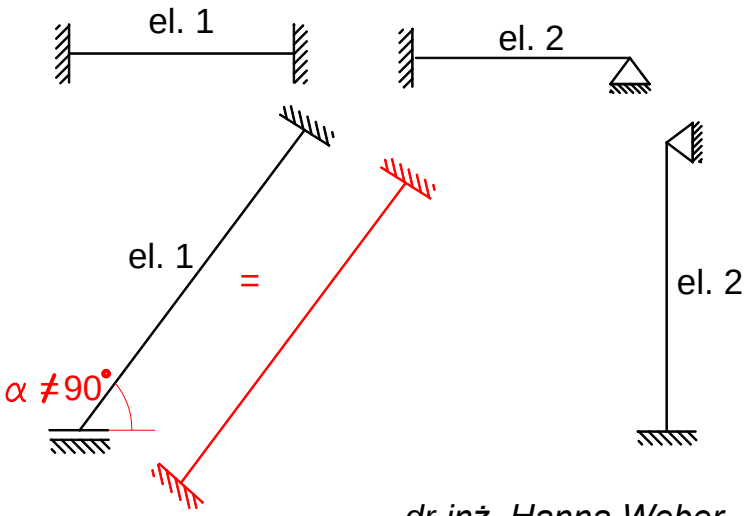

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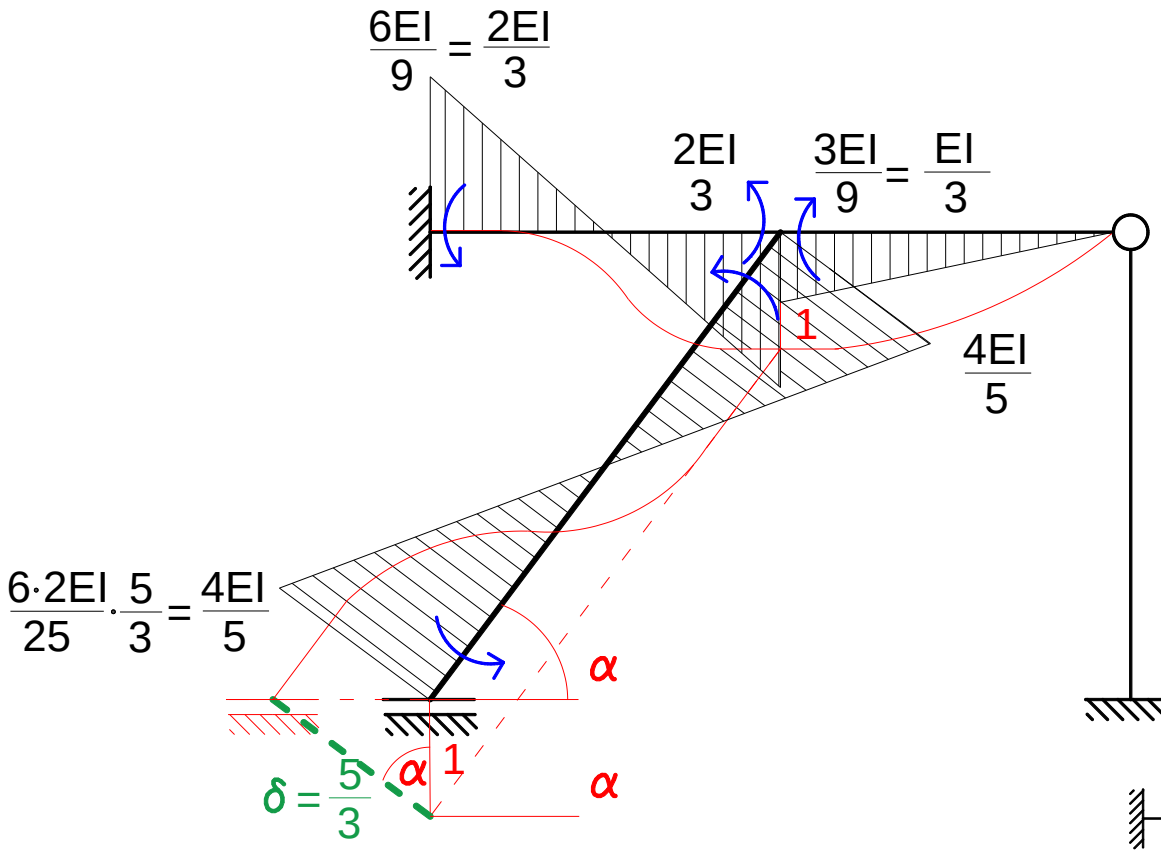

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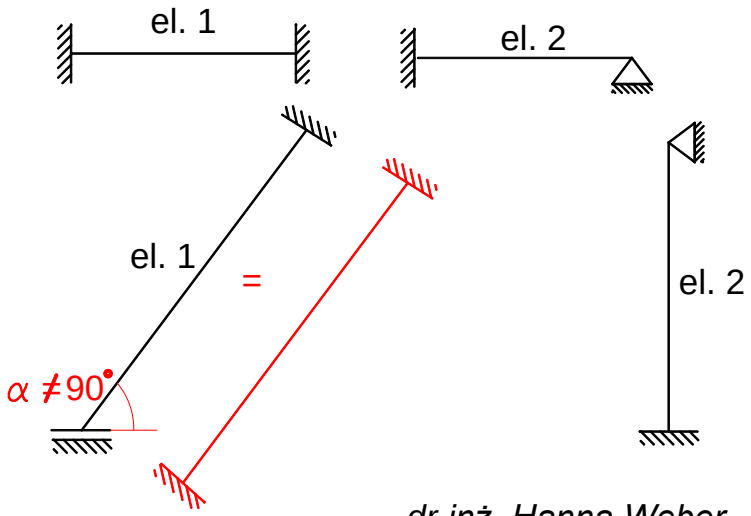

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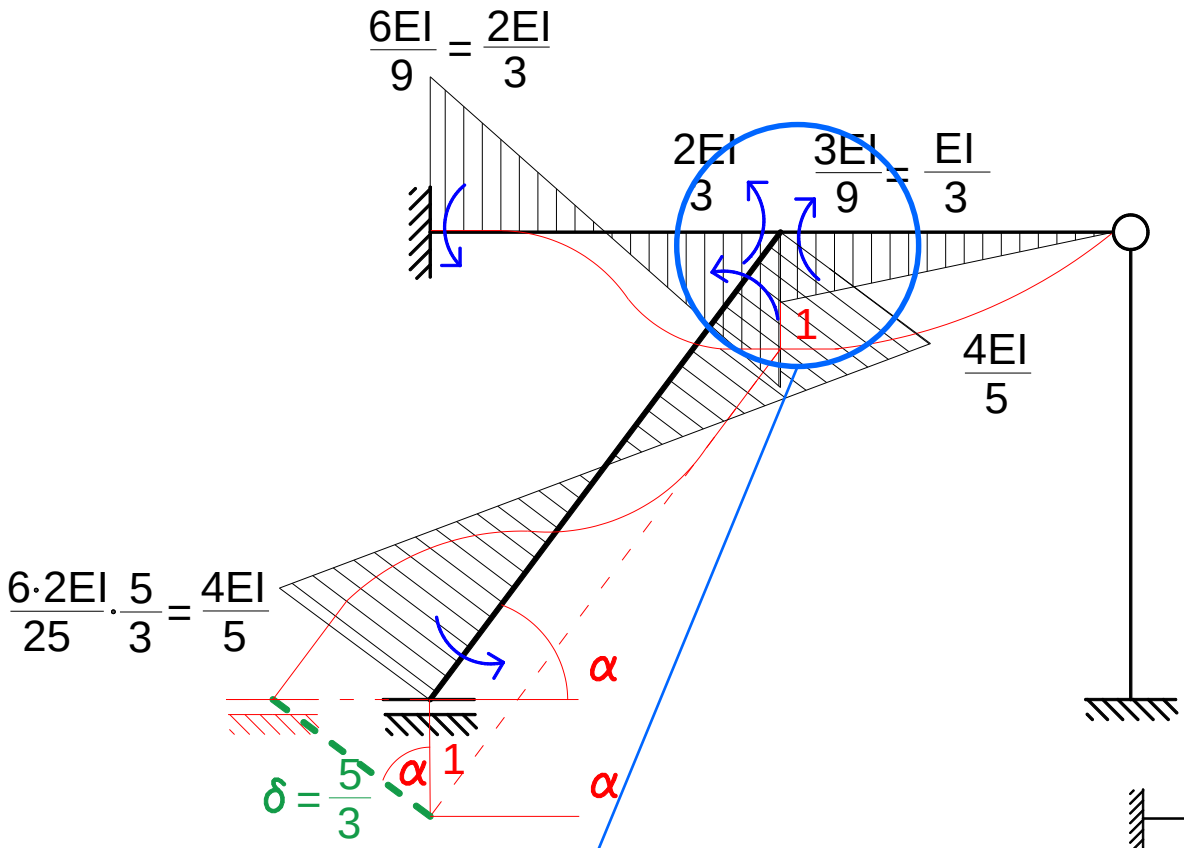

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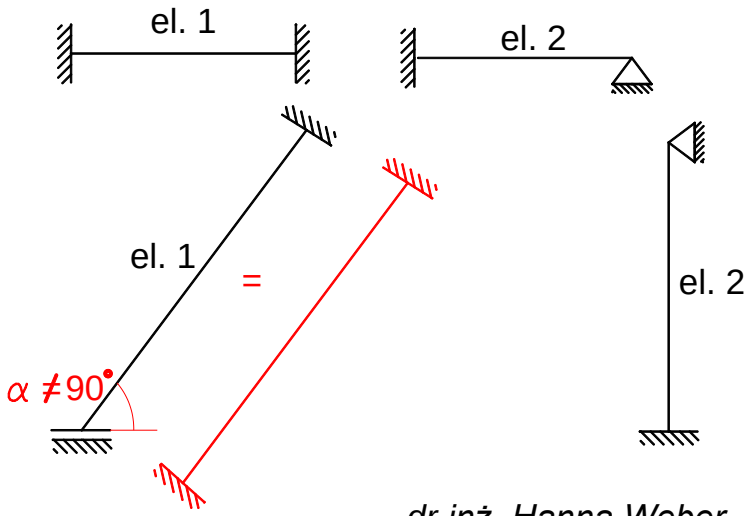
$$k_{12} = -\frac{2EI}{3} + \frac{EI}{3} - \frac{4EI}{5} = -\frac{17EI}{15}$$



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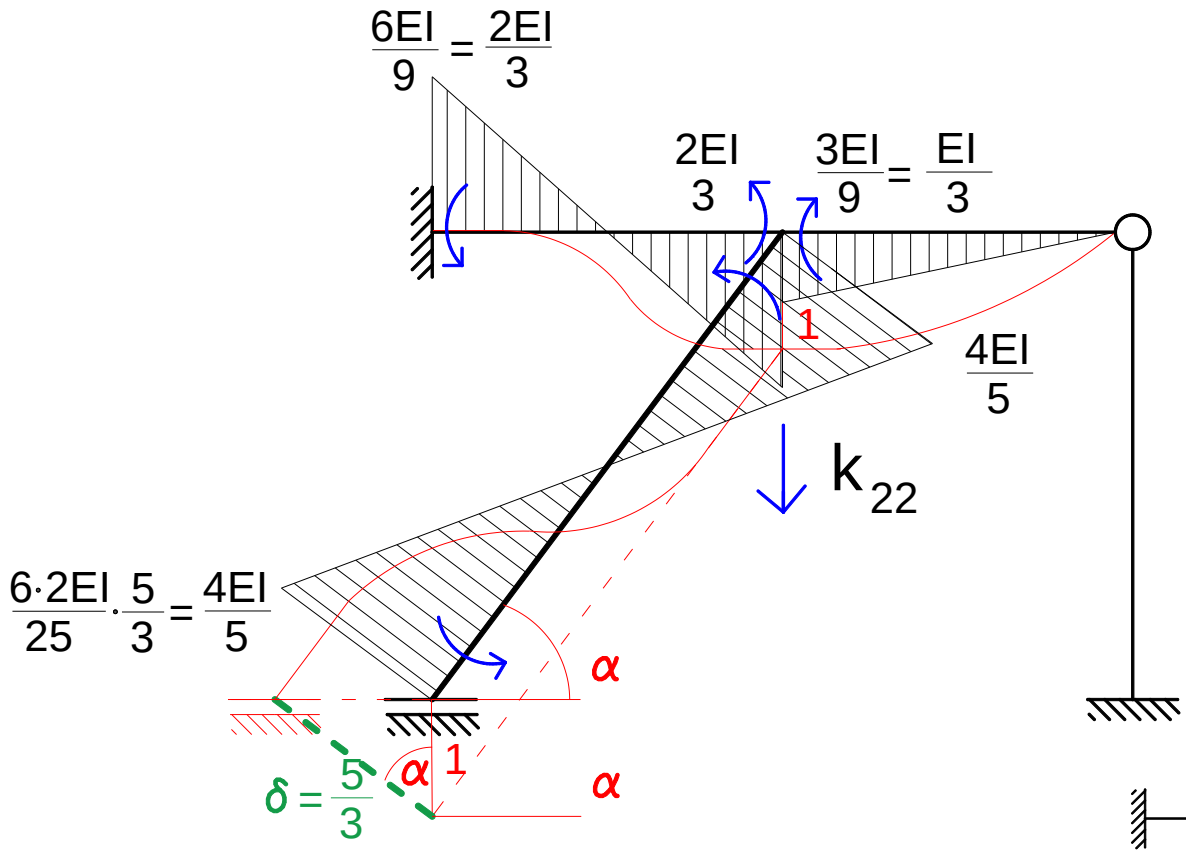
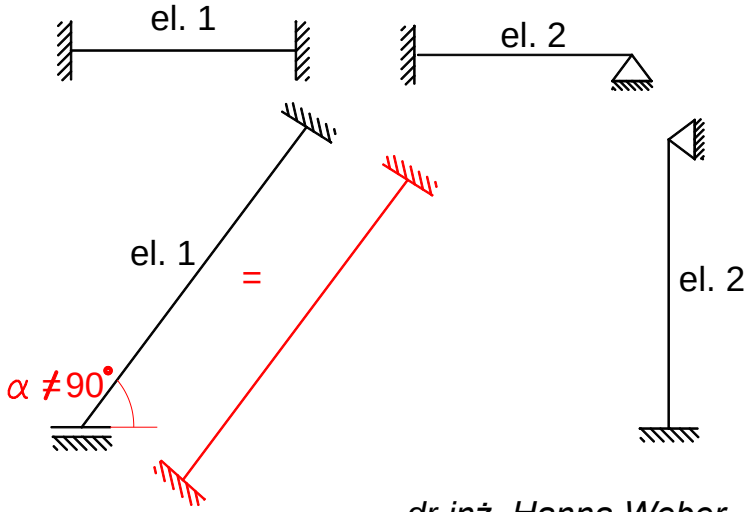
Stan $\Delta_2=1$





$$k_{11} = \frac{4EI}{3} + \frac{8EI}{5} + EI = \frac{59EI}{15}$$

$$k_{12} = -\frac{2EI}{3} + \frac{EI}{3} - \frac{4EI}{5} = -\frac{17EI}{15}$$

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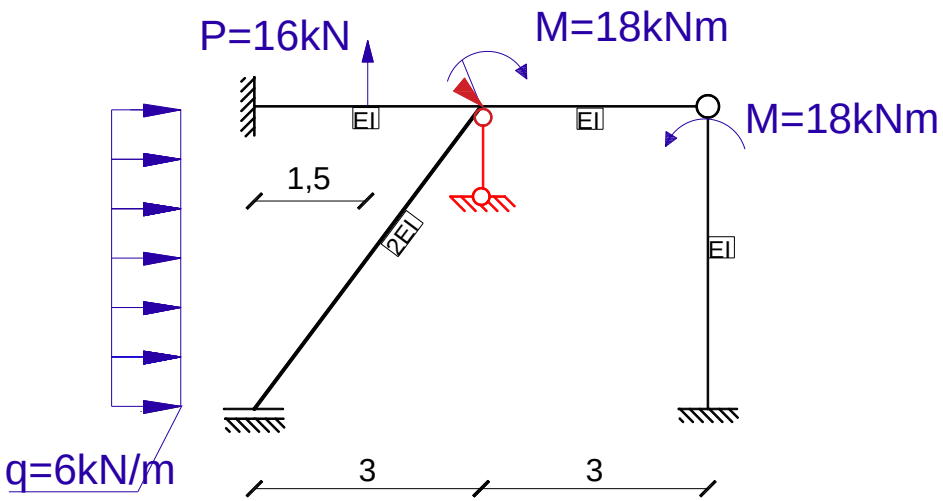
Obciążenie zewnętrzne



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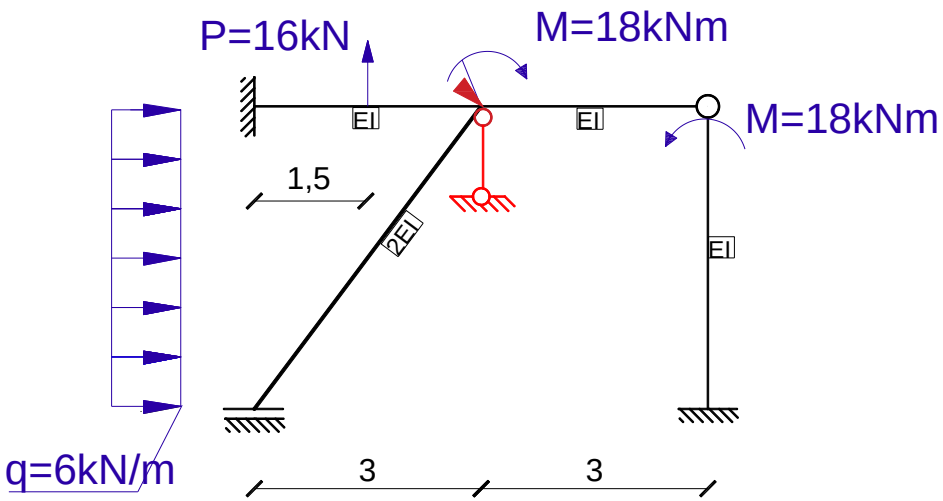
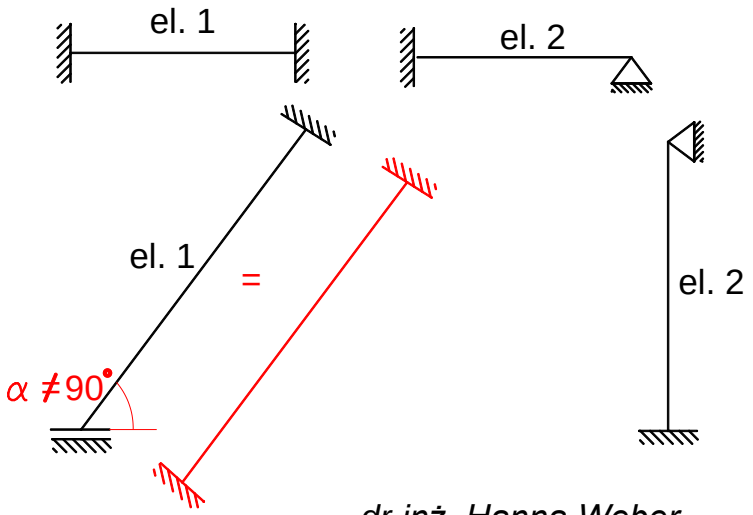
Obciążenie zewnętrzne





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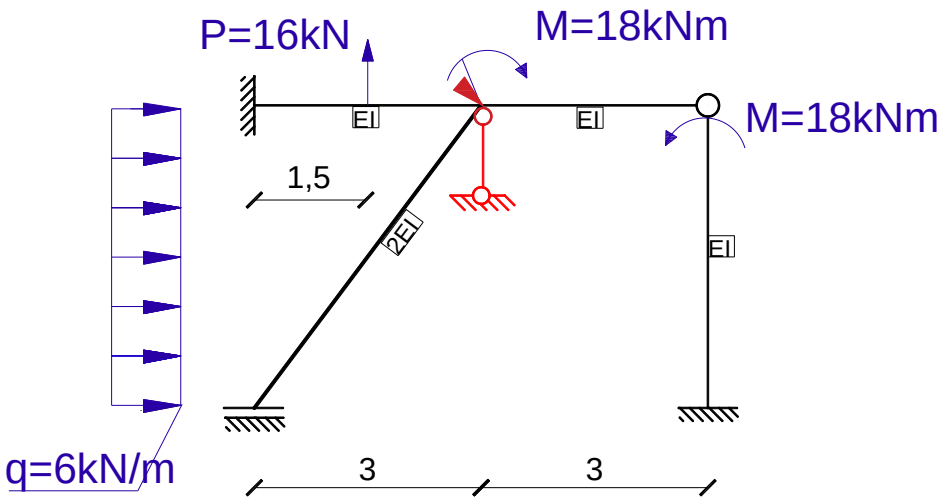
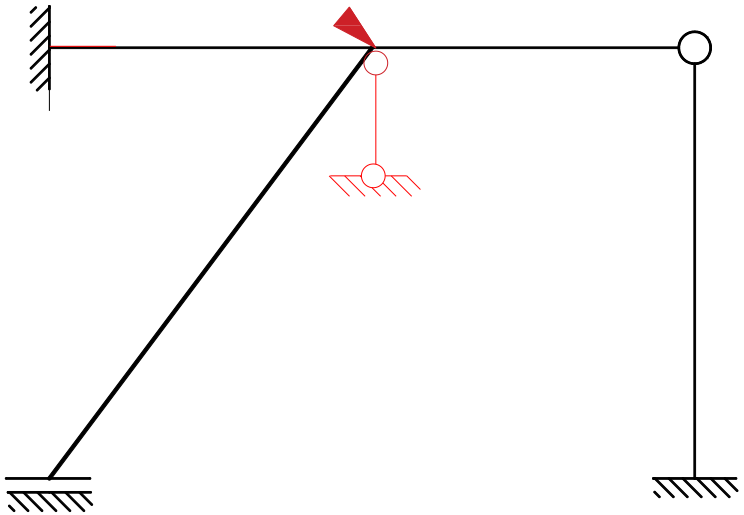
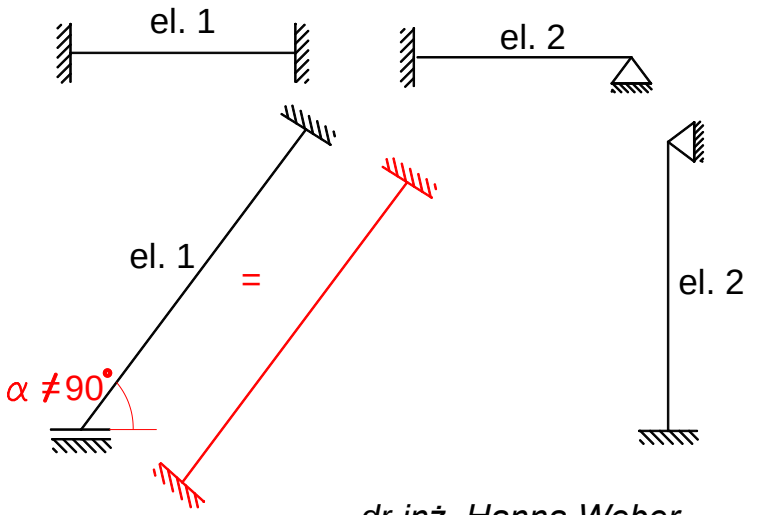
Obciążenie zewnętrzne







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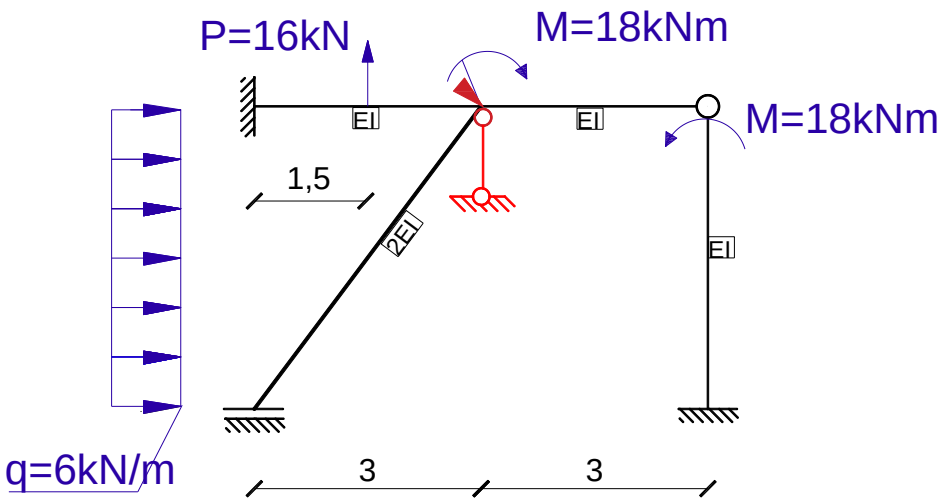
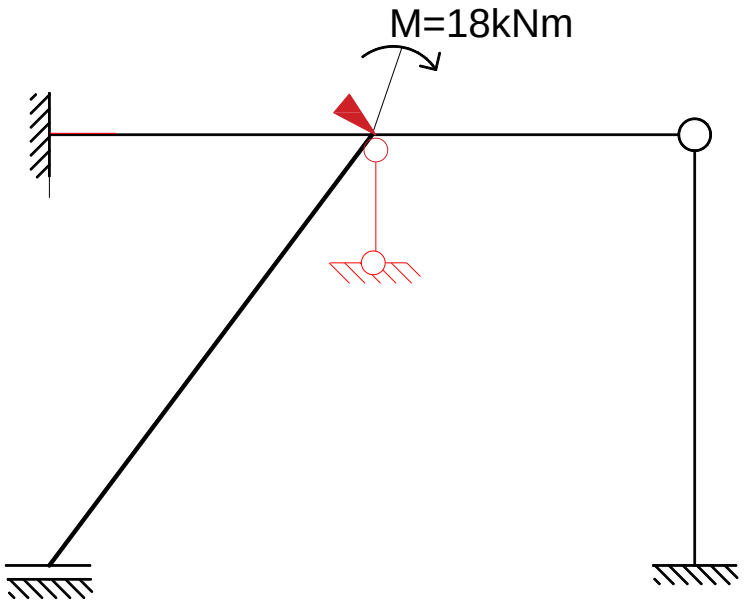
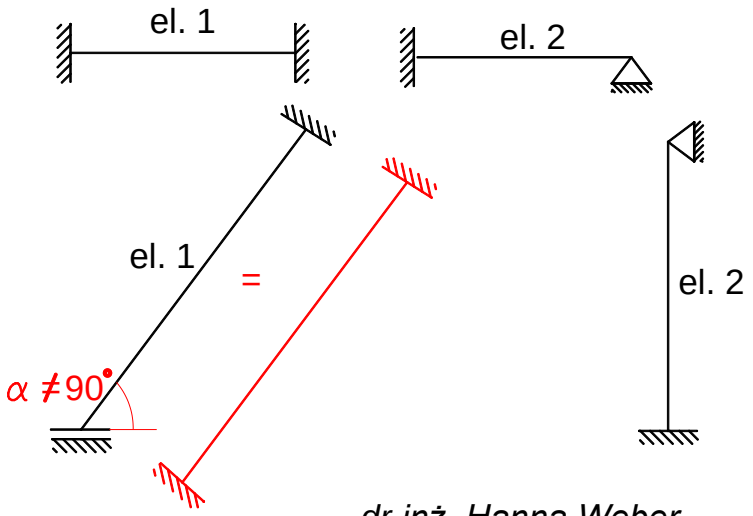
Obciążenie zewnętrzne







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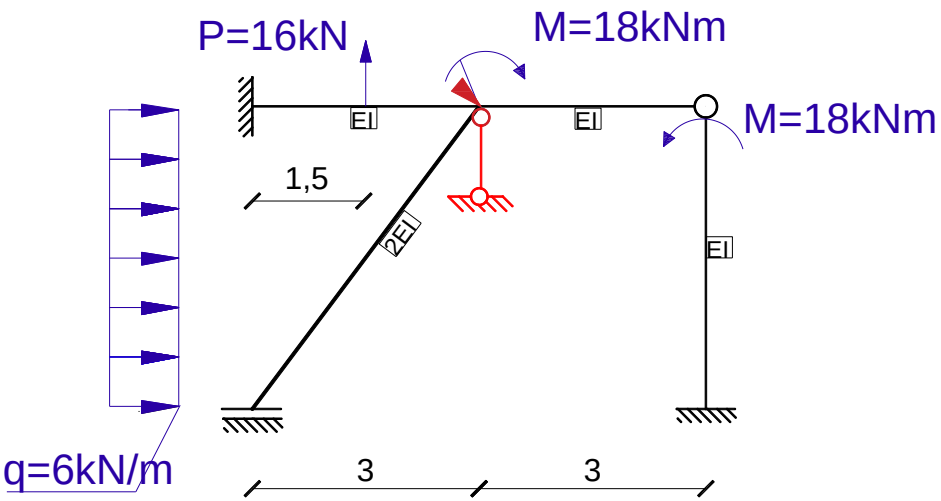
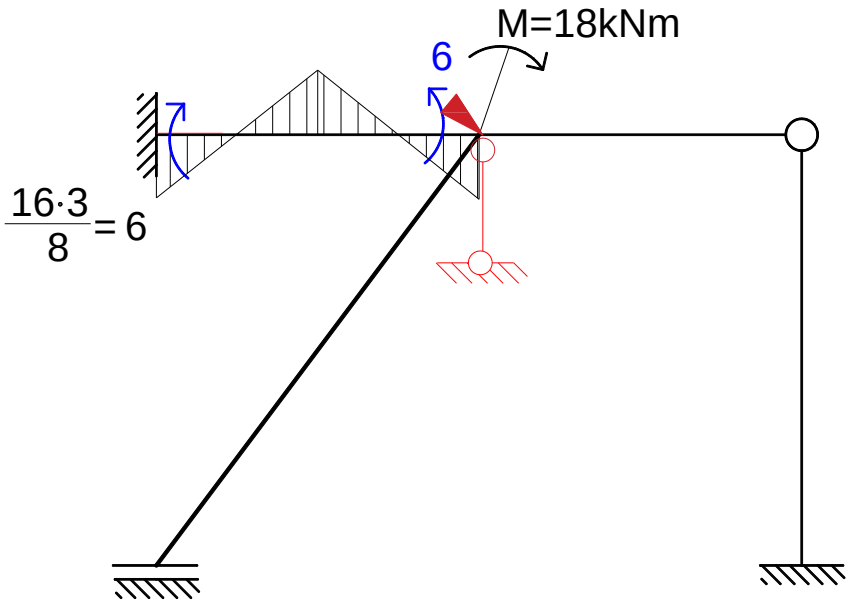
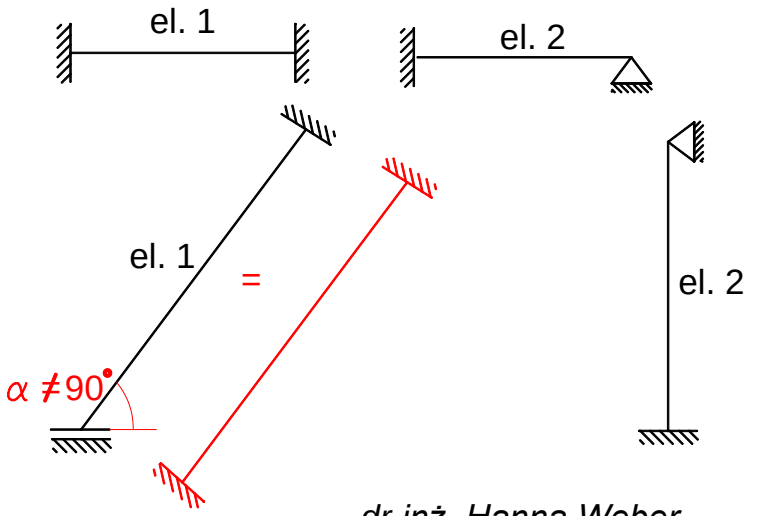
Obciążenie zewnętrzne







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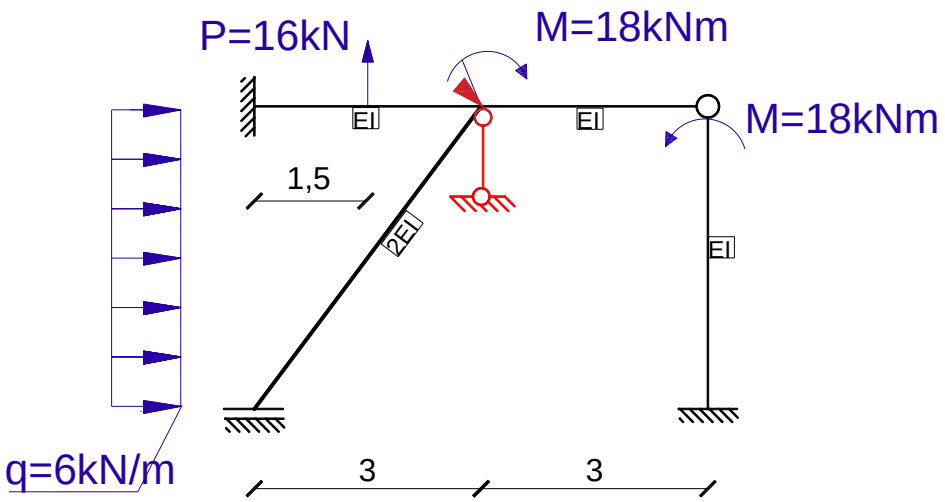
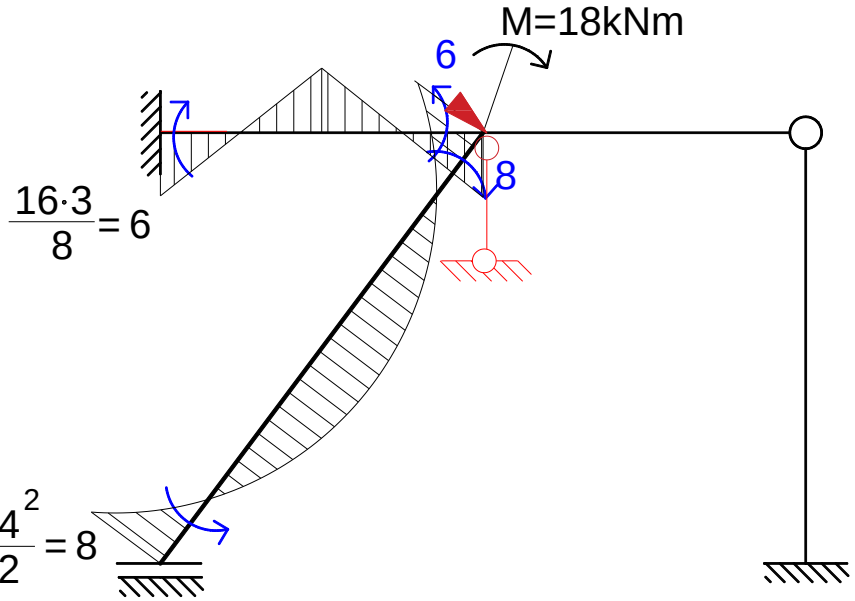
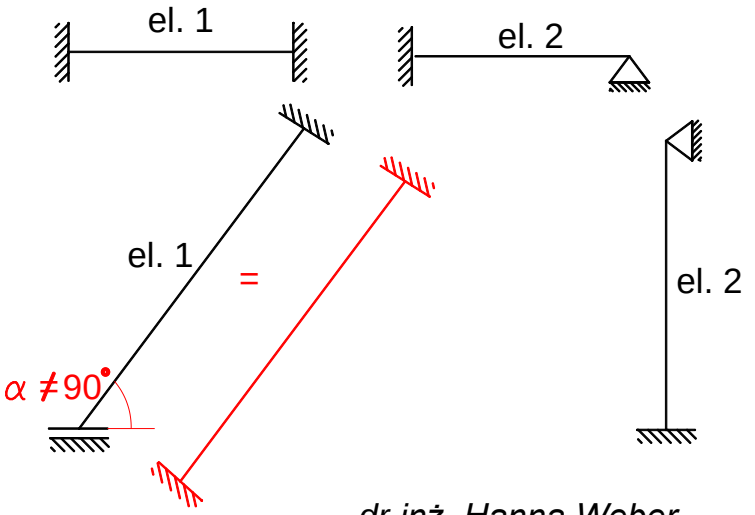
Obciążenie zewnętrzne







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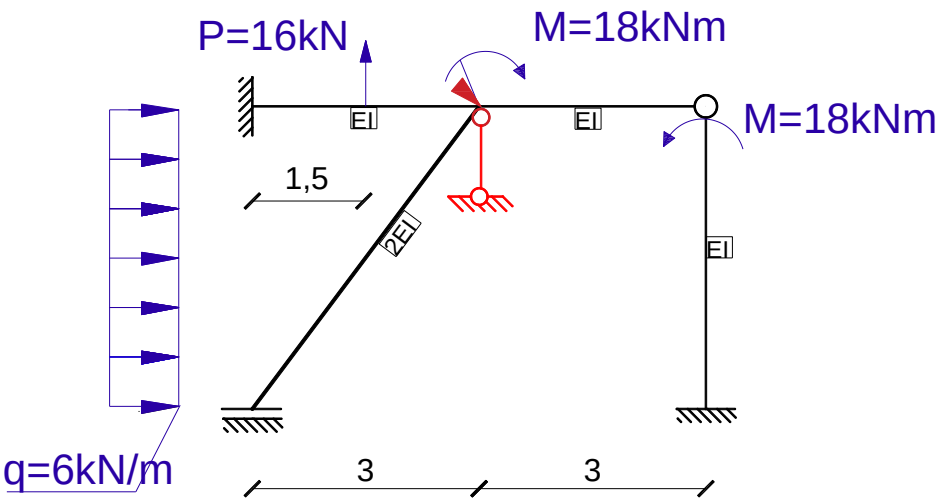
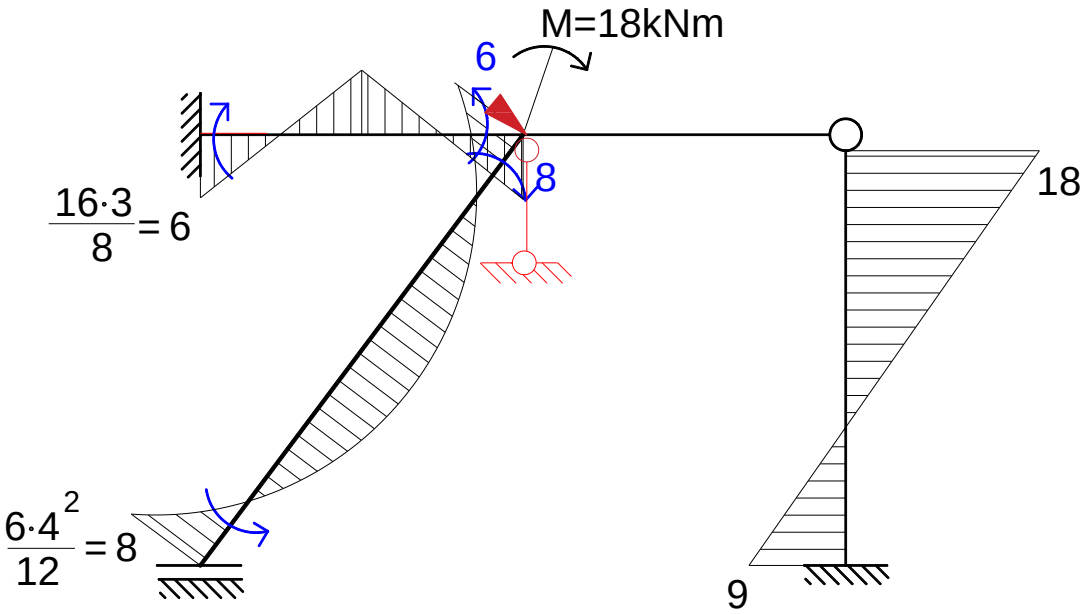
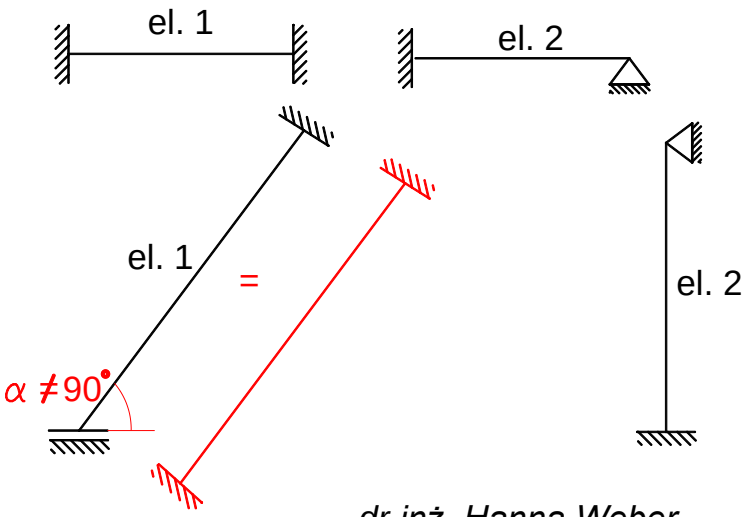
Obciążenie zewnętrzne







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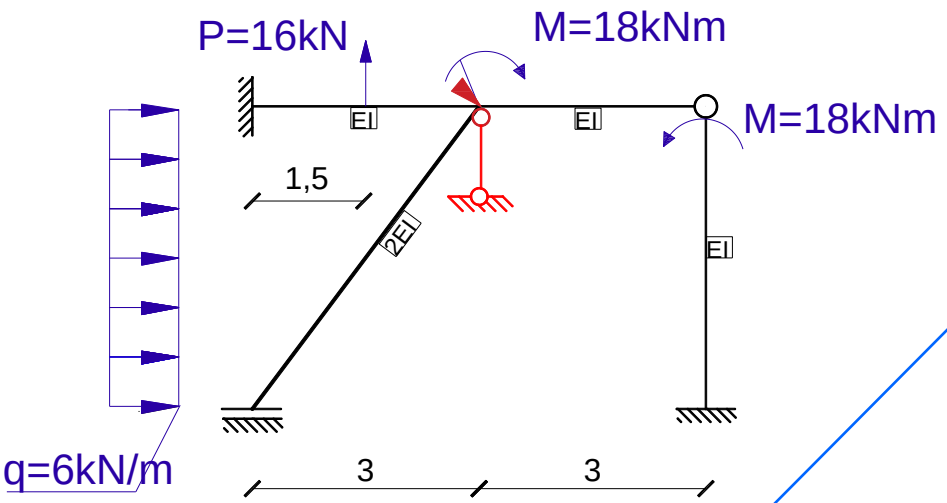
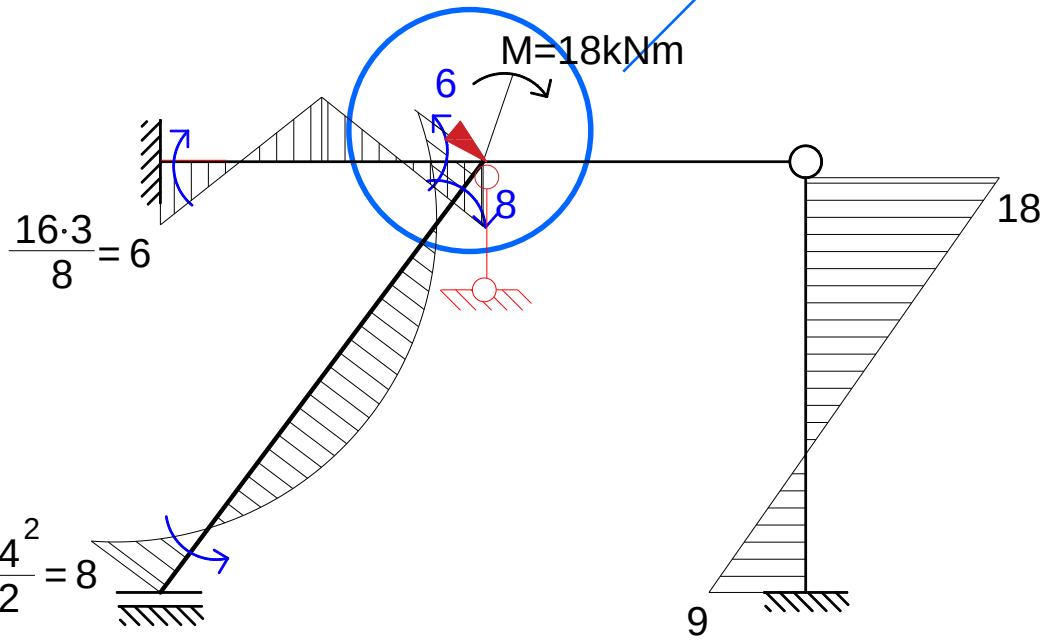
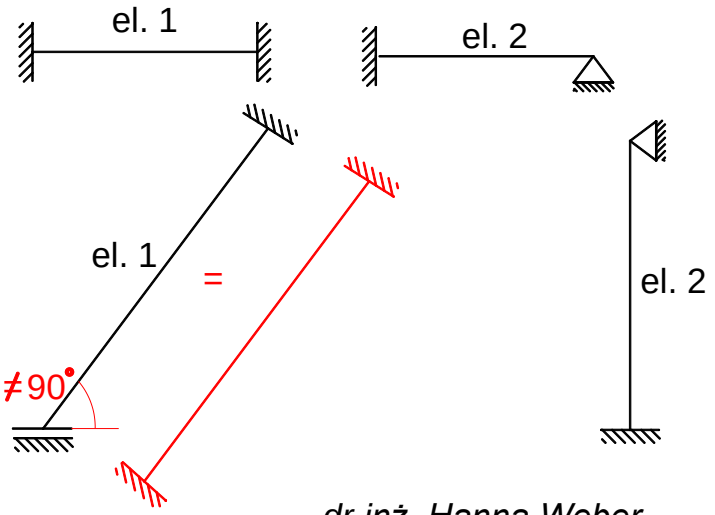
Obciążenie zewnętrzne



$$k_{10} = -6 + 8 - 18 = -16kNm$$





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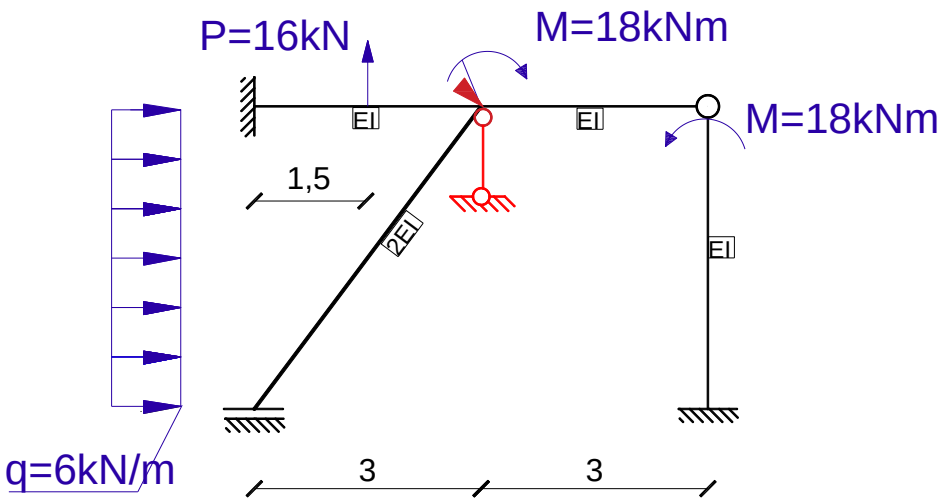
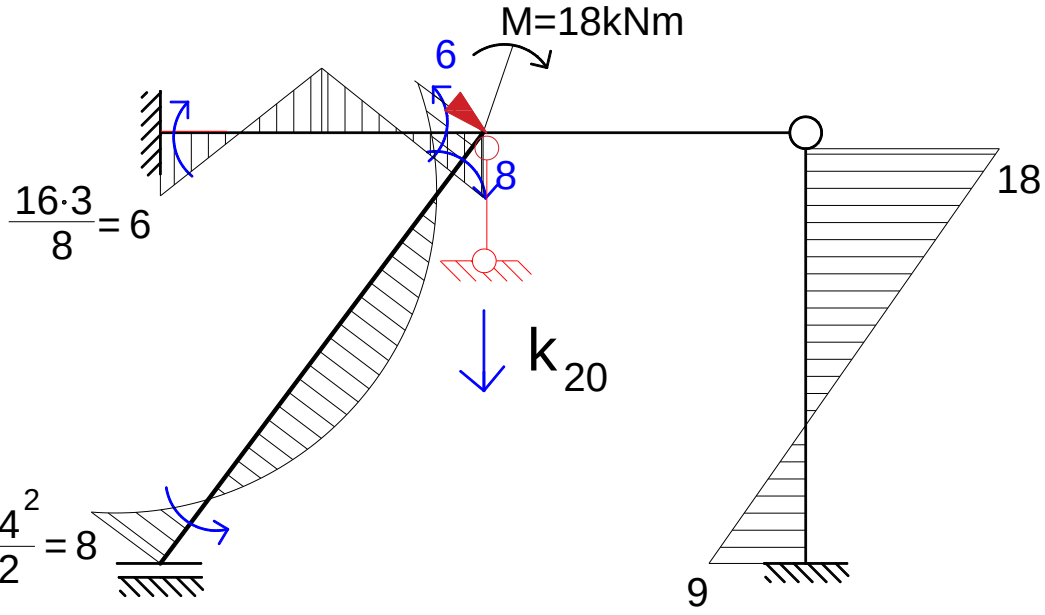
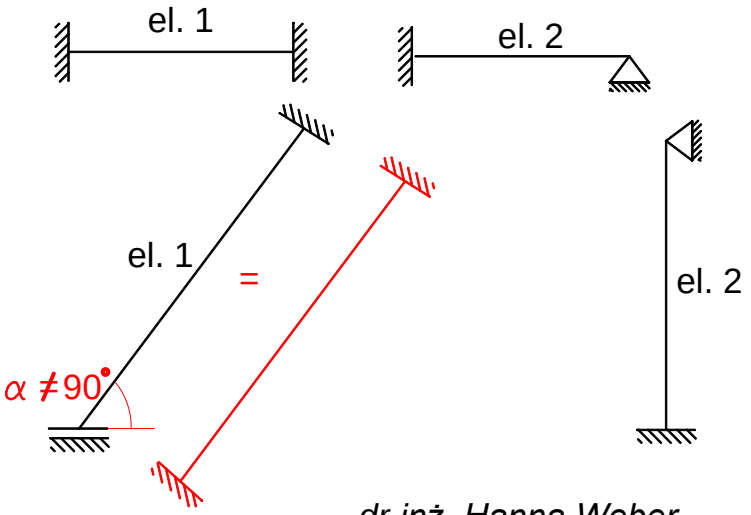
Obciążenie zewnętrzne



$$k_{10} = -6 + 8 - 18 = -16kNm$$





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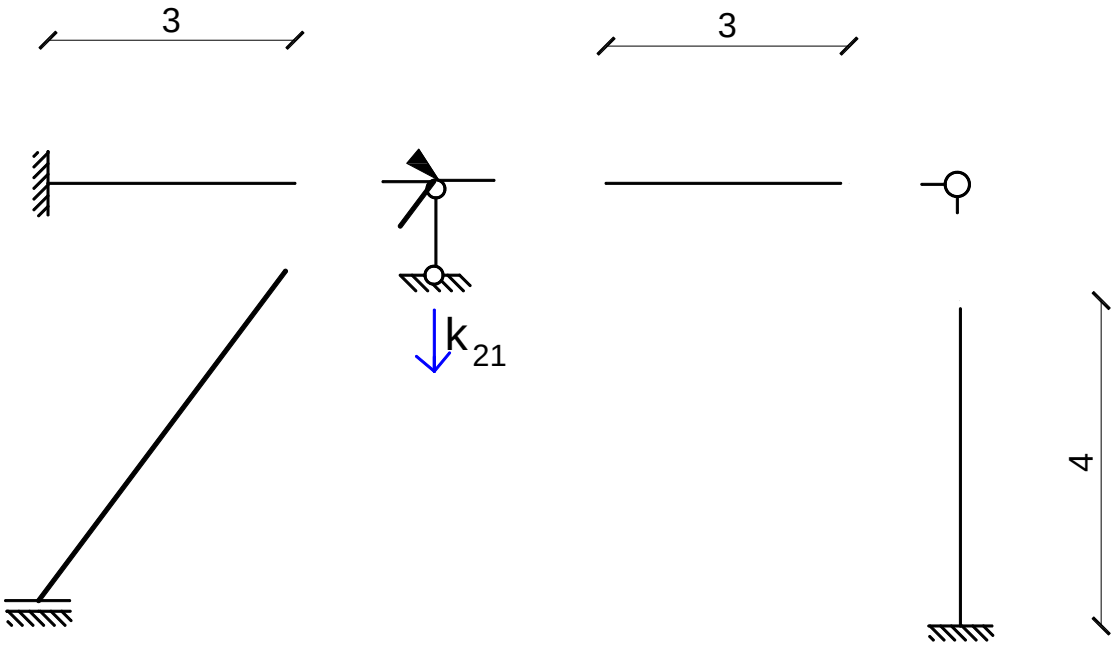
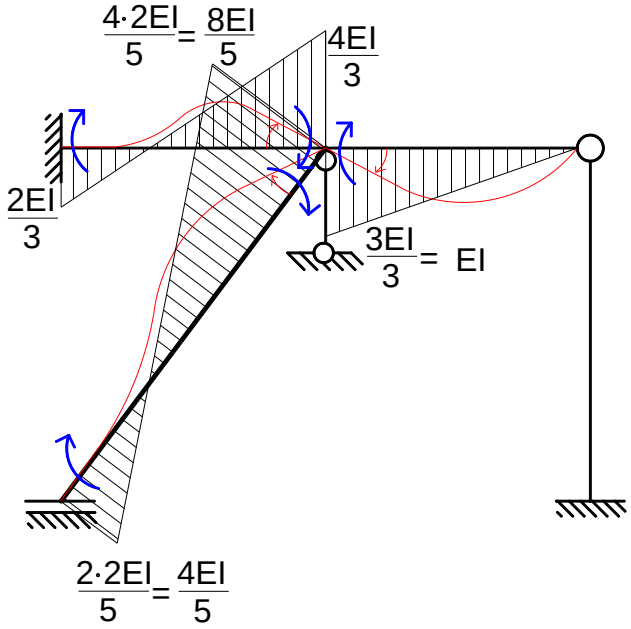
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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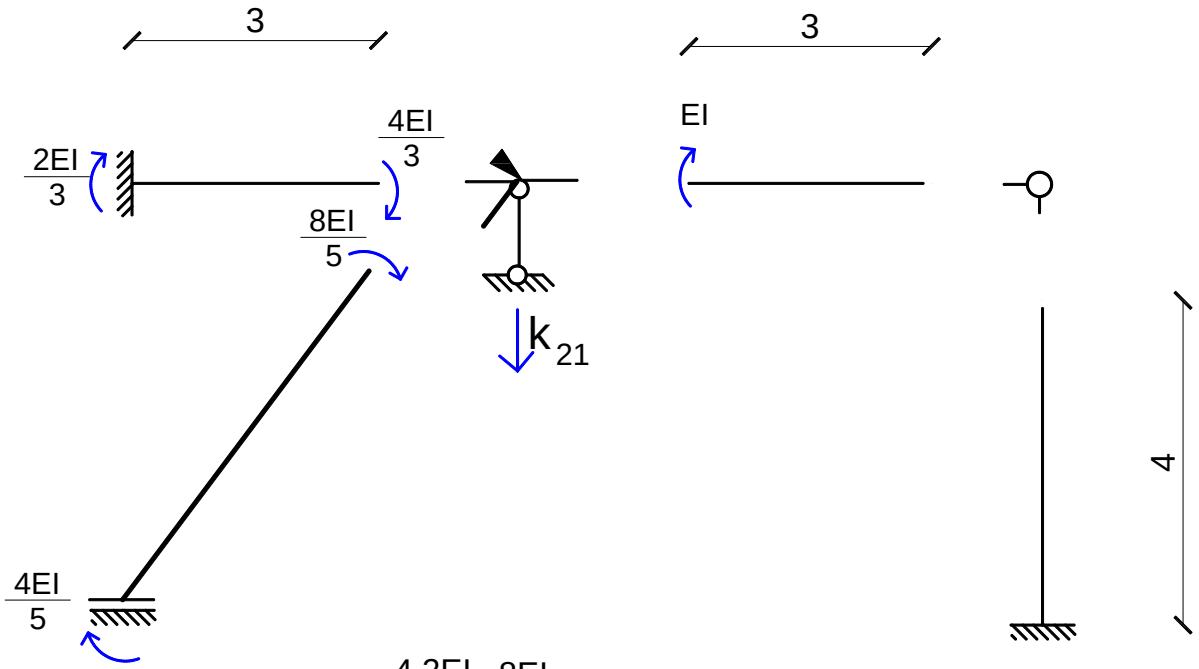
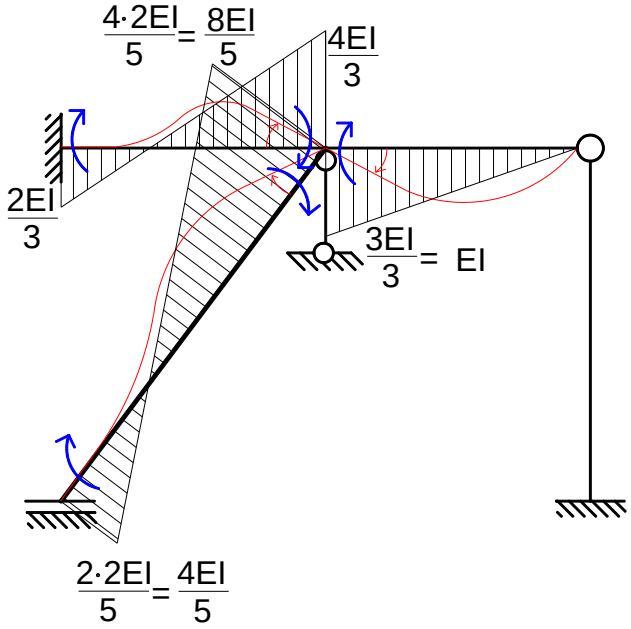
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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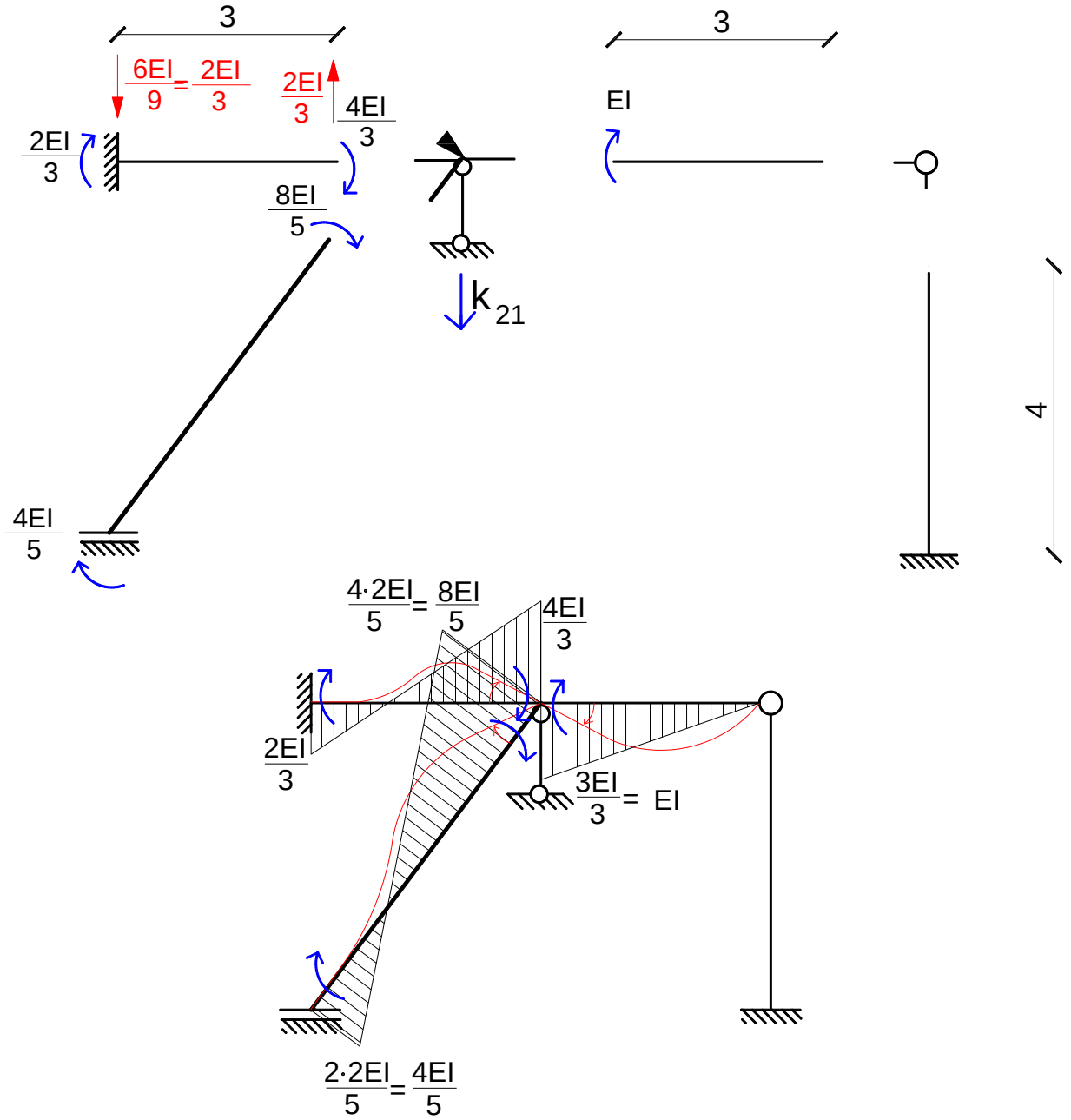
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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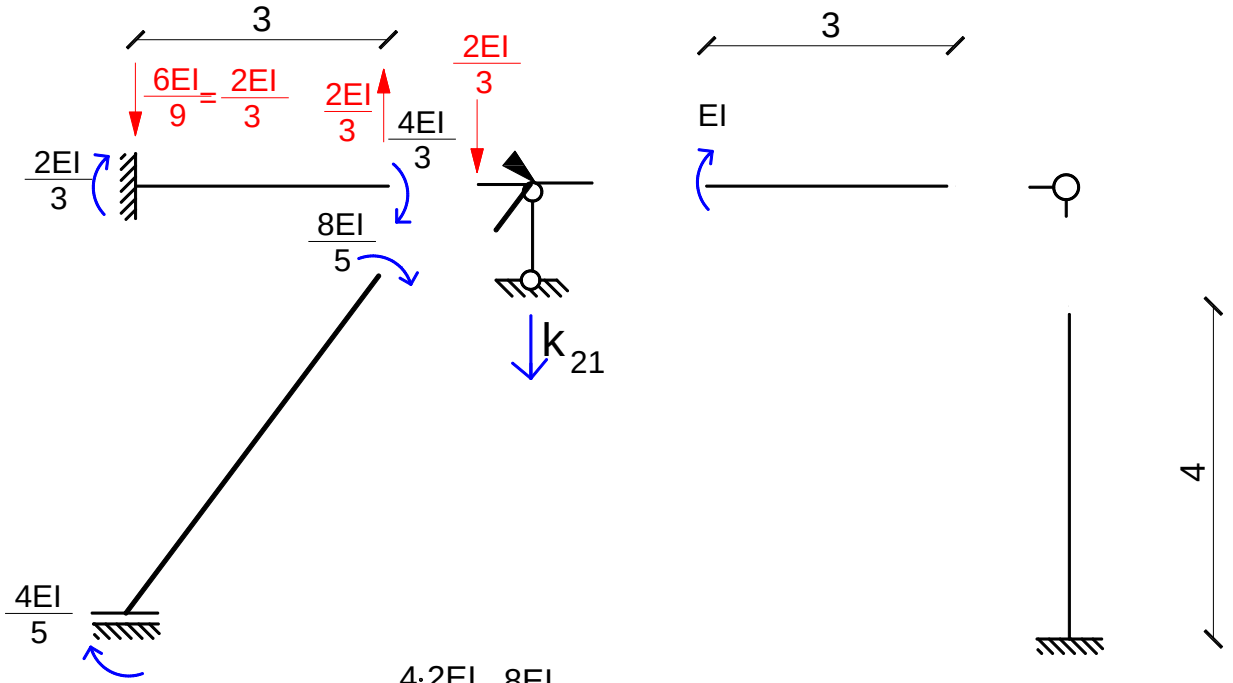
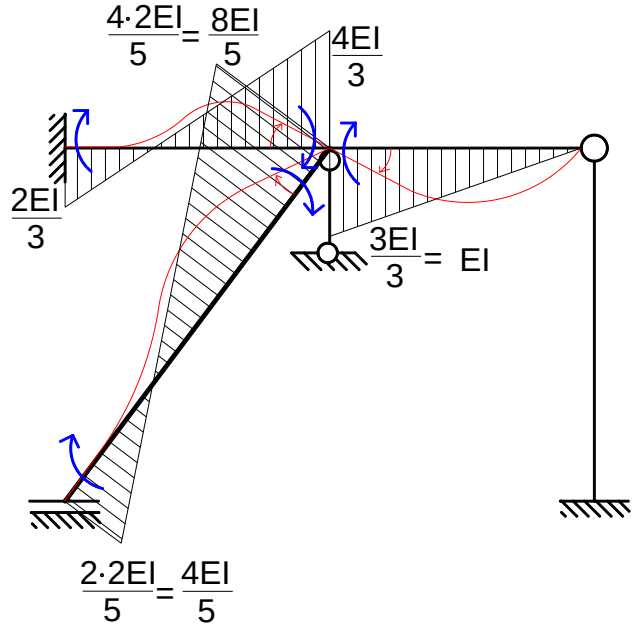
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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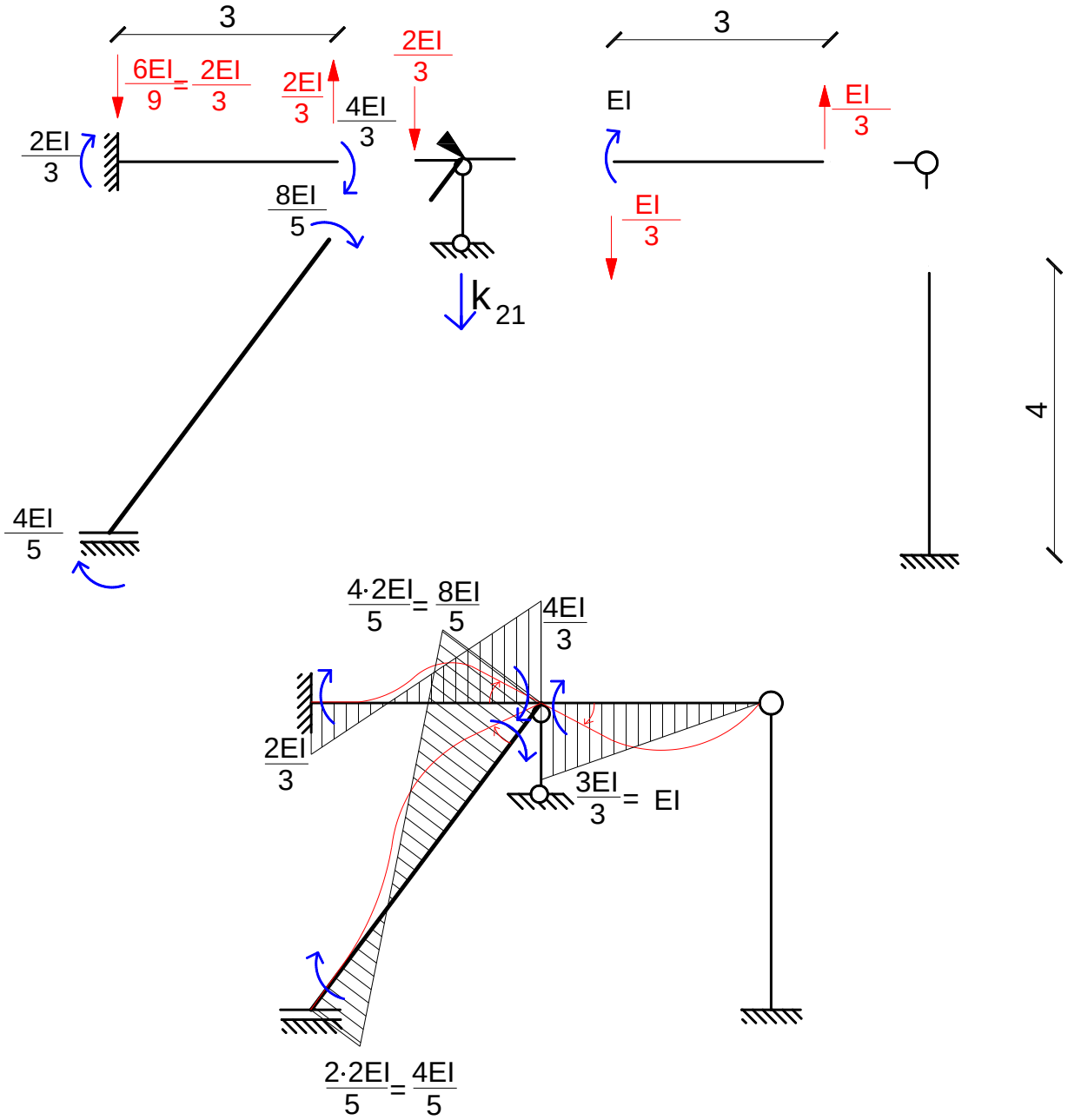
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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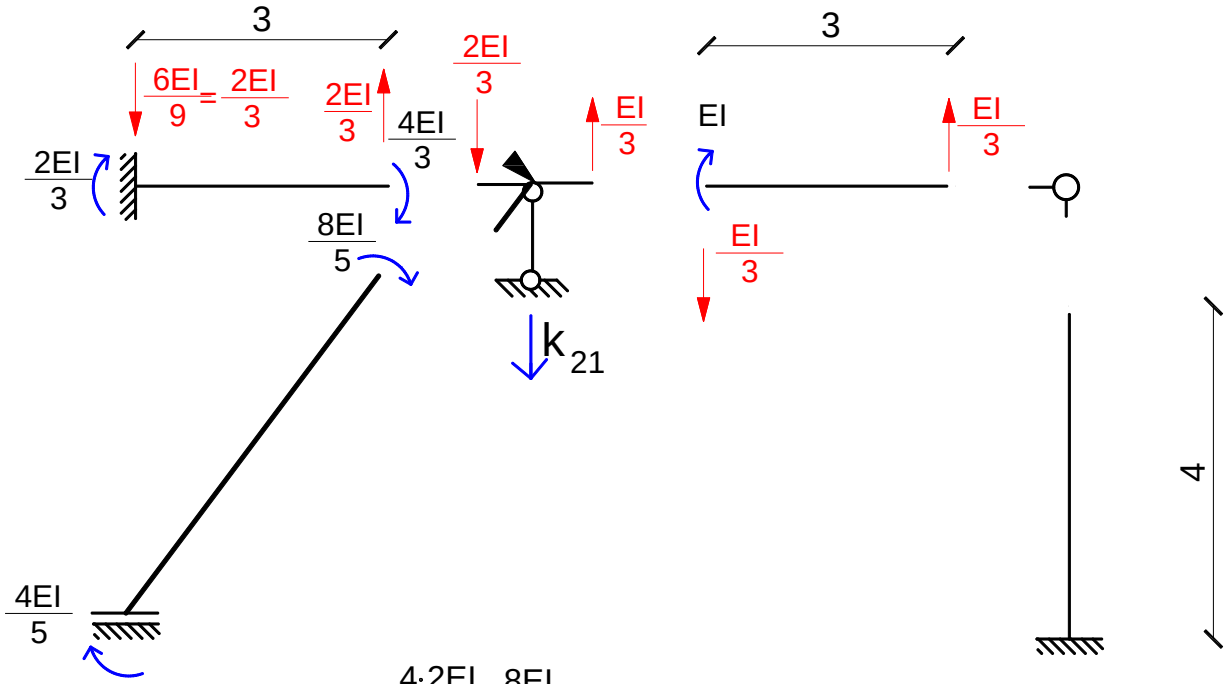
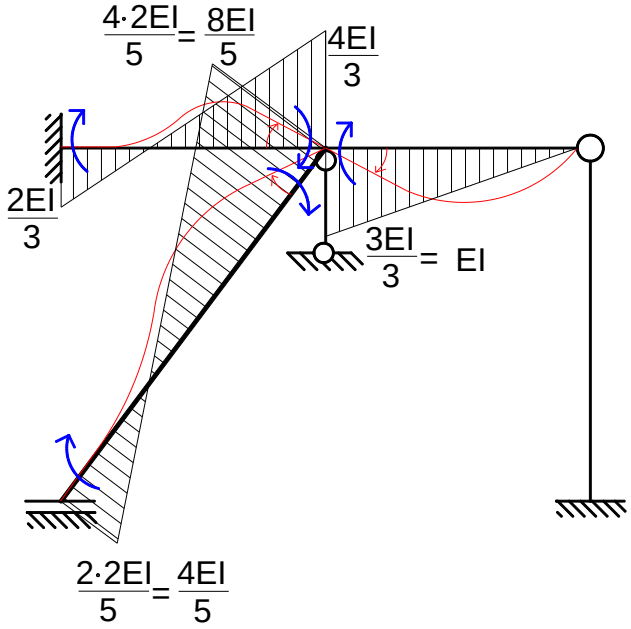
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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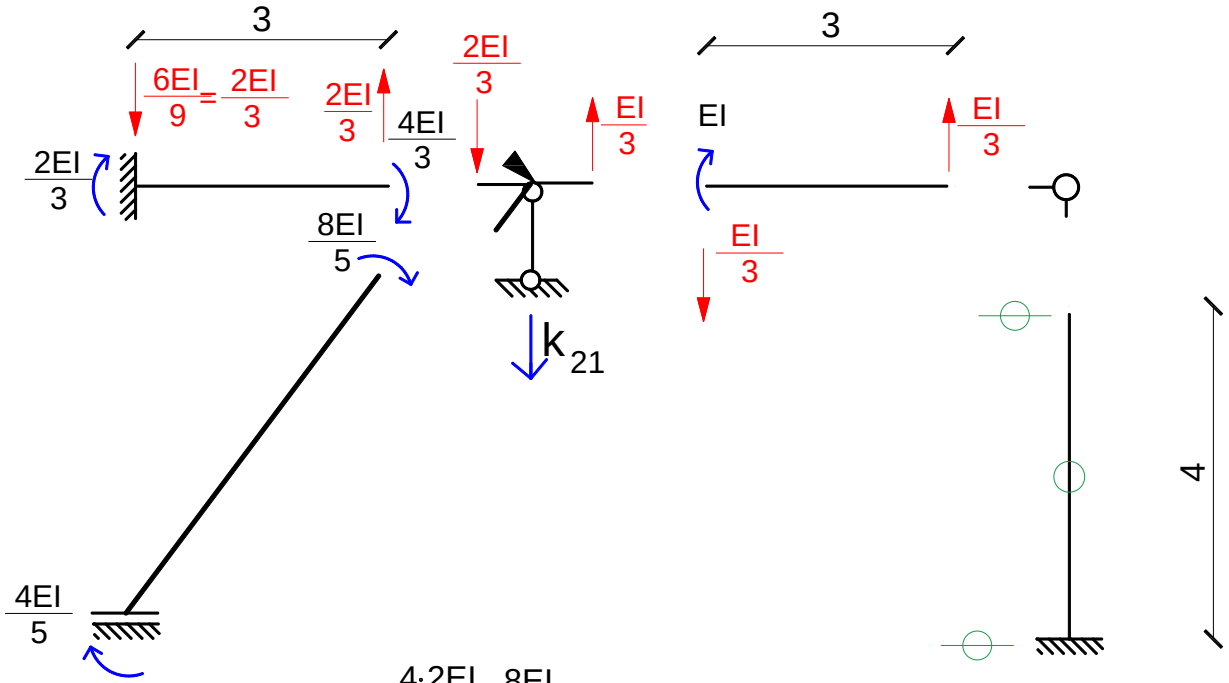
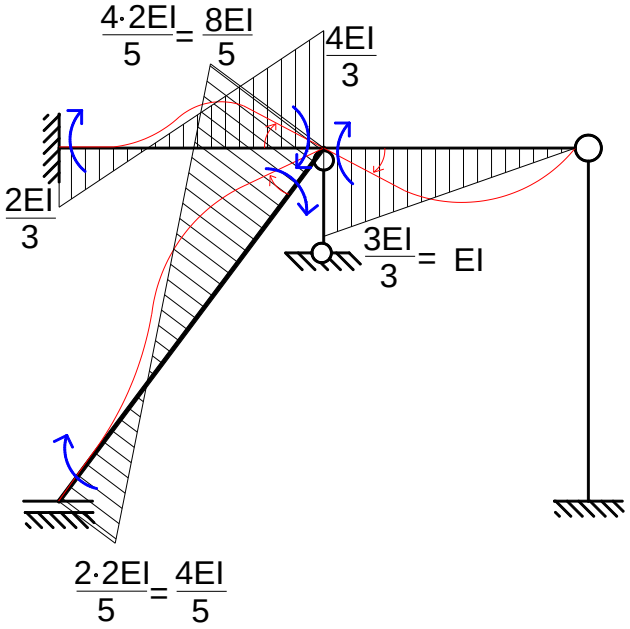
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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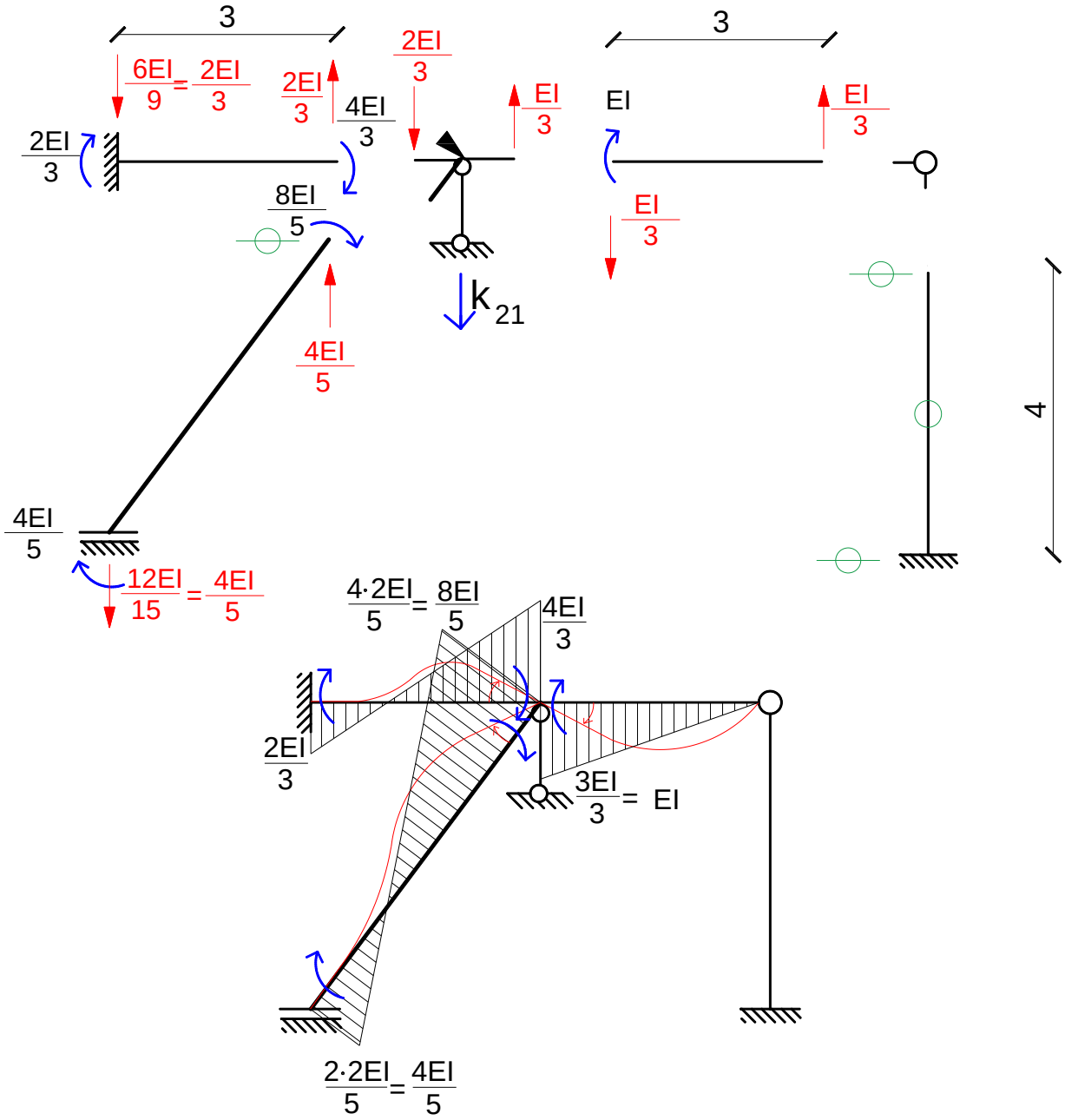
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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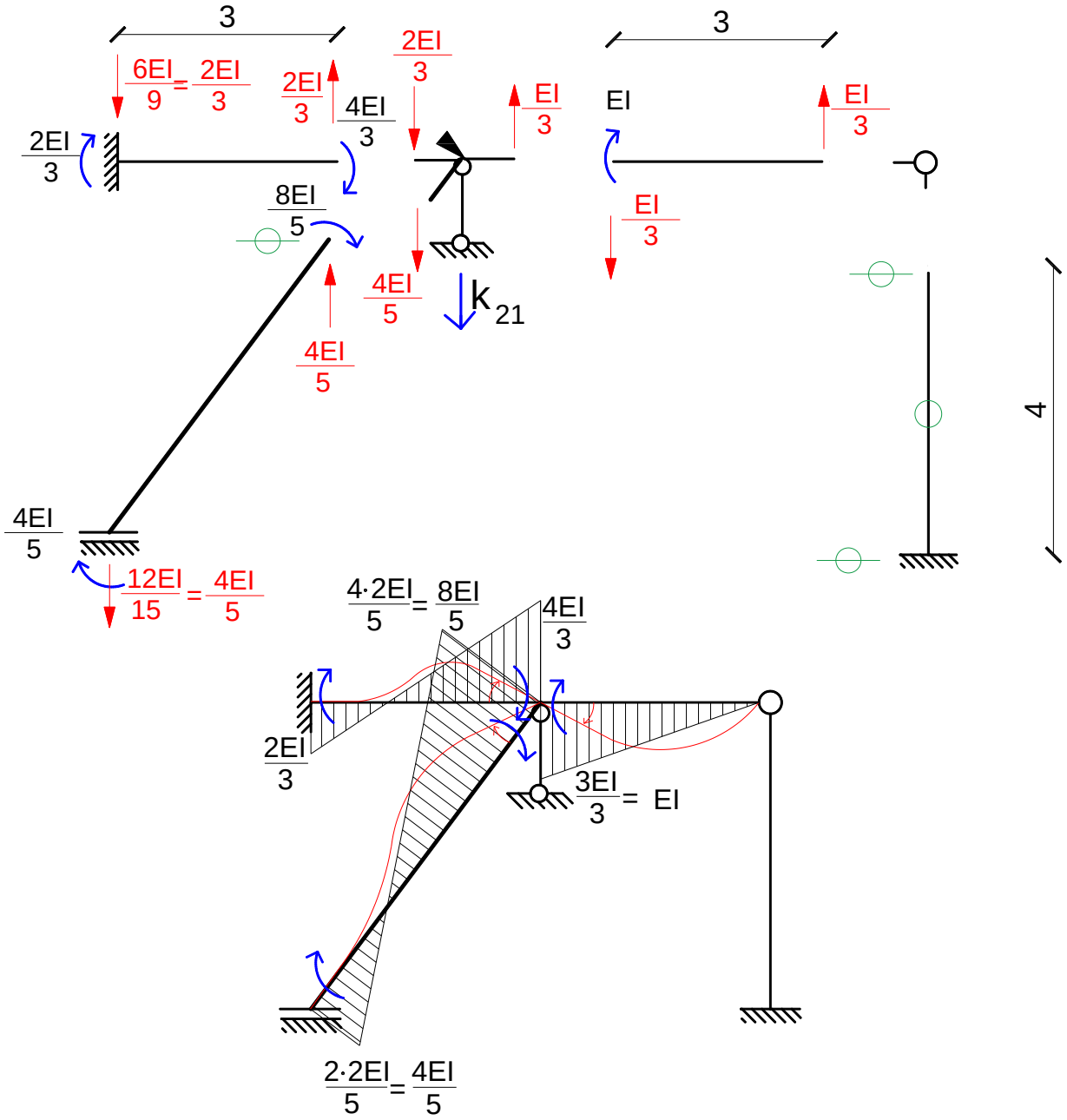
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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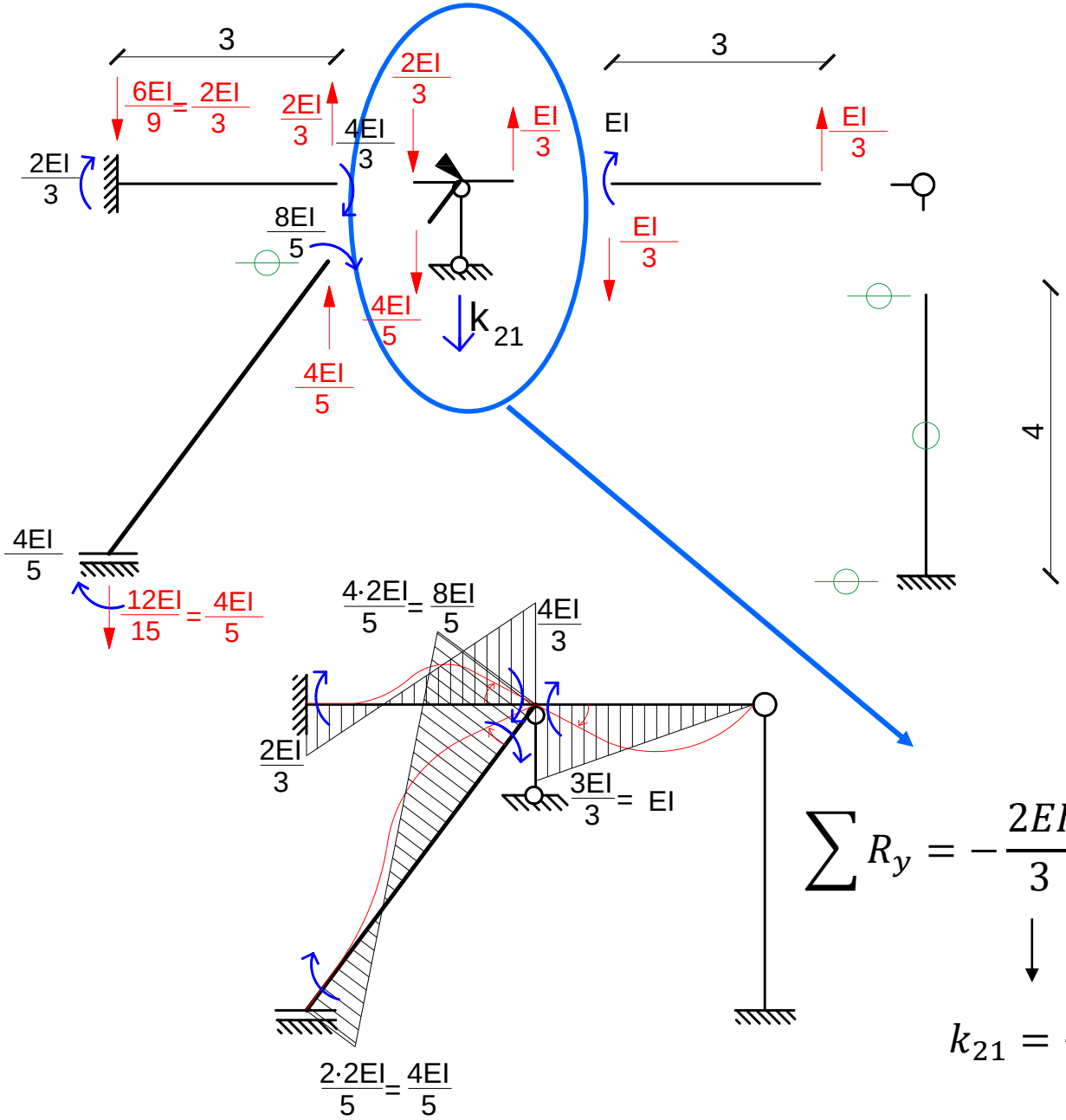
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



$$\sum R_y = -\frac{2EI}{3} + \frac{EI}{3} - \frac{4EI}{5} - k_{21} = 0$$

$$\downarrow$$

$$k_{21} = -\frac{17EI}{15} = k_{12}$$

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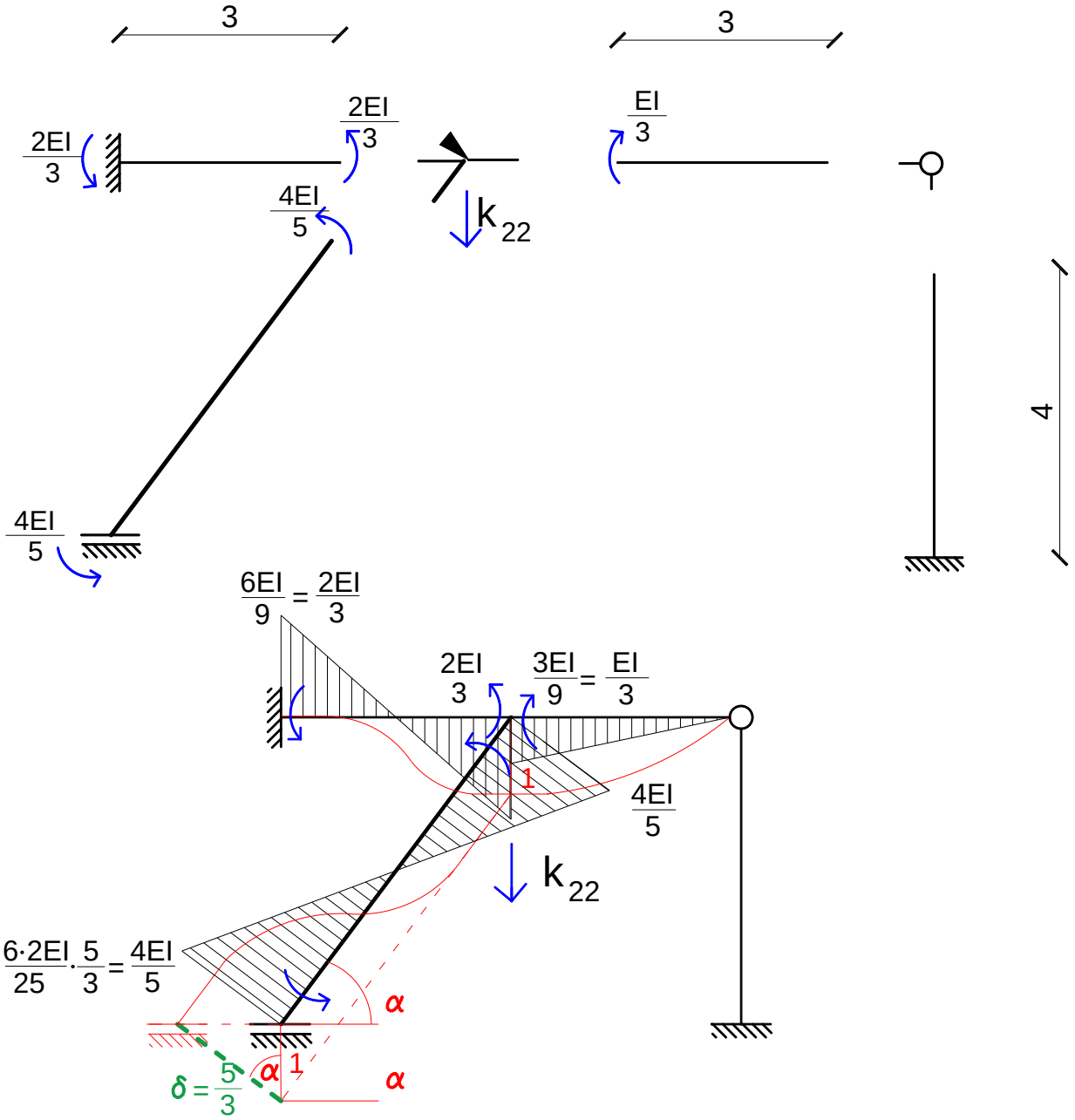
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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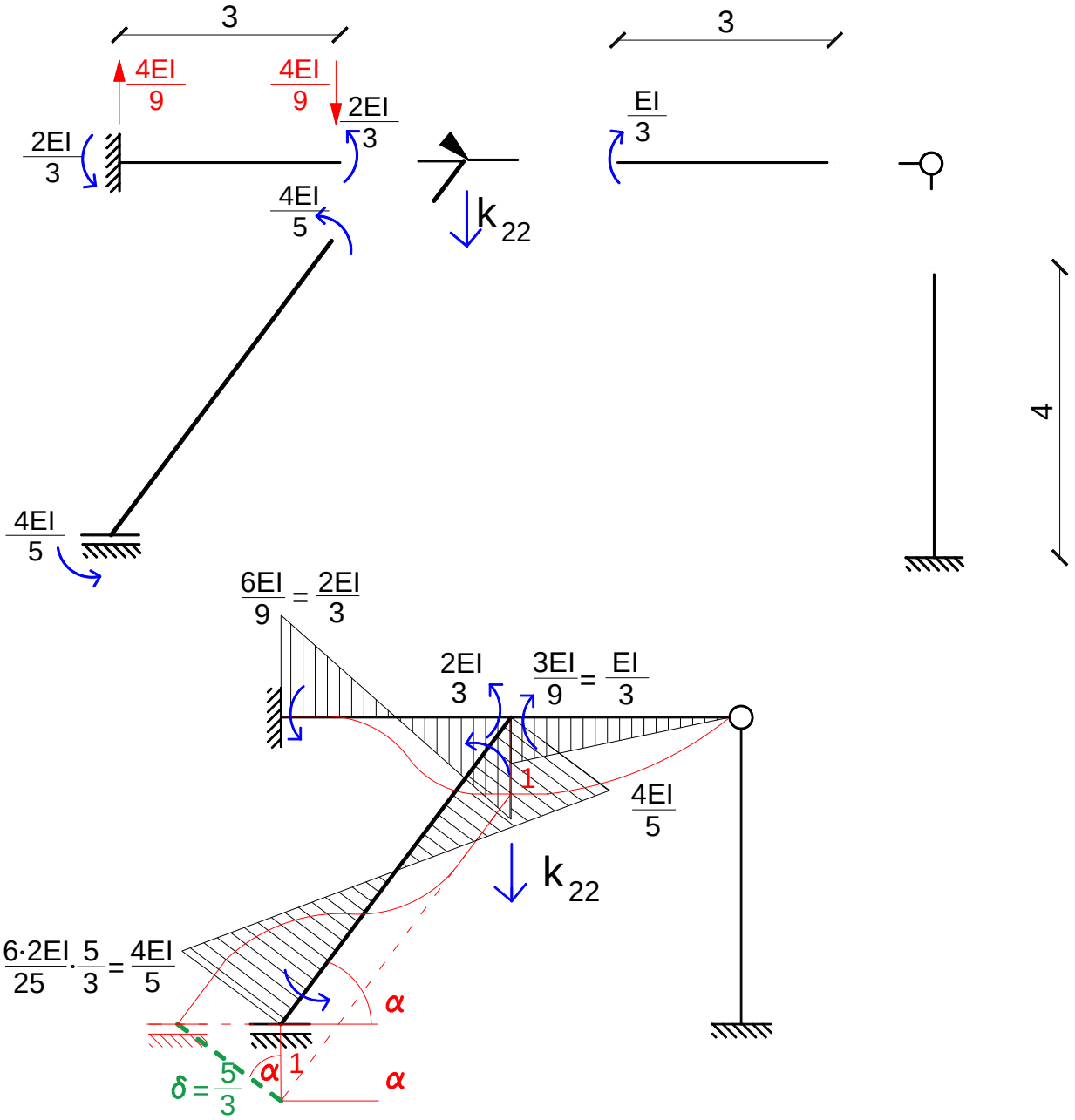
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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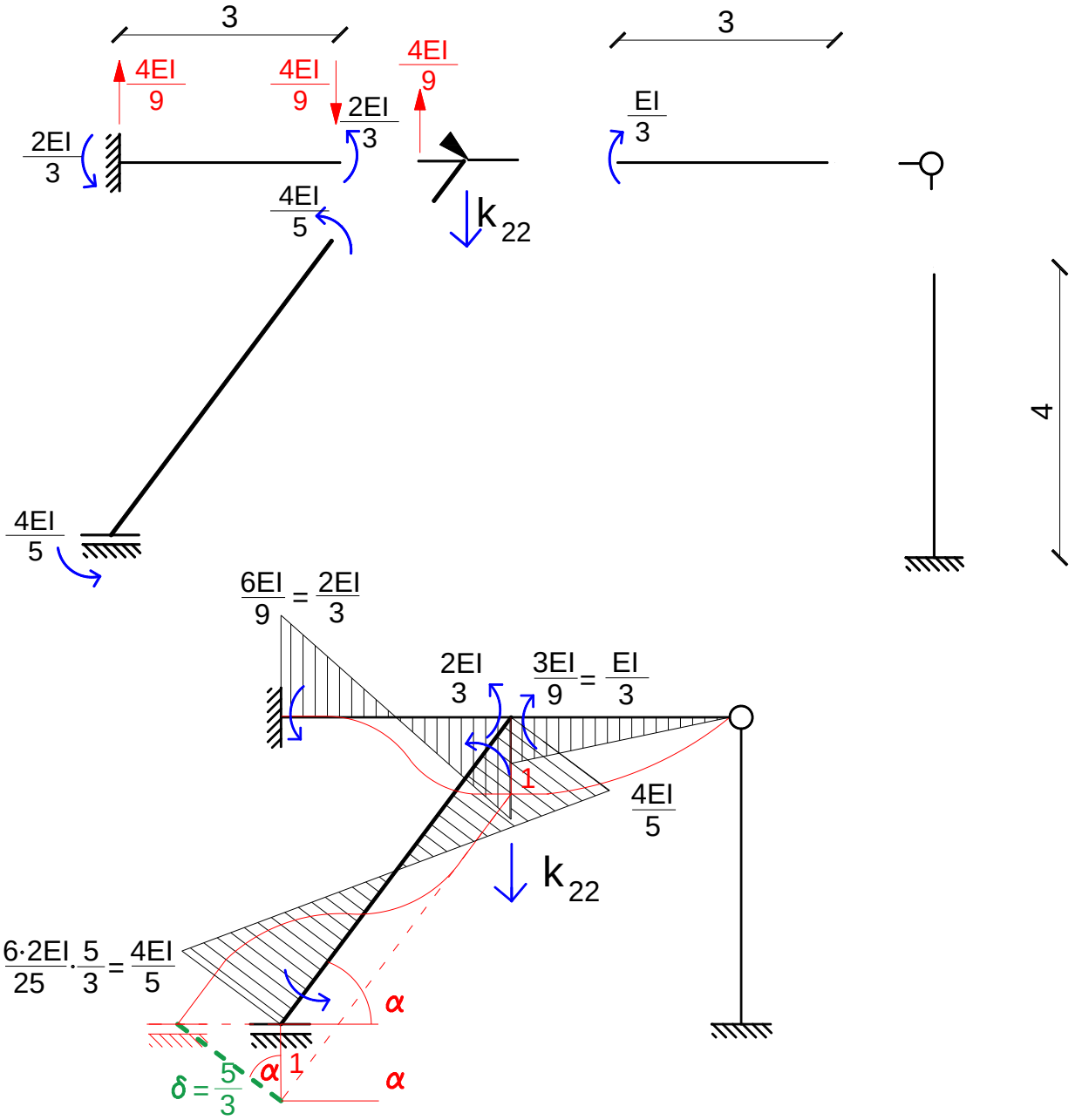
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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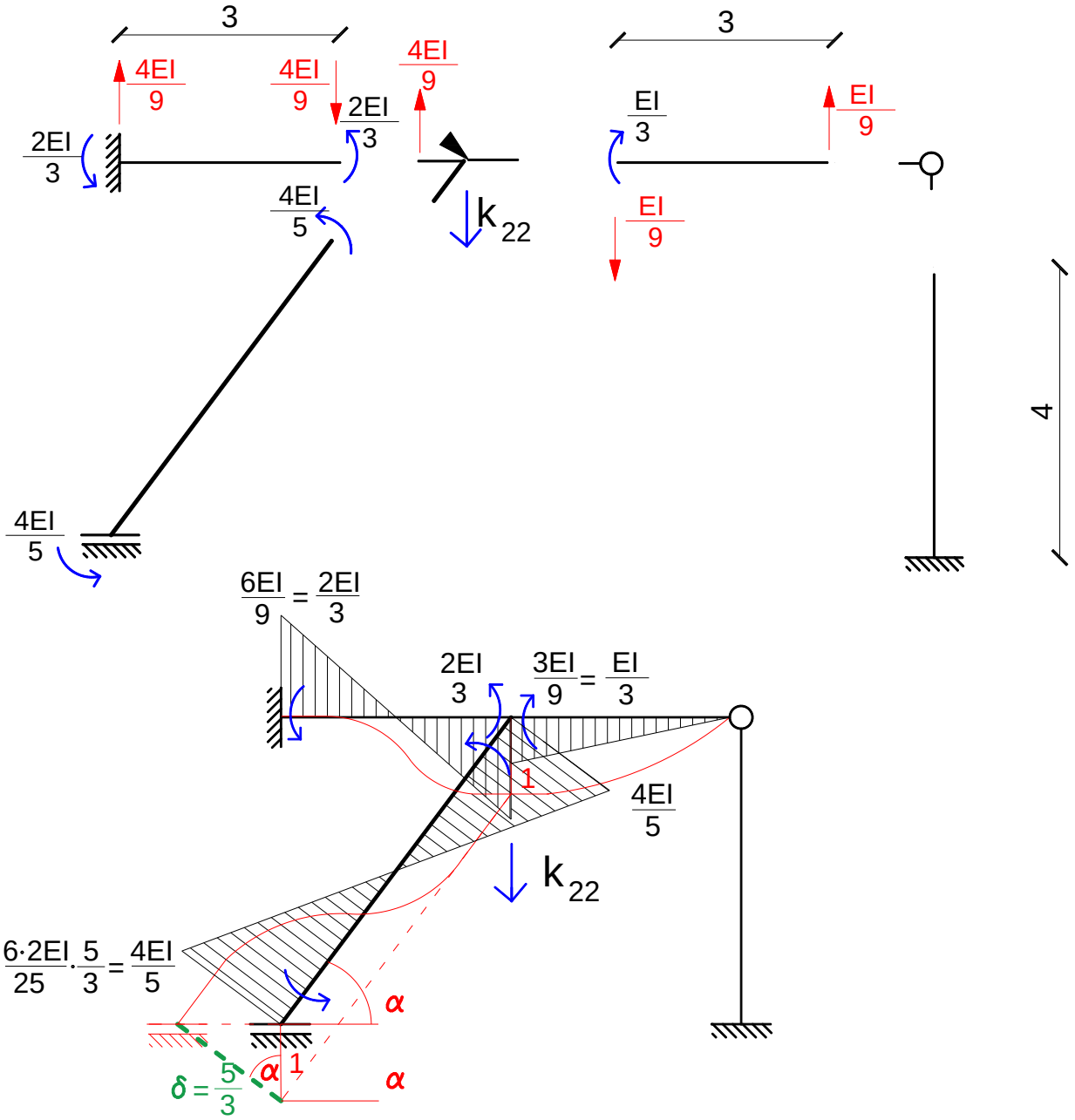
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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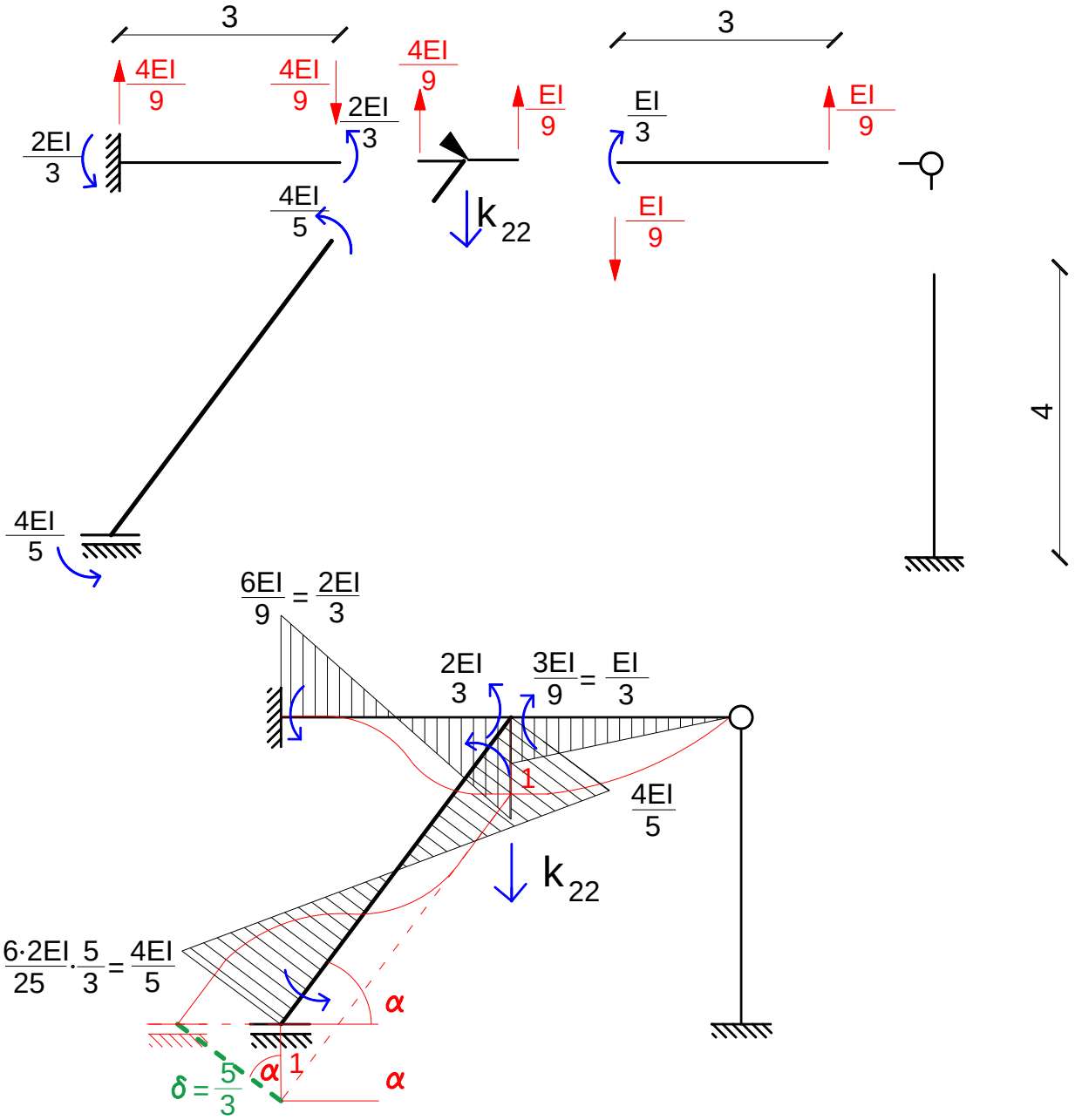
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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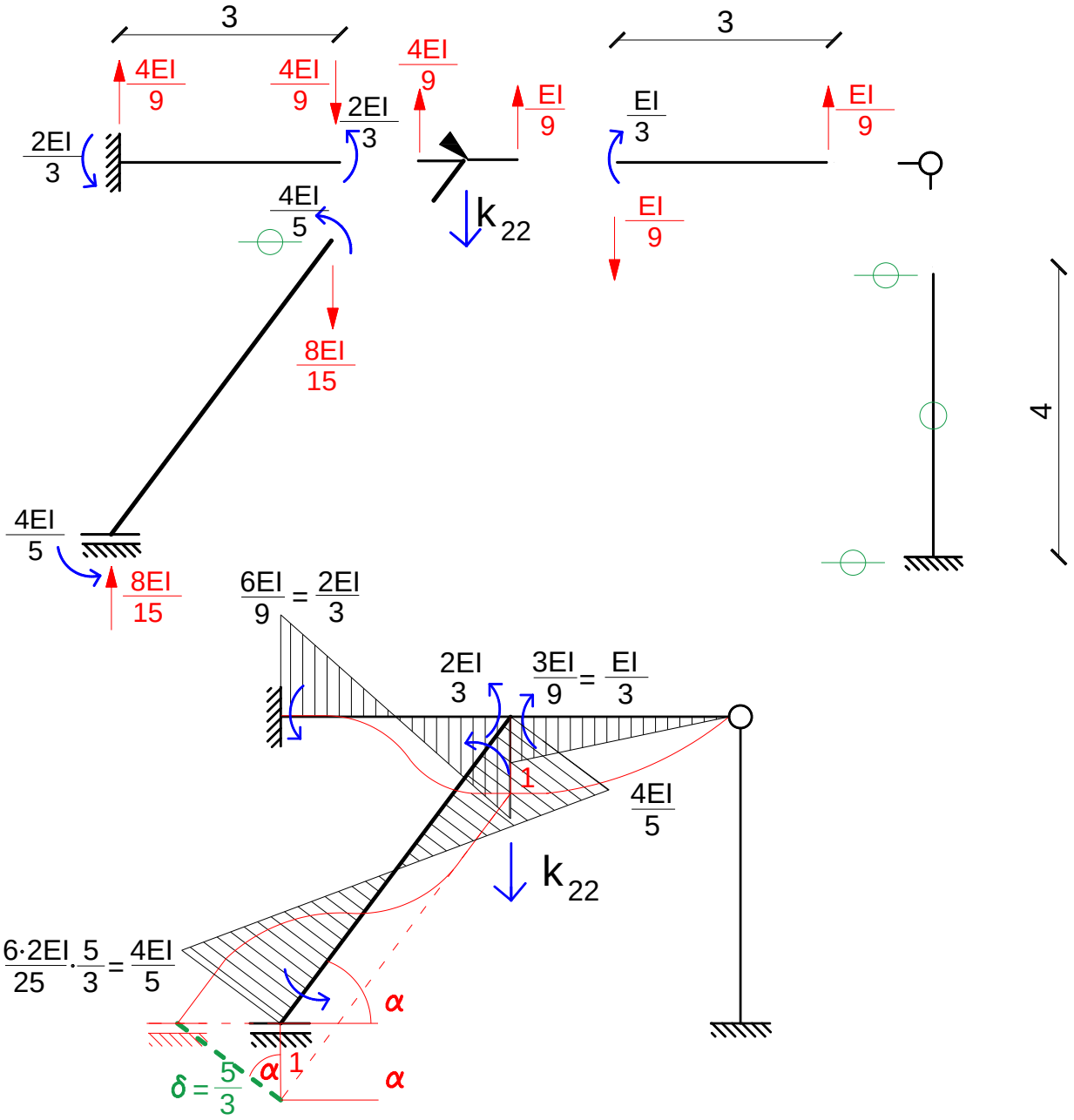
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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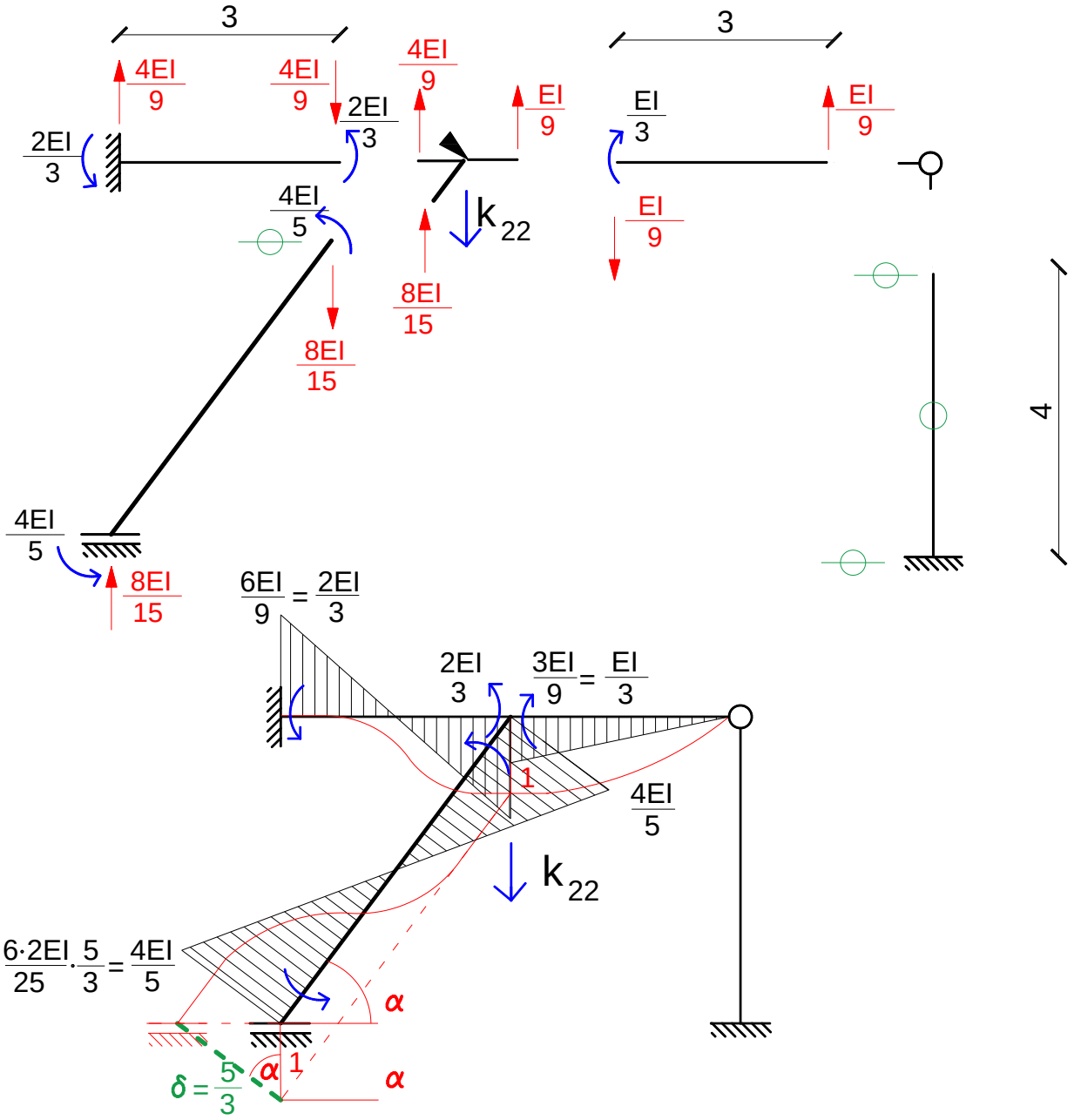
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



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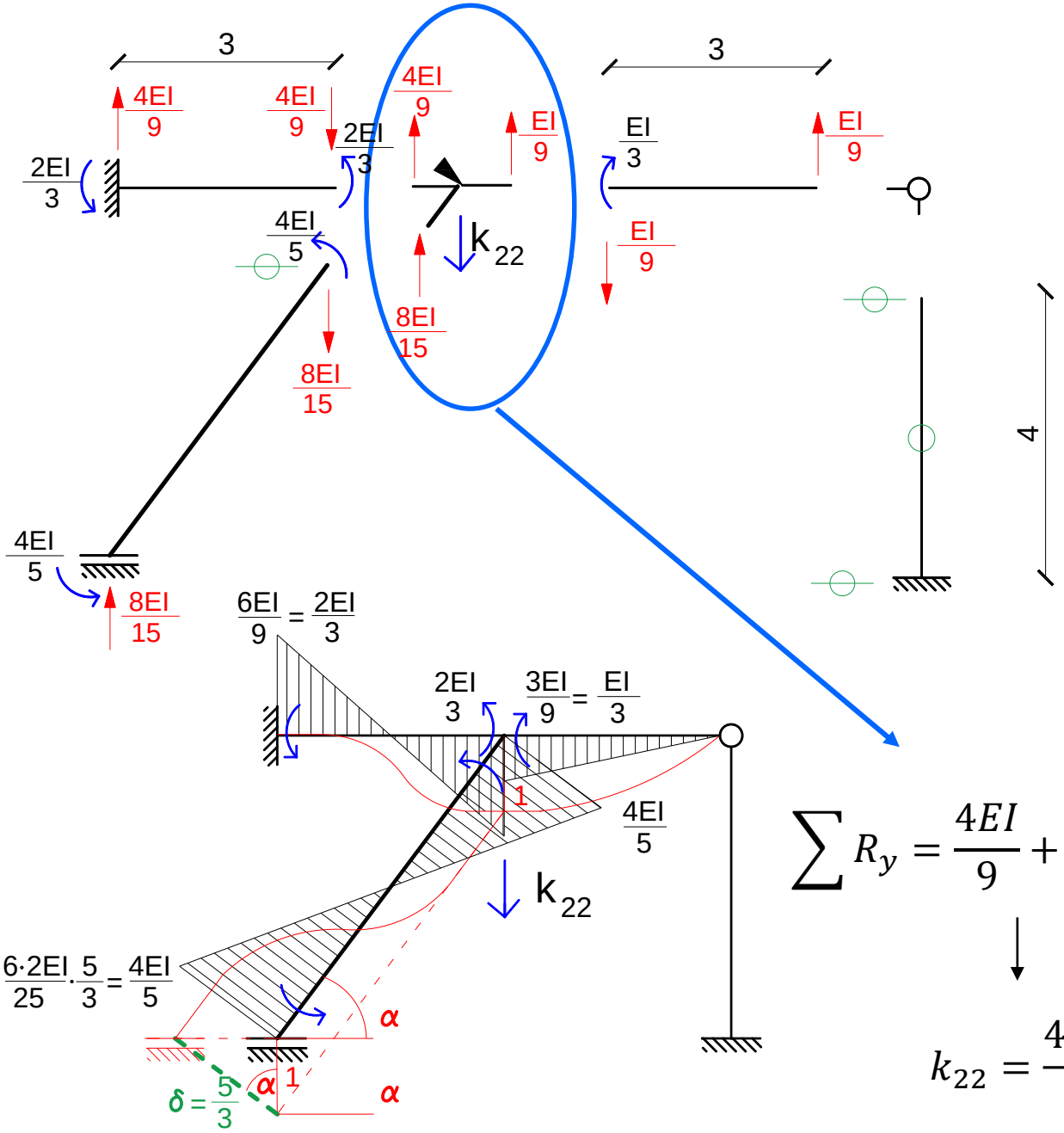
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}



$$\sum R_y = \frac{4EI}{9} + \frac{EI}{9} + \frac{8EI}{15} - k_{22} = 0$$

$$\downarrow$$

$$k_{22} = \frac{49EI}{45}$$

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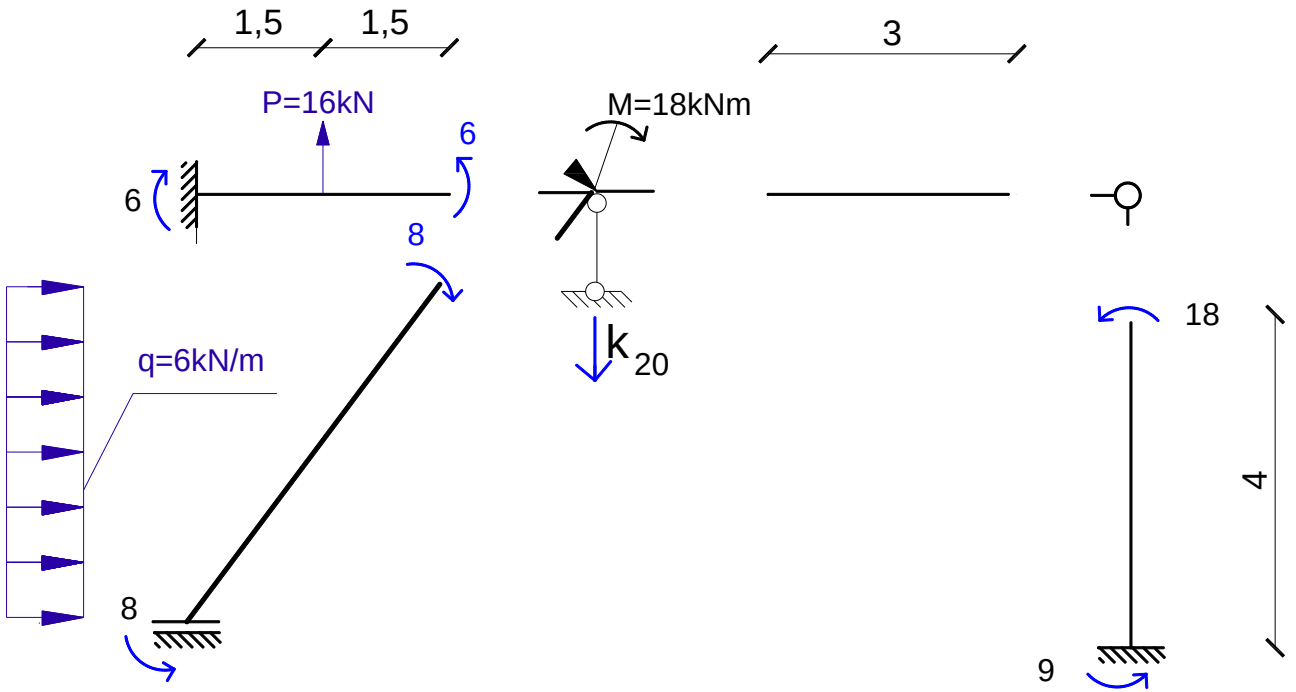
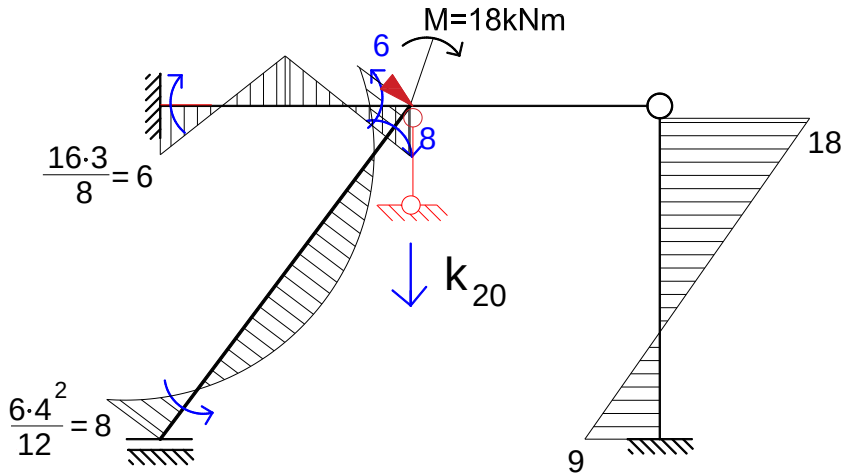
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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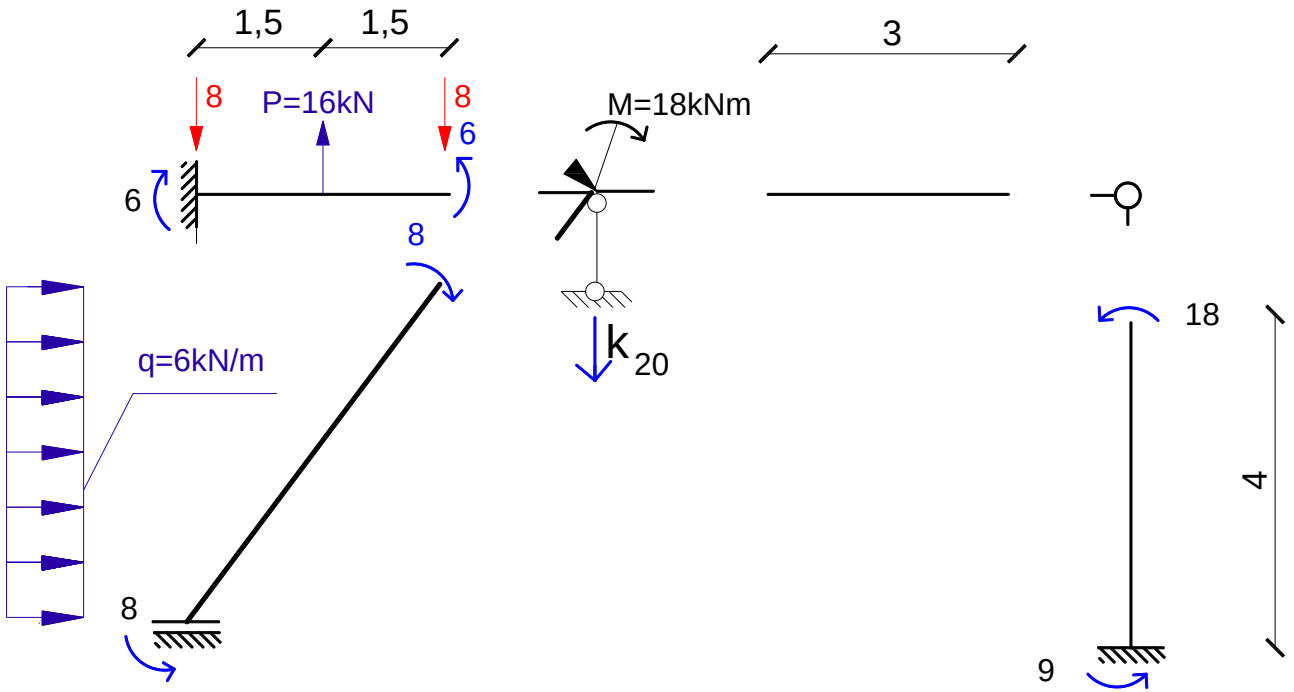
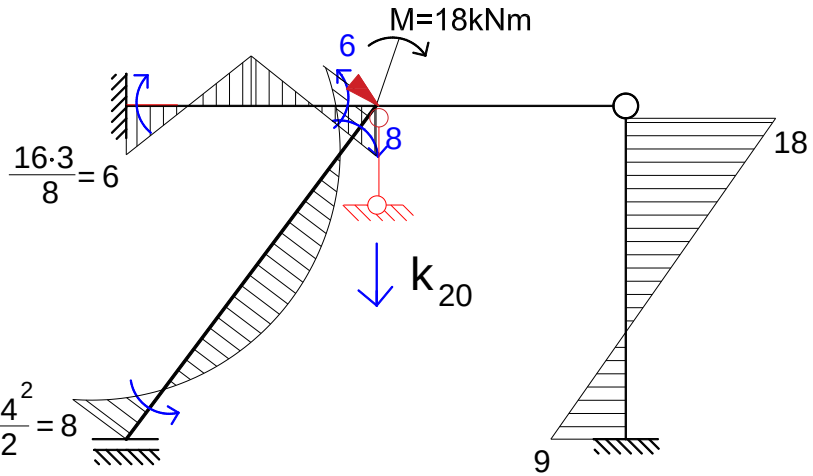
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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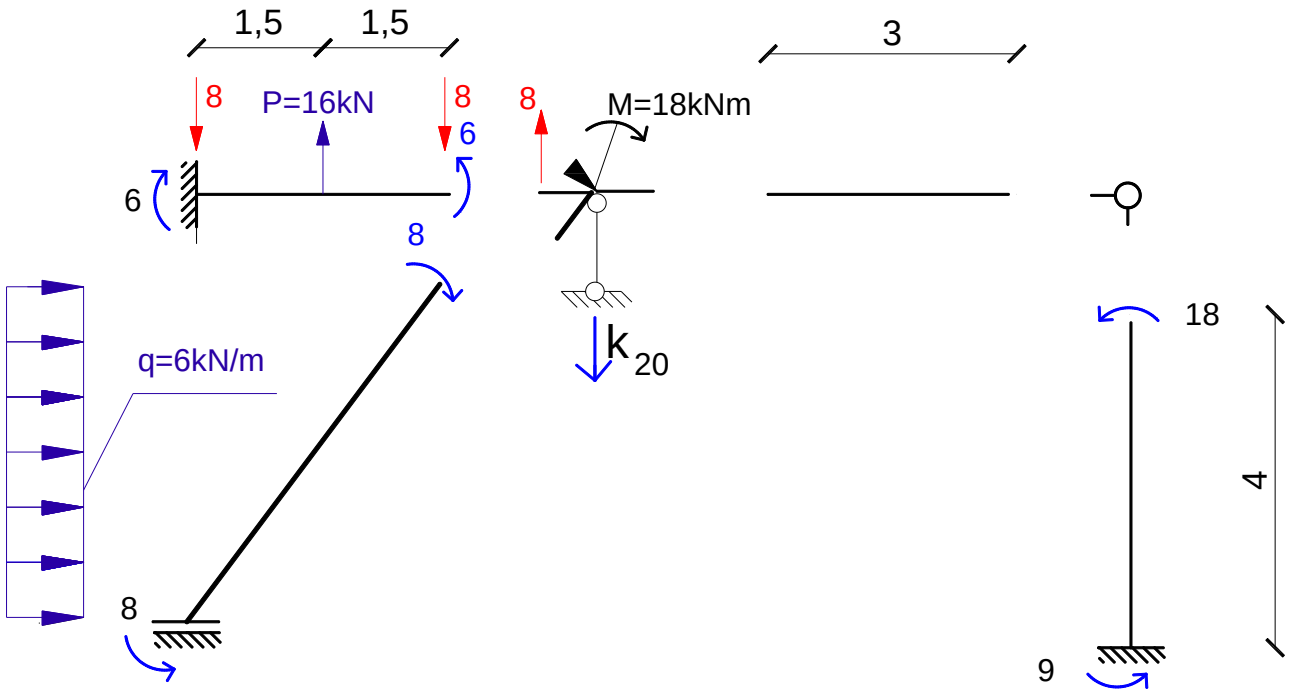
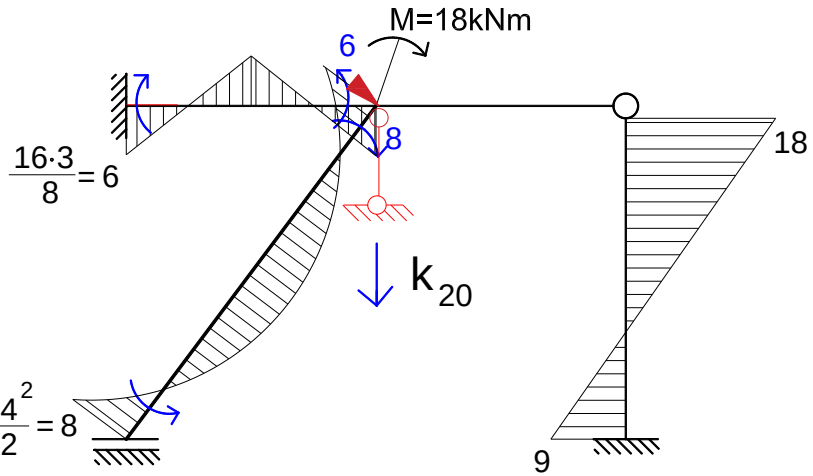
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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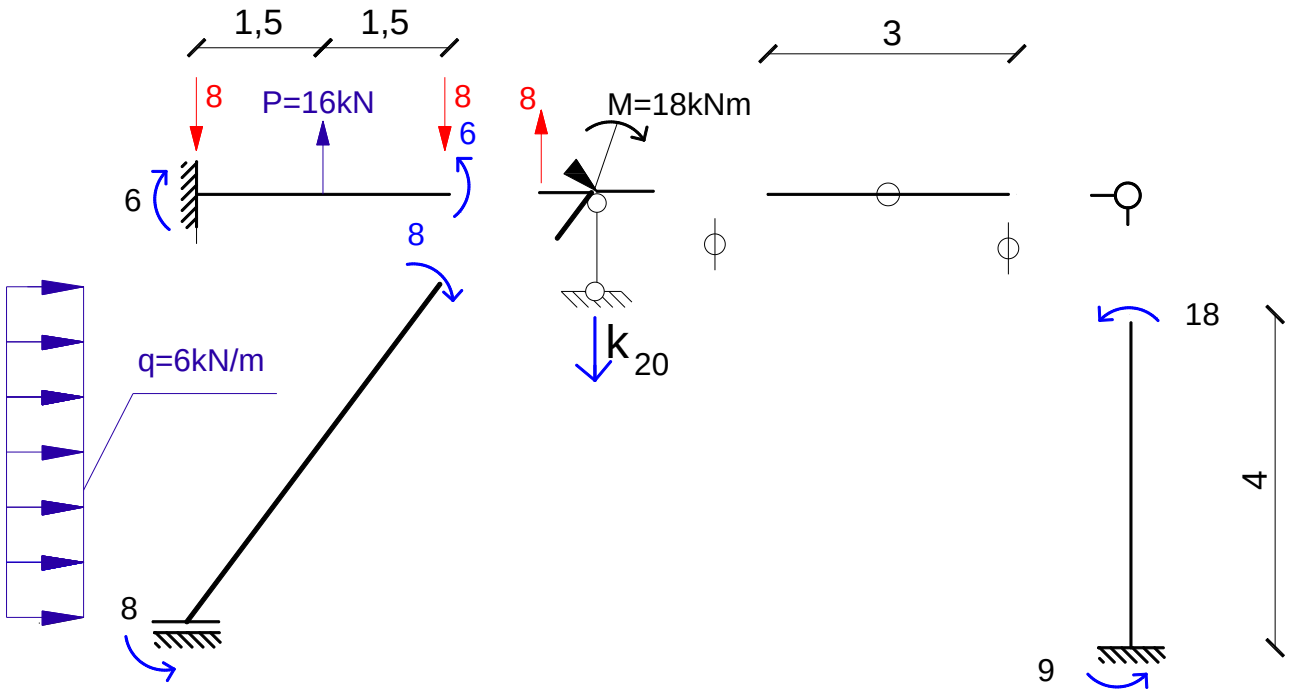
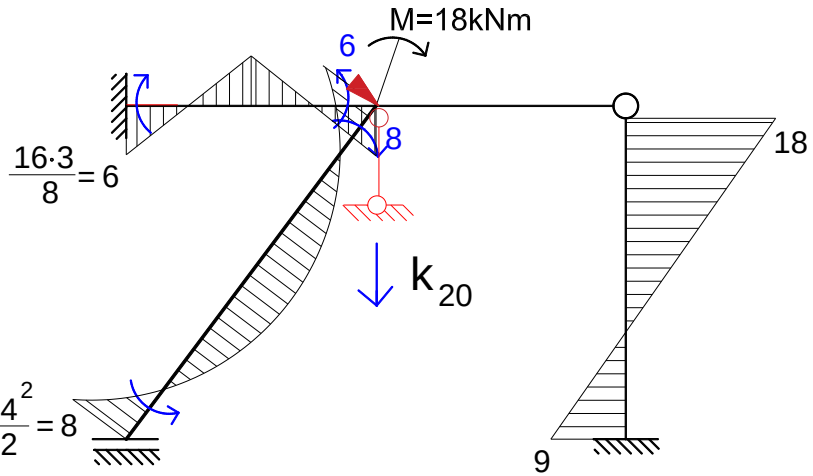
Wyznaczenie reakcji od przemieszczeń k_{21}, k_{22}, k_{20}





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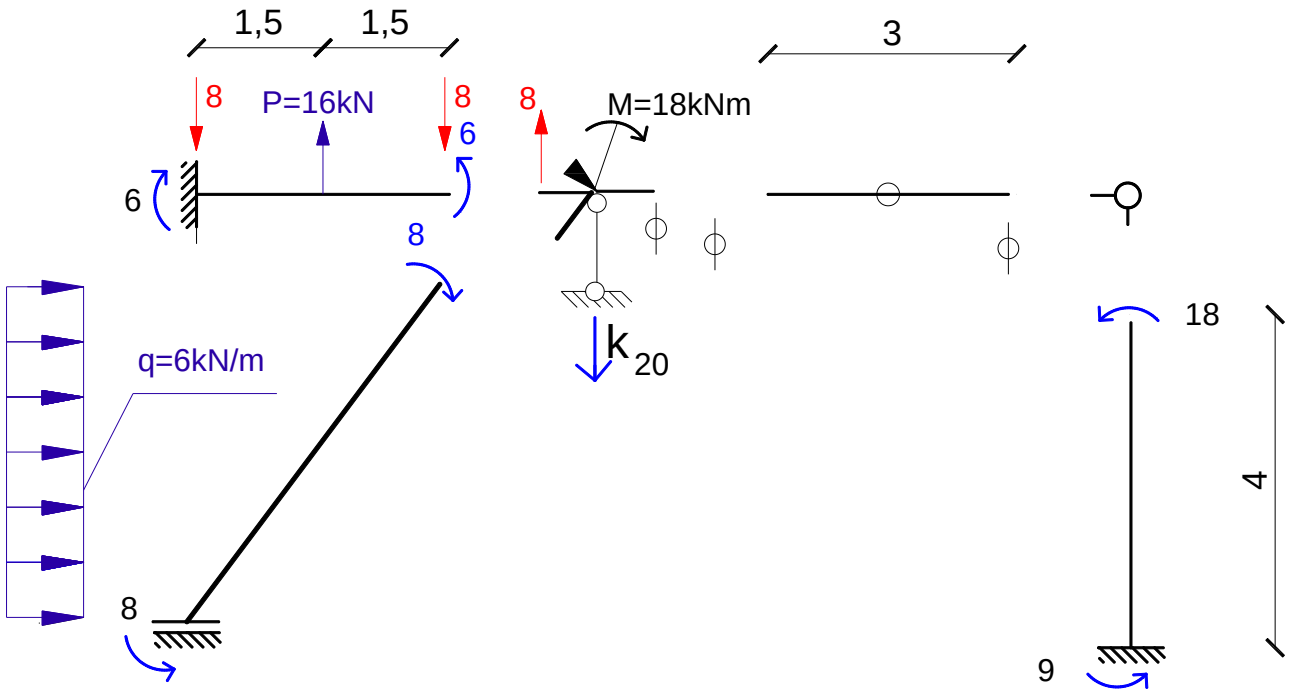

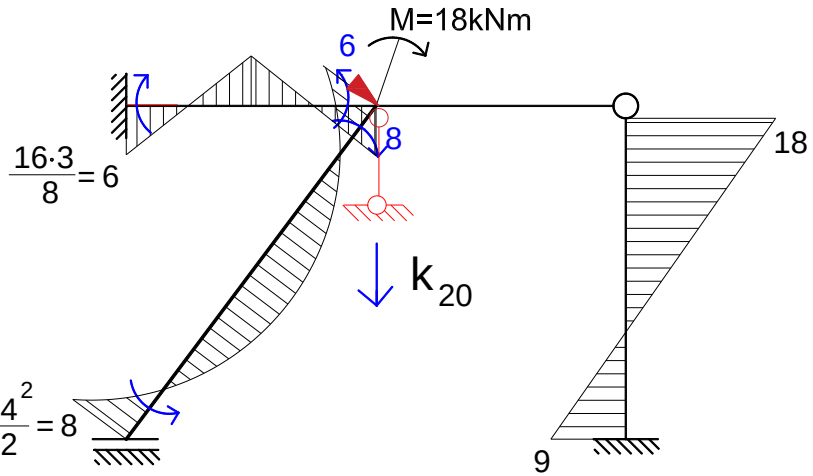

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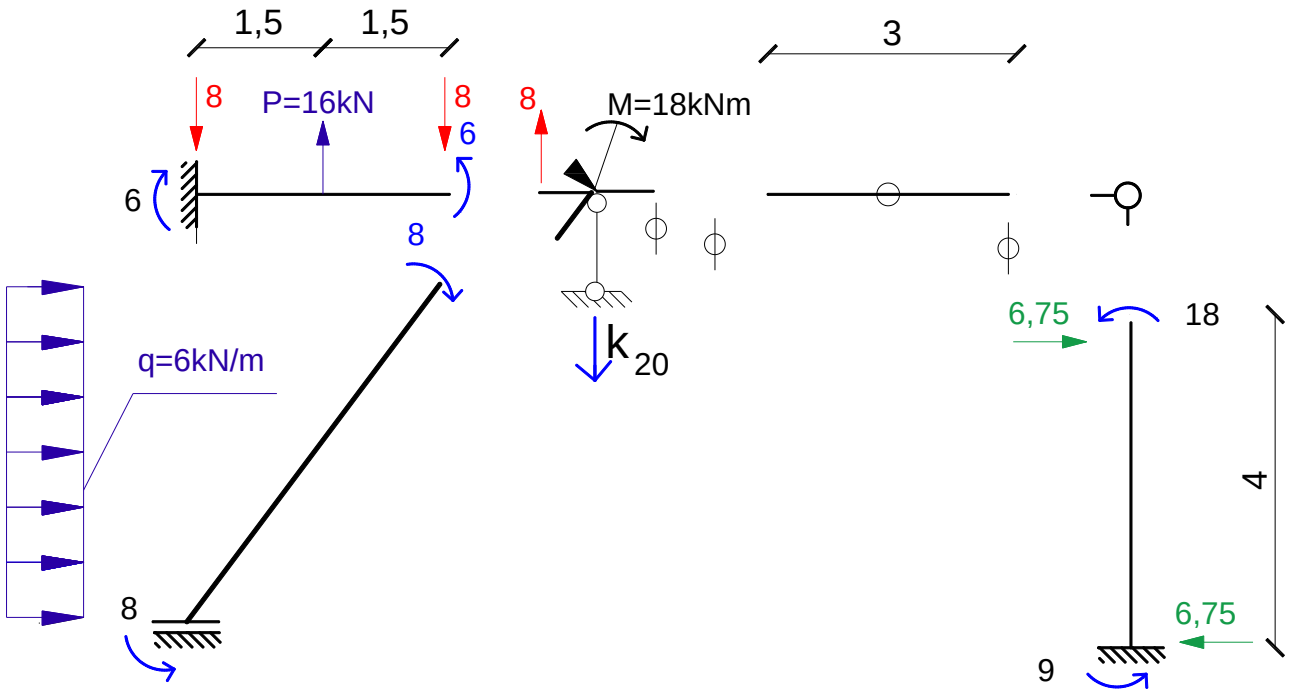

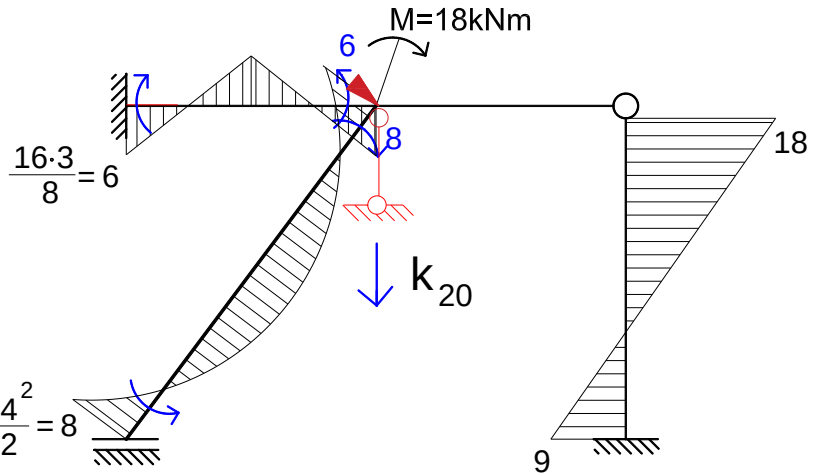

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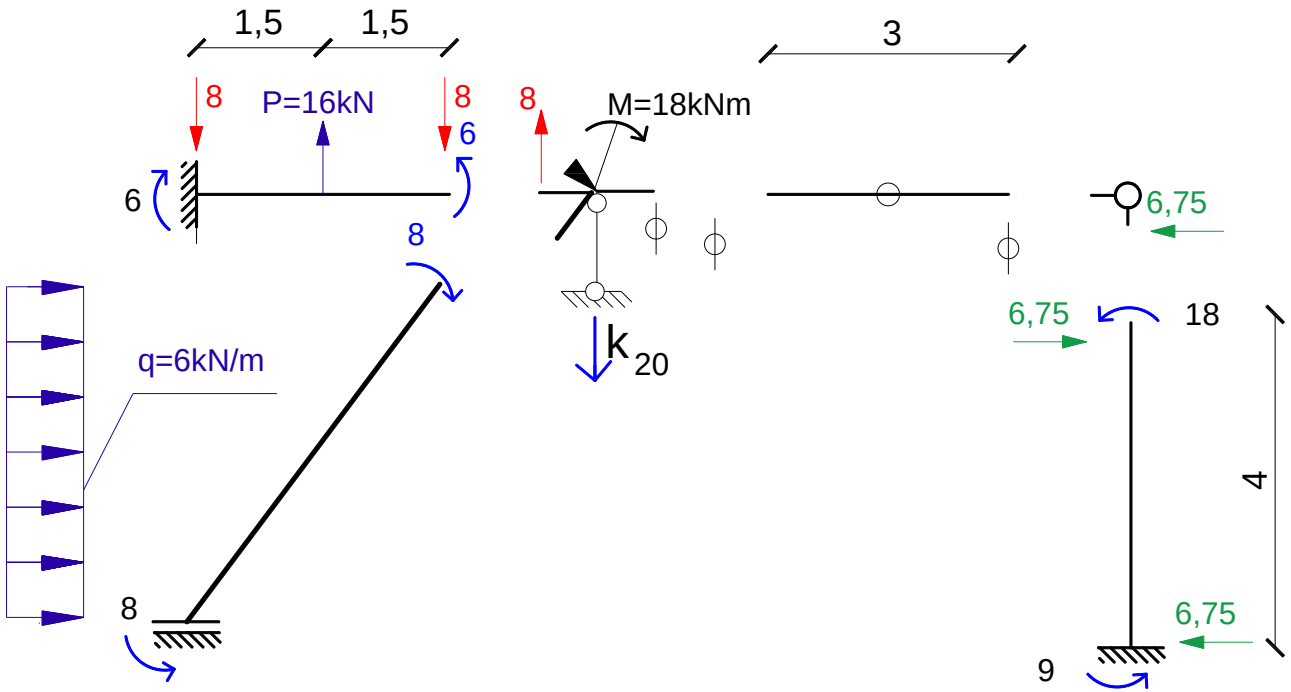

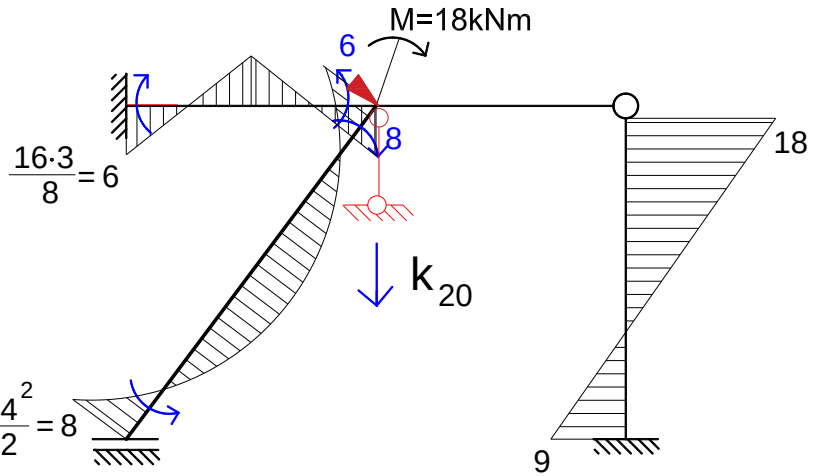

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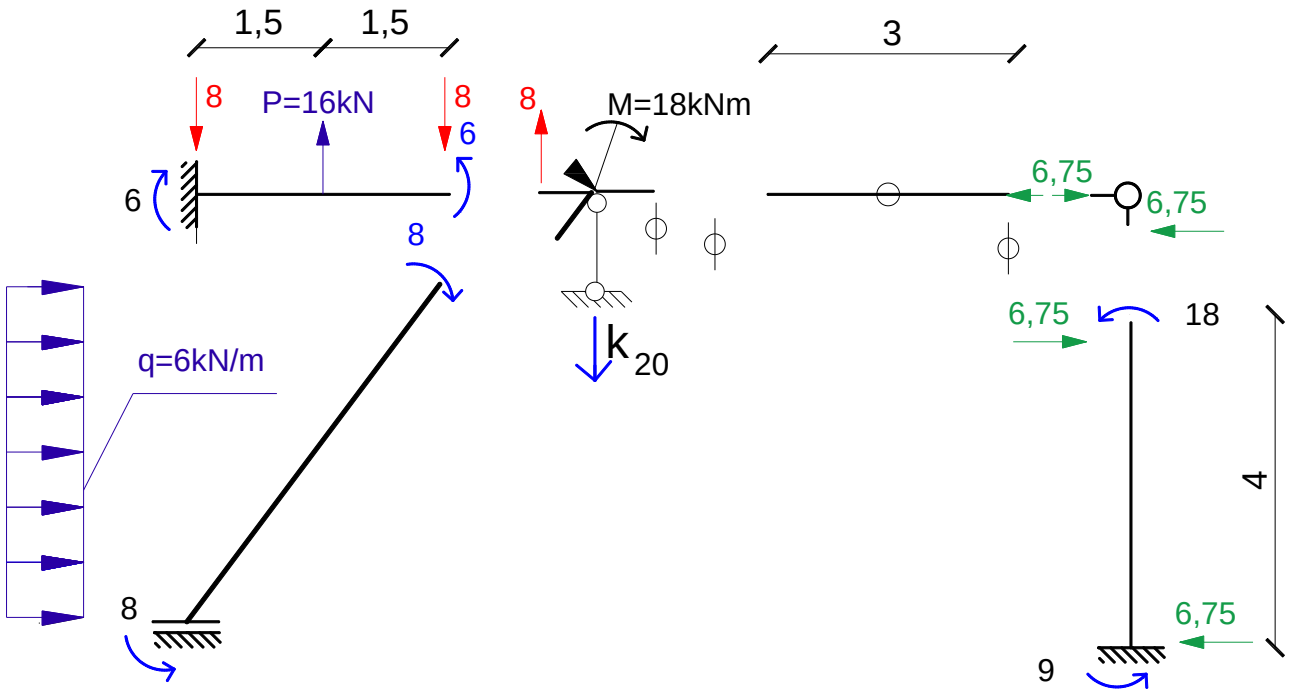

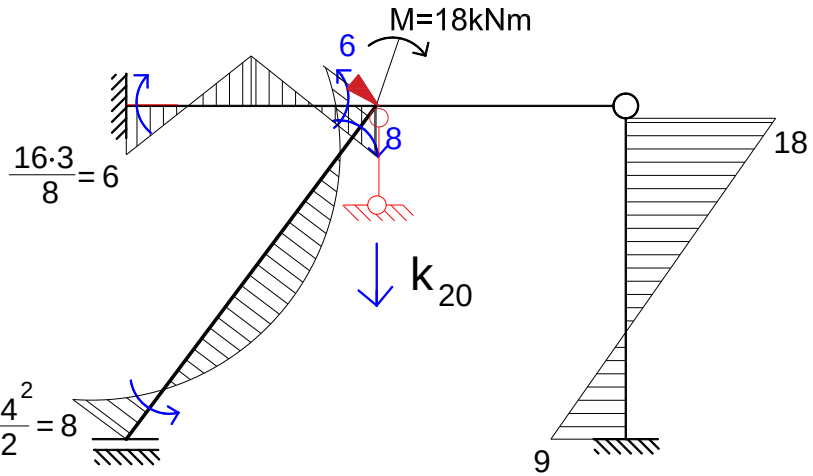

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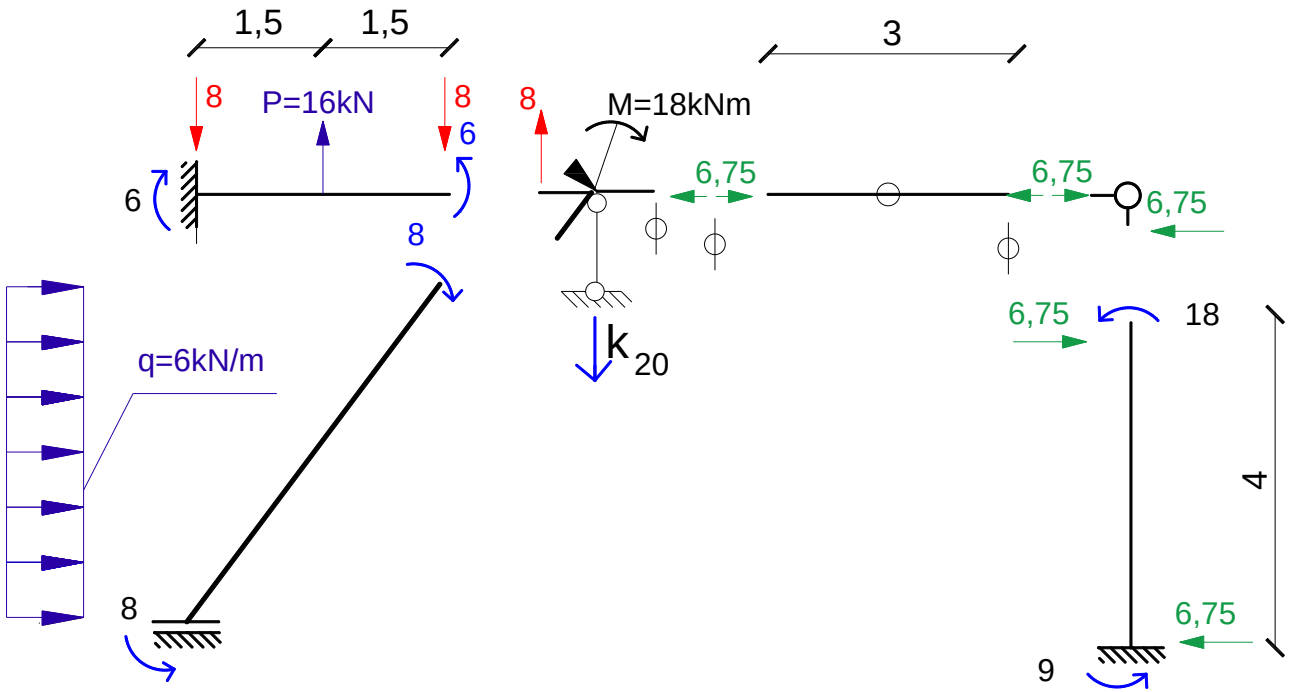

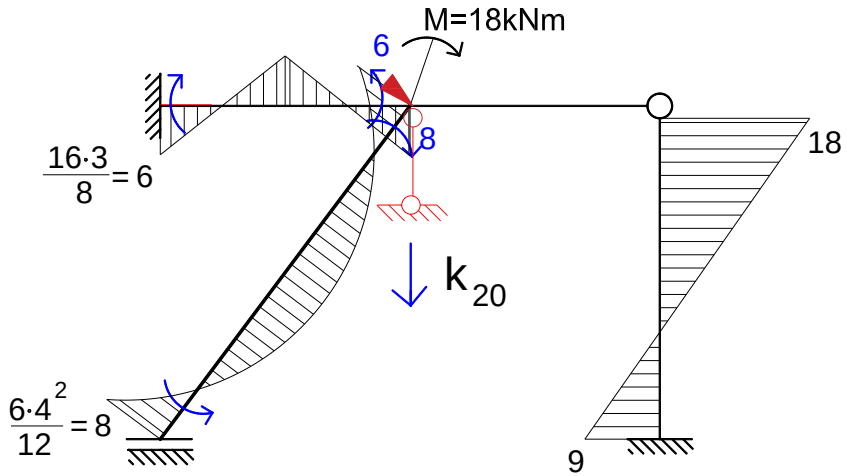

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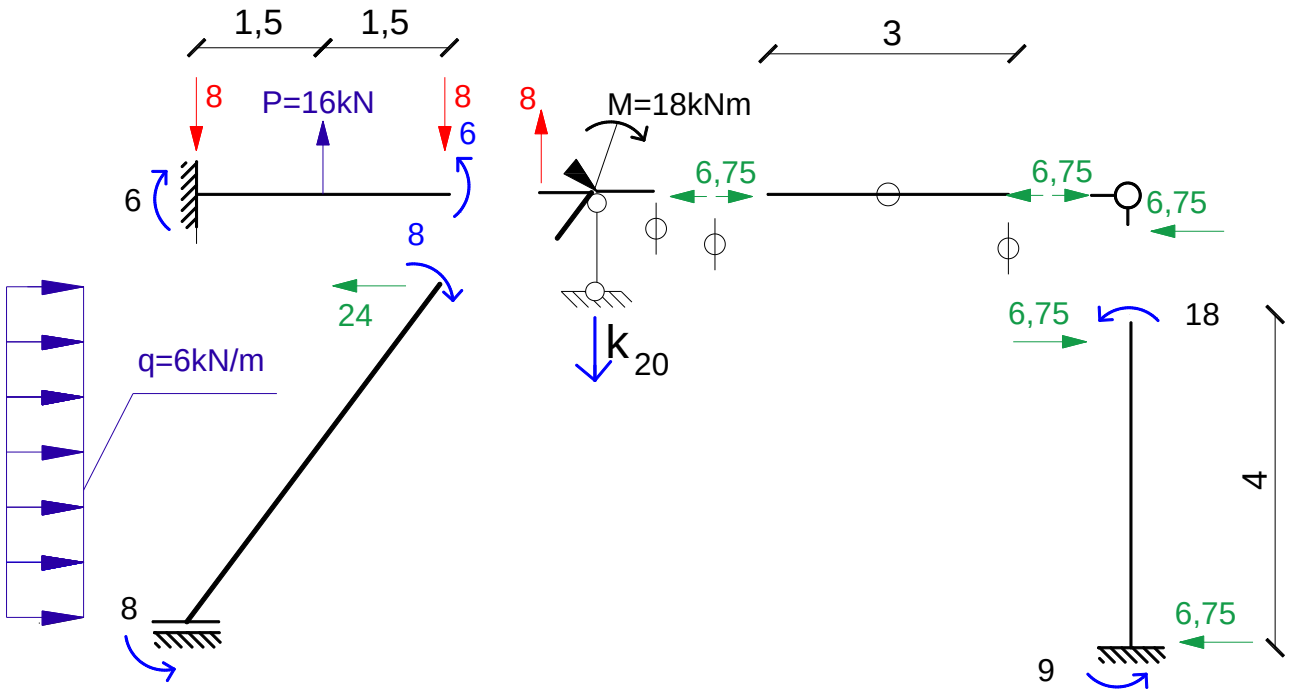

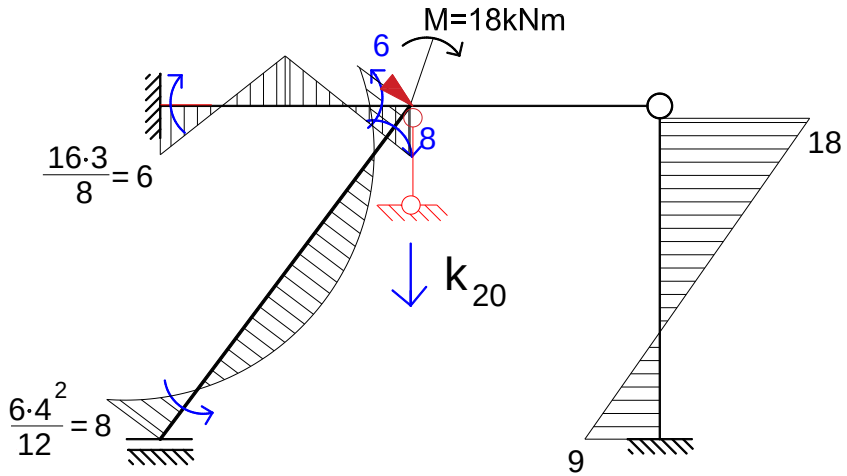

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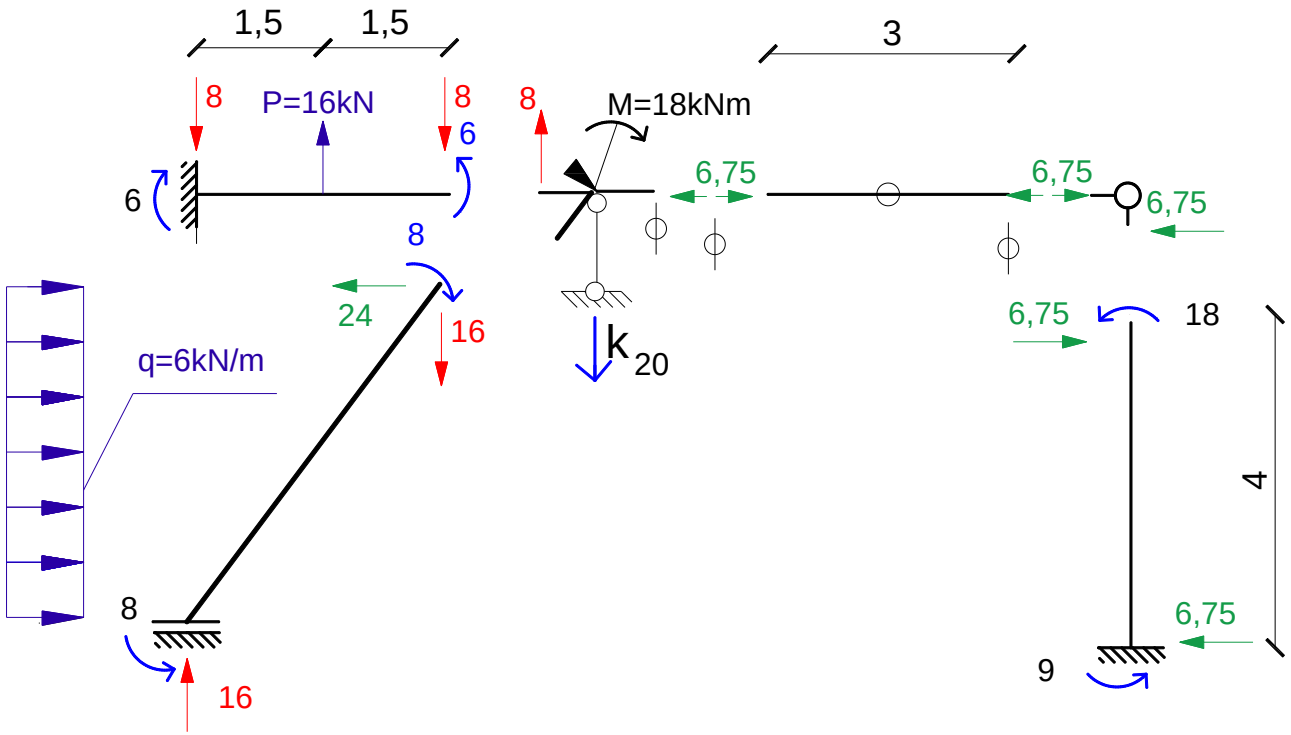

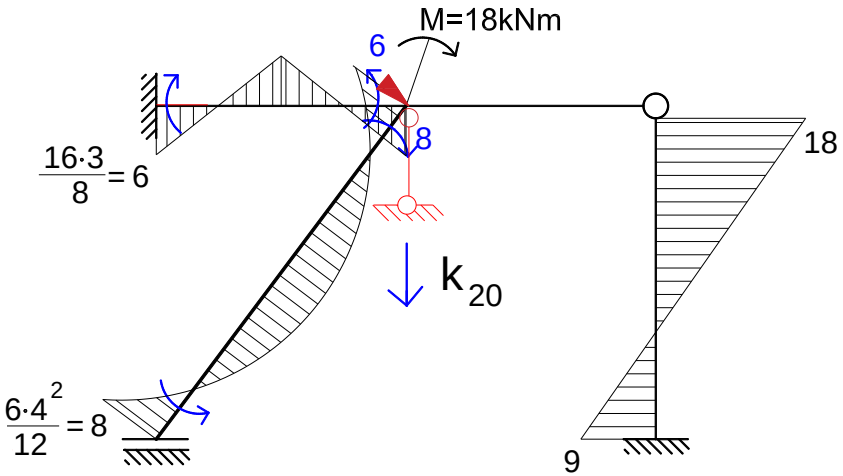

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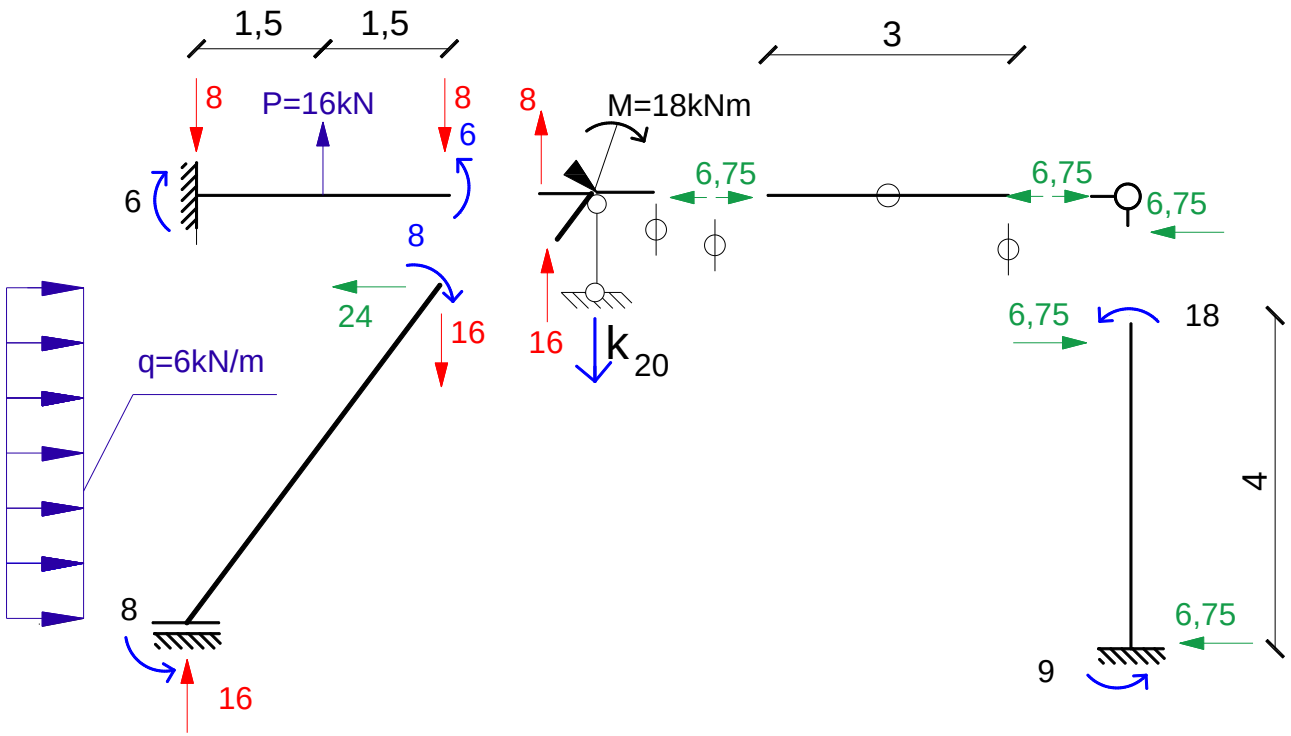

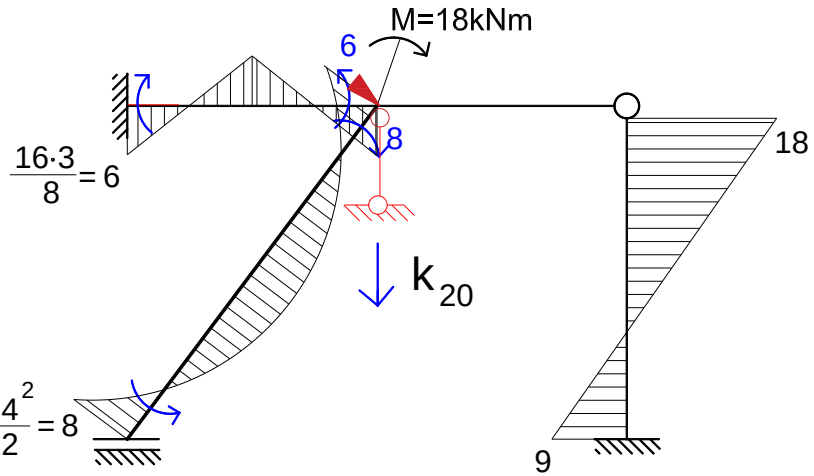

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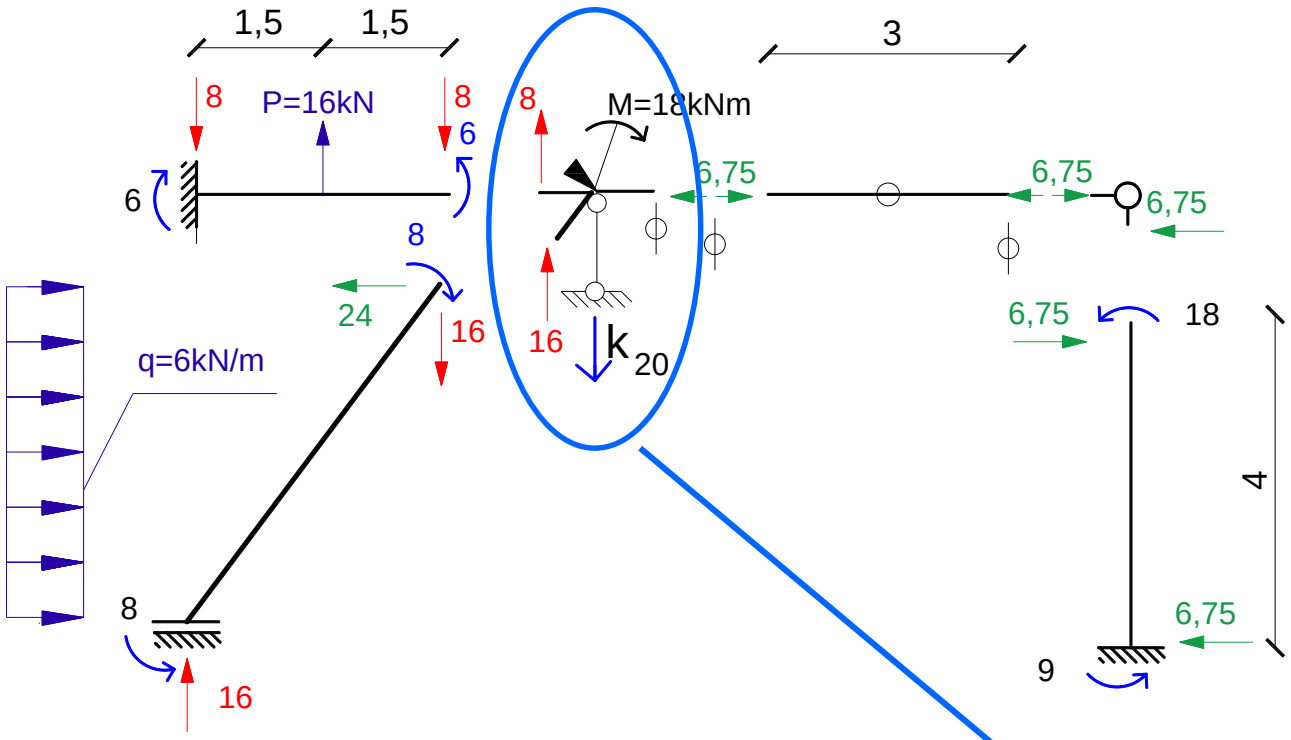

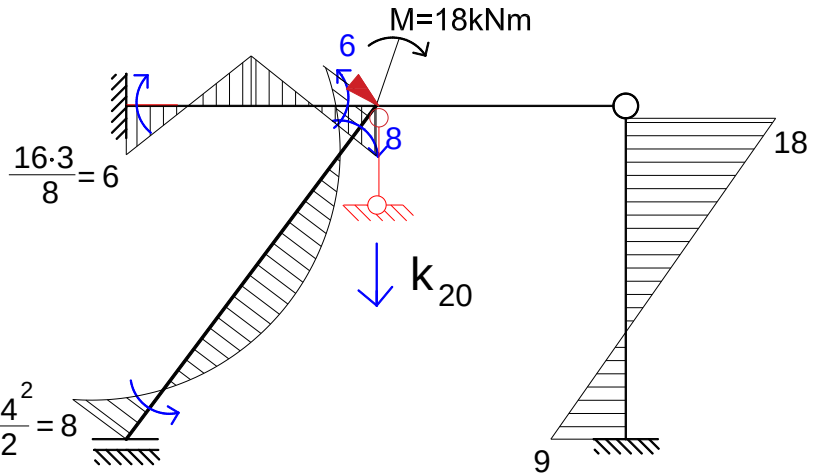

$$\sum R_y = 8 + 16 - k_{20} = 0$$

$$\downarrow$$

$$k_{20} = 24kN$$

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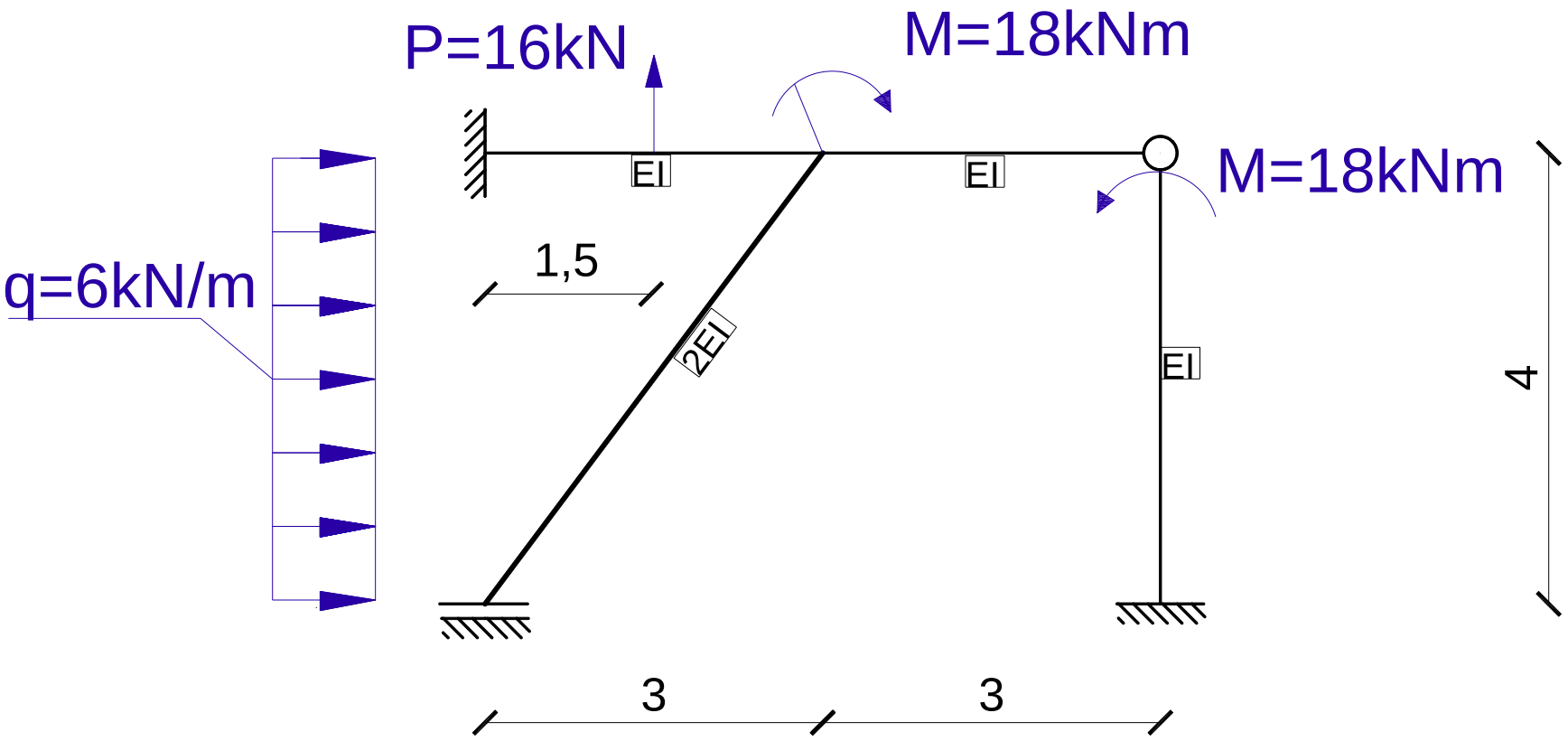
Zadanie 2: Wykorzystując wyniki z zadania 1 narysuj końcowy wykres momentów dla ramy statycznie niewyznaczalnej i wykonaj sprawdzenie poprawności wykresu korzystając z twierdzenia redukcyjnego.



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Układ równań metody przemieszczeń:

$$k_{11}\varphi_1 + k_{12}\Delta_2 + k_{10} = 0$$
$$k_{21}\varphi_1 + k_{22}\Delta_2 + k_{10} = 0$$

Podstawiając wyliczone wcześniej wartości otrzymujemy:

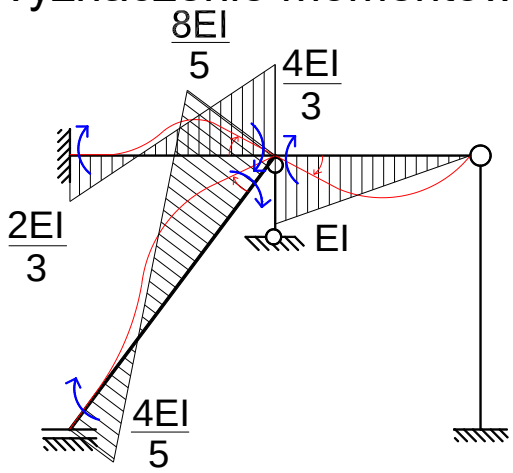
$$\frac{59EI}{15}\varphi_1 - \frac{17EI}{15}\Delta_2 - 16 = 0$$
$$-\frac{17EI}{15}\varphi_1 + \frac{49EI}{45}\Delta_2 + 24 = 0$$

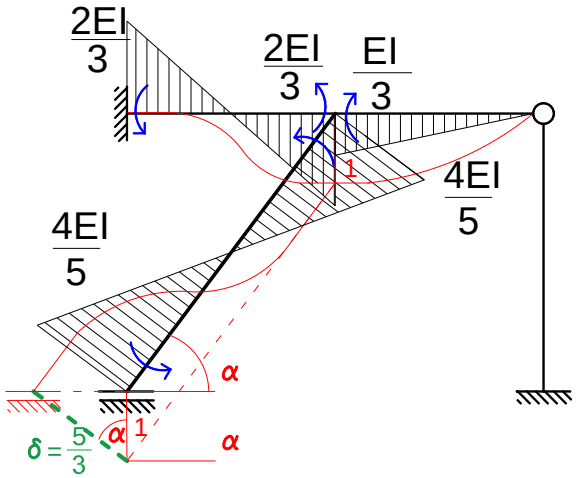
Rozwiązanie układu równań:

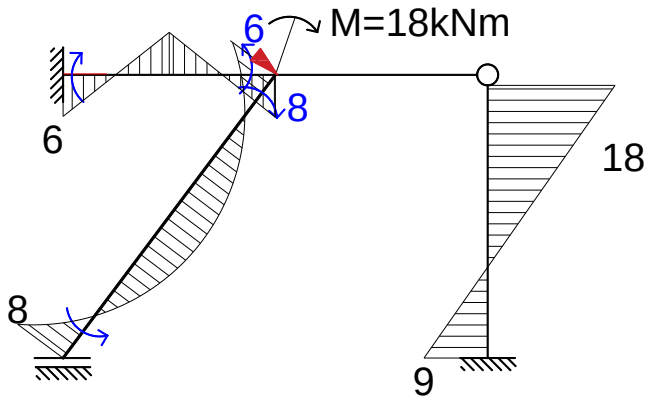
$$\varphi_1 = \frac{-3{,}261}{EI}$$

$$\Delta_2 = \frac{-25{,}435}{EI}$$

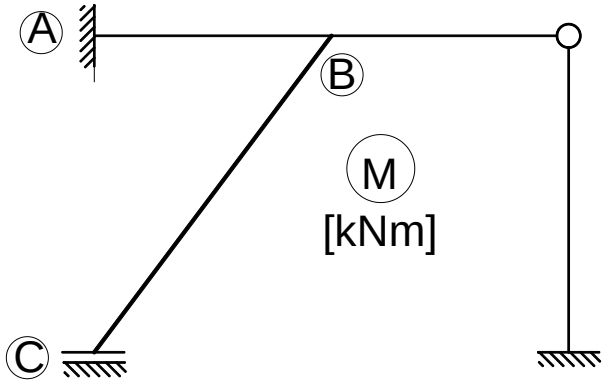
Wyznaczenie momentów na końcach elementów: $\varphi_1 = \frac{-3{,}261}{EI}$ $\Delta_2 = \frac{-25{,}435}{EI}$







$$M_i = M_i^{\varphi 1=1} \cdot \varphi_1 + M_i^{\Delta 2=1} \cdot \Delta_2 + M_i^{obc.zewn.}$$



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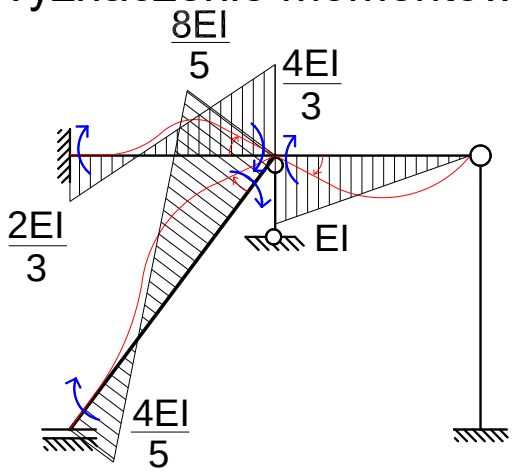

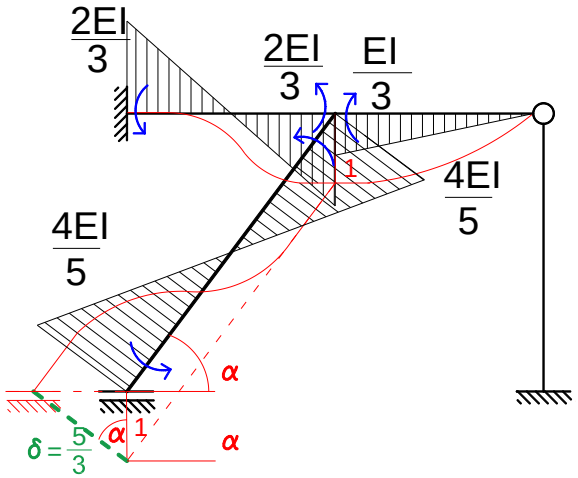

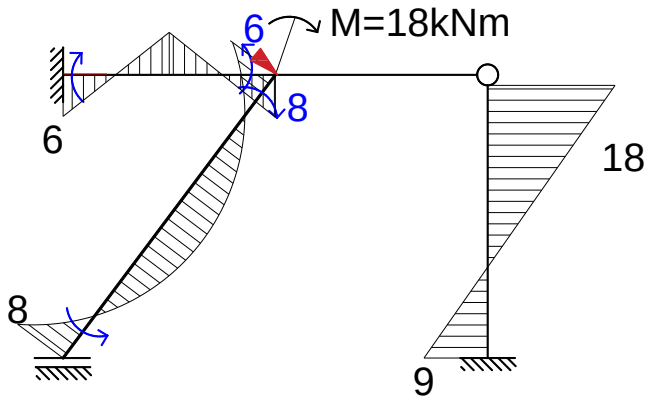

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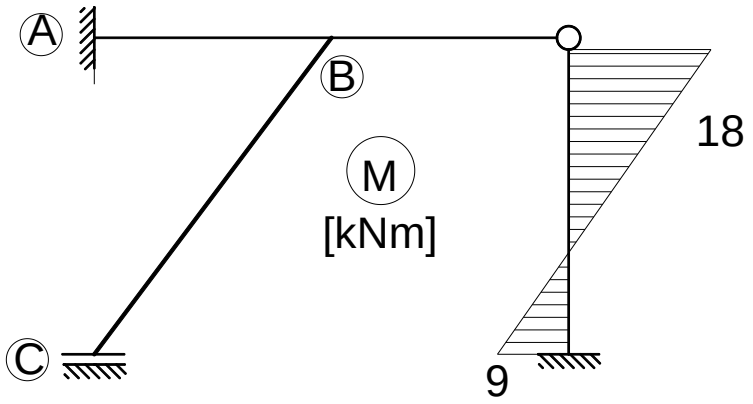

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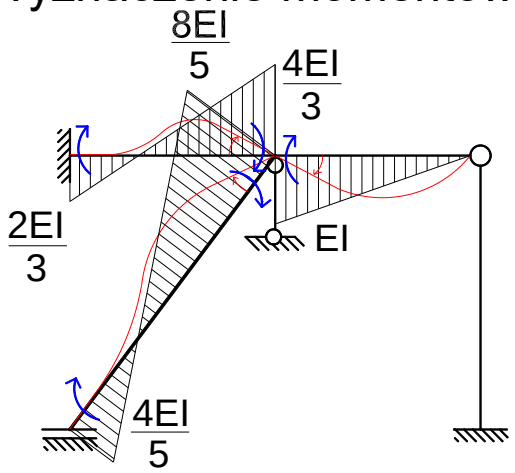
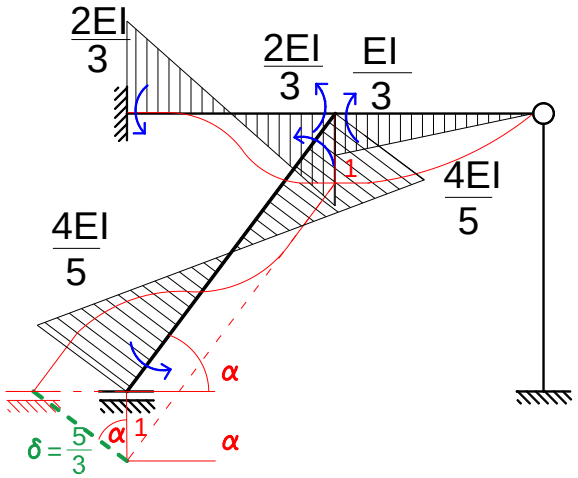
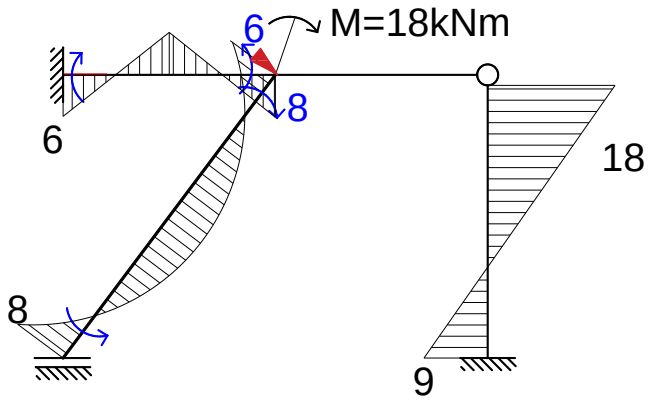
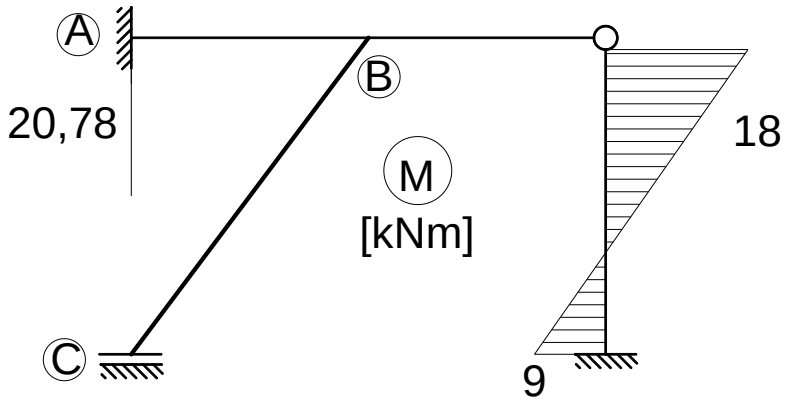






$$M_i = M_i^{\varphi 1=1} \cdot \varphi_1 + M_i^{\Delta 2=1} \cdot \Delta_2 + M_i^{obc.zewn.}$$

$$M_A = \frac{2EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} + 6 = 20{,}78kNm$$



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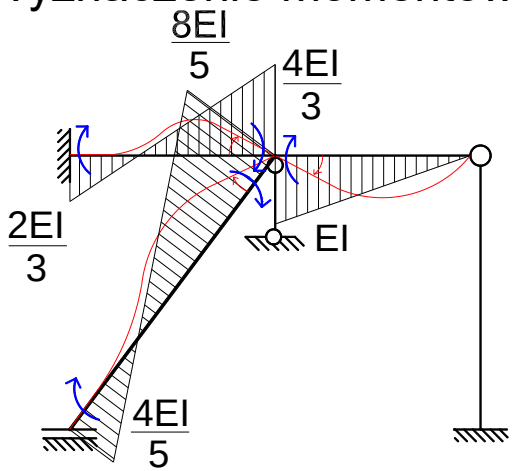
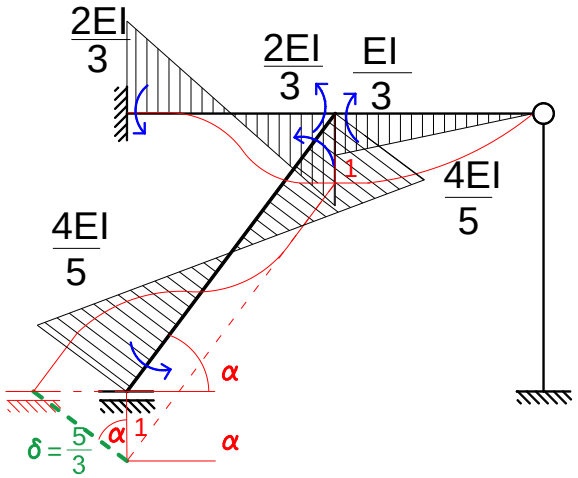
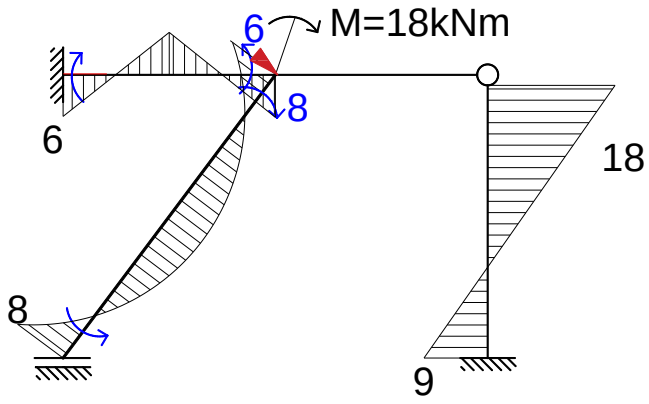


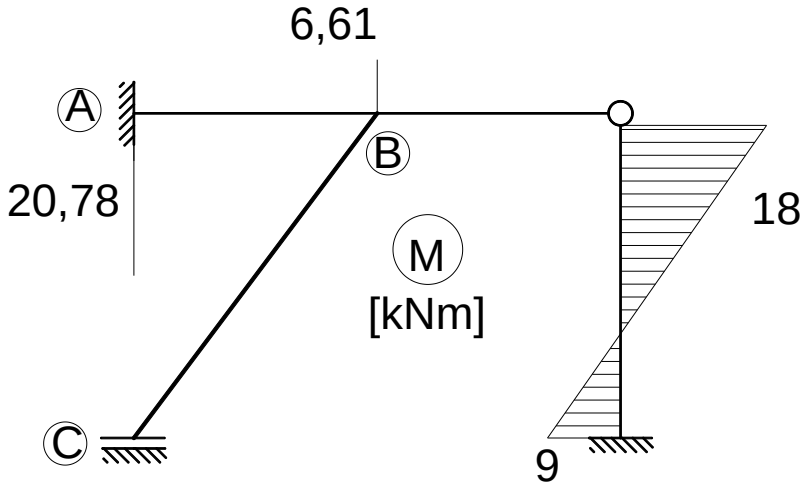




$$M_i = M_i^{\varphi 1=1} \cdot \varphi_1 + M_i^{\Delta 2=1} \cdot \Delta_2 + M_i^{obc.zewn.}$$

$$M_A = \frac{2EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} + 6 = 20{,}78kNm$$

$$M_{BL} = \frac{4EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} - 6 = 6{,}61kNm$$



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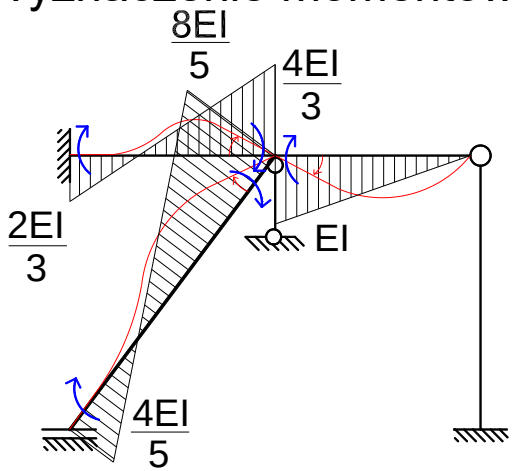

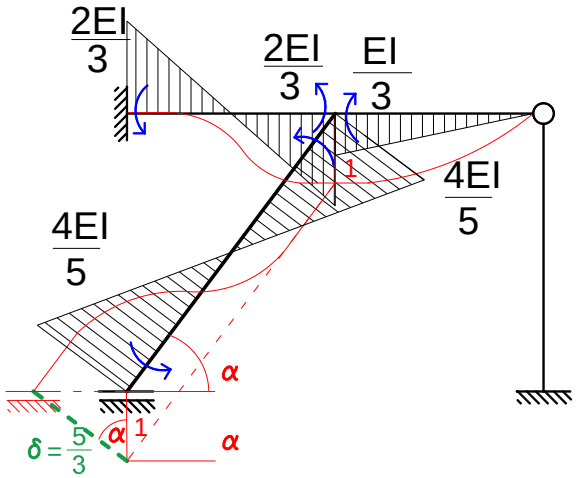

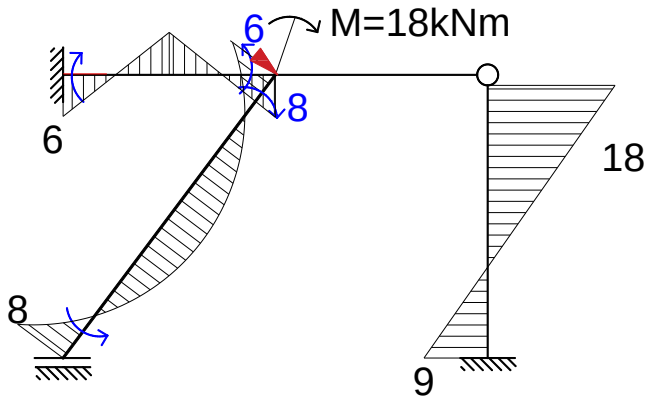

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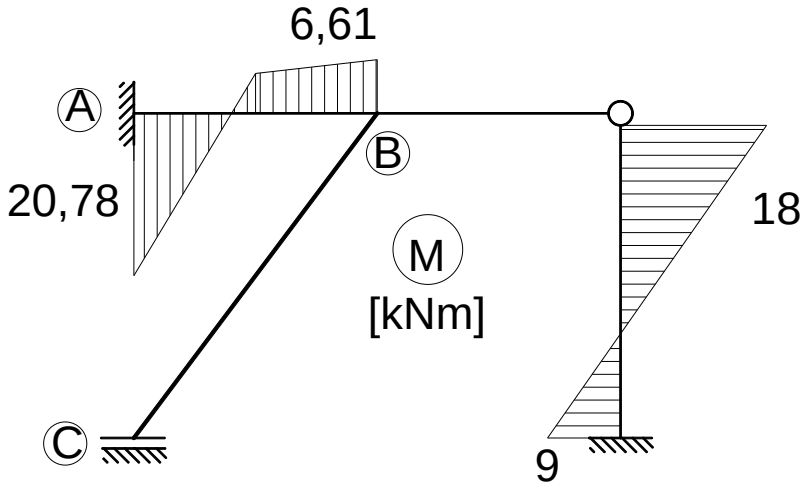

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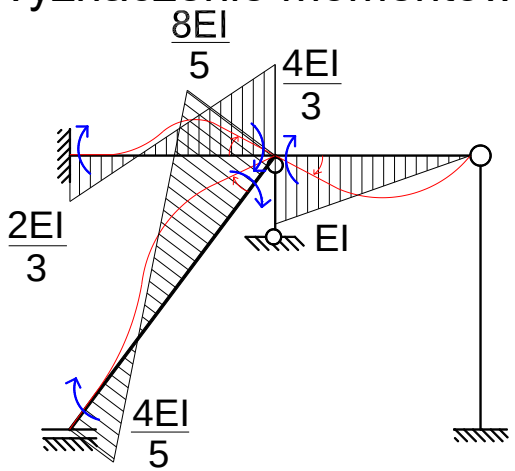

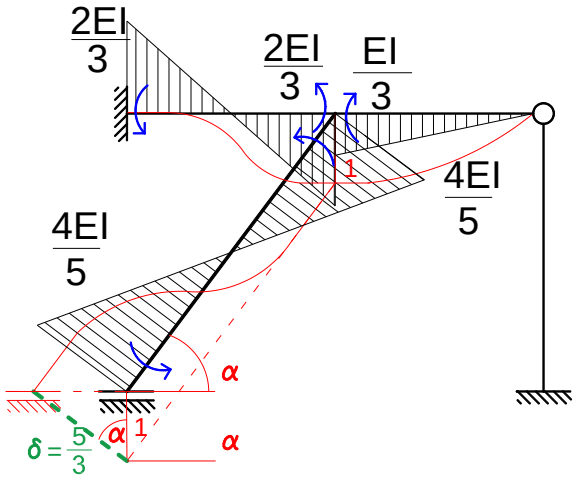

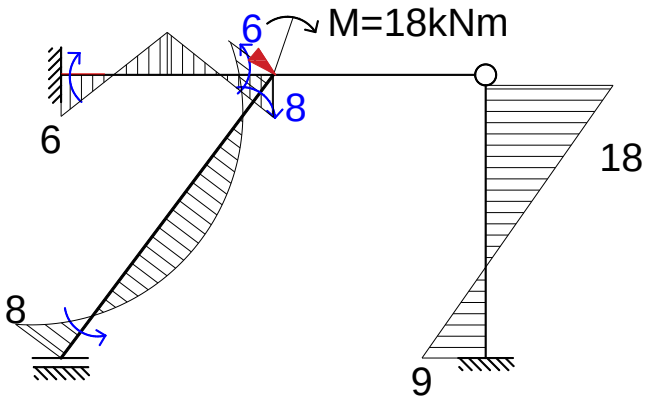

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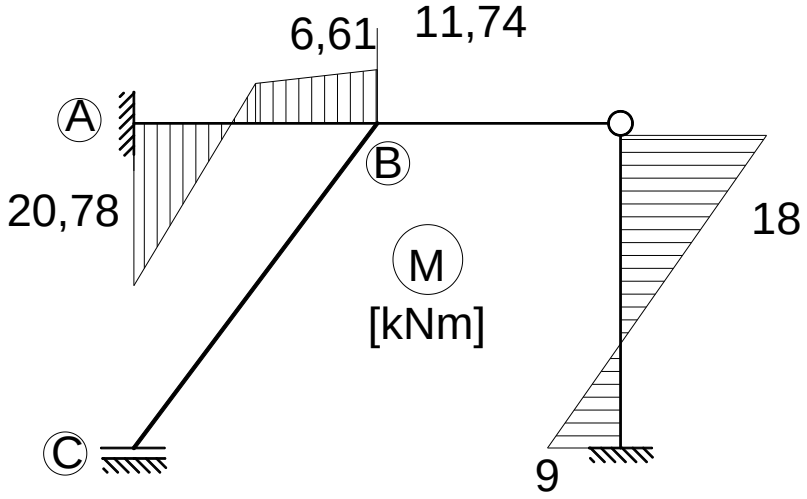
$$M_{Bp} = EI \cdot \frac{(-3{,}261)}{EI} + \frac{EI}{3} \cdot \frac{(-25{,}435)}{EI} = -11{,}74kNm$$



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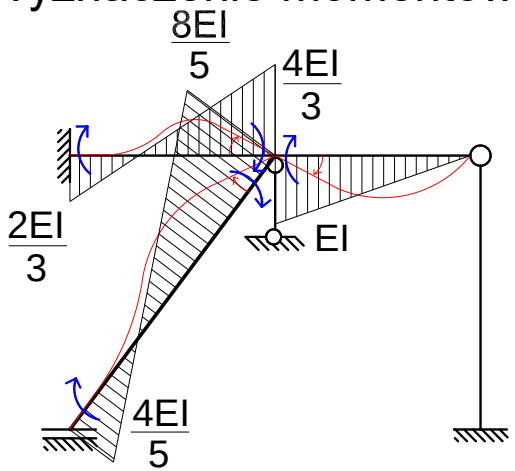

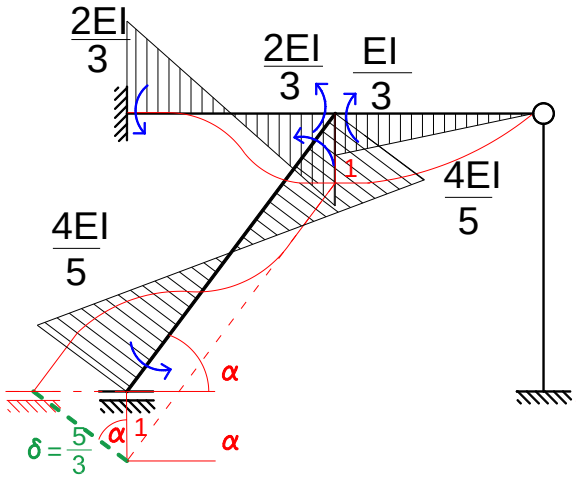

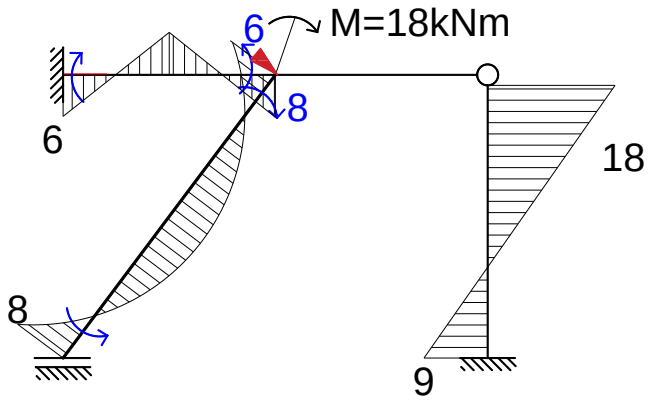

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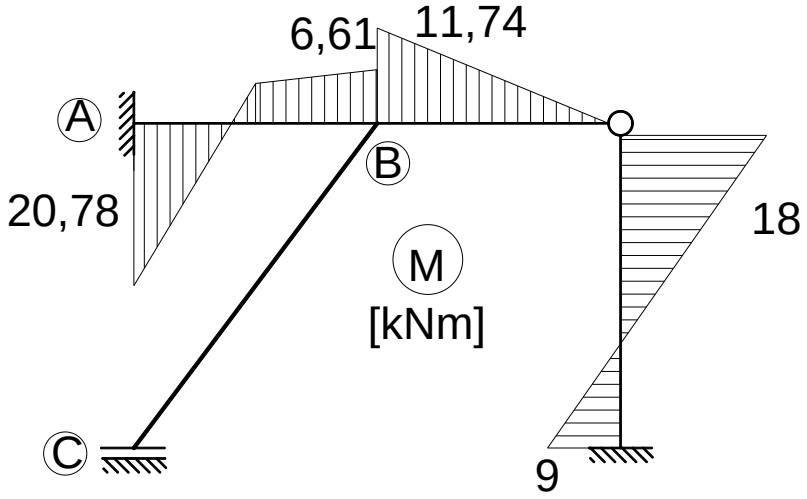
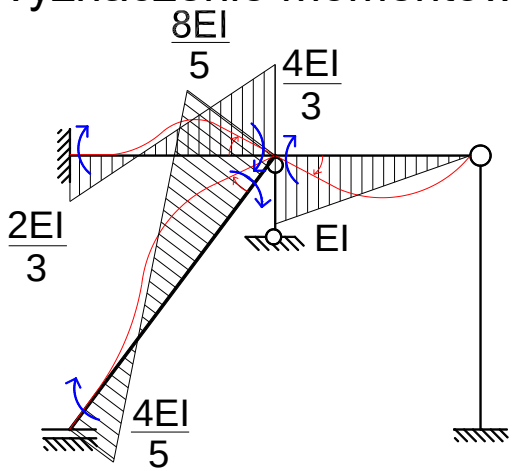
$$M_A = \frac{2EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} + 6 = 20{,}78kNm$$

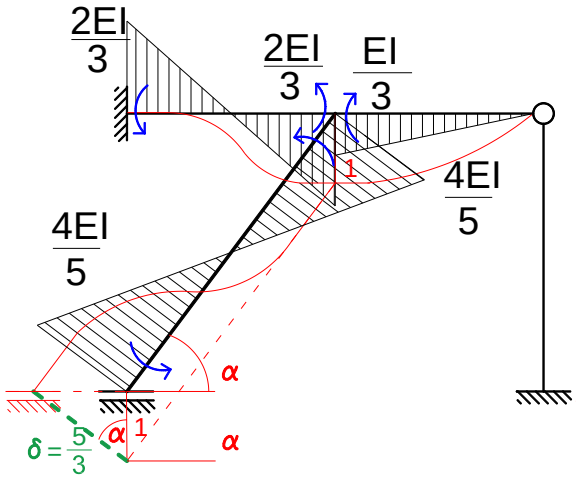
$$M_{BL} = \frac{4EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} - 6 = 6{,}61kNm$$

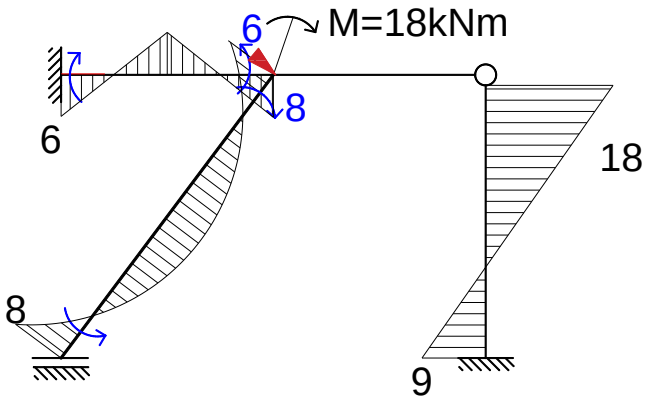
$$M_{Bp} = EI \cdot \frac{(-3{,}261)}{EI} + \frac{EI}{3} \cdot \frac{(-25{,}435)}{EI} = -11{,}74kNm$$



Wyznaczenie momentów na końcach elementów: $\varphi_1 = \frac{-3{,}261}{EI}$ $\Delta_2 = \frac{-25{,}435}{EI}$







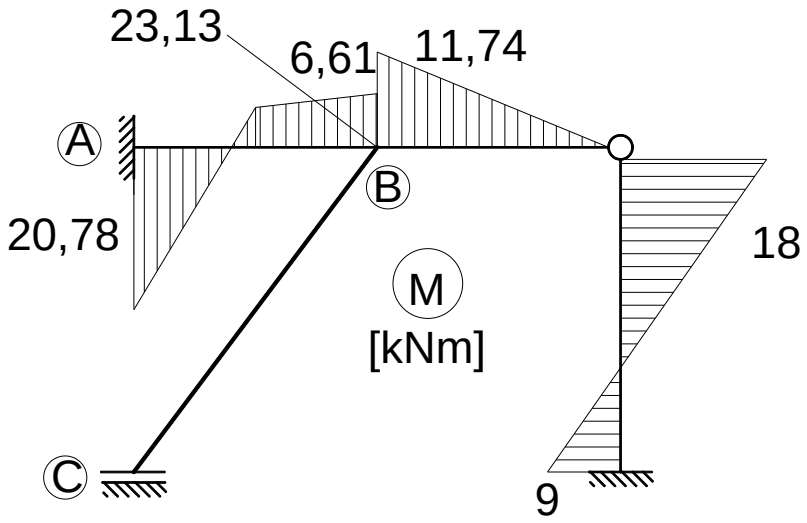
$$M_i = M_i^{\varphi 1=1} \cdot \varphi_1 + M_i^{\Delta 2=1} \cdot \Delta_2 + M_i^{obc.zewn.}$$

$$M_A = \frac{2EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} + 6 = 20{,}78kNm$$

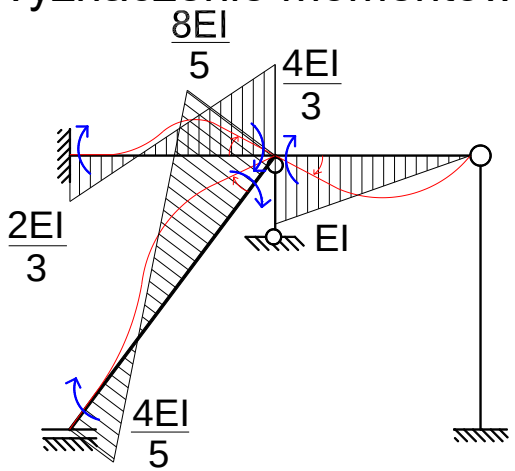
$$M_{BL} = \frac{4EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} - 6 = 6{,}61kNm$$

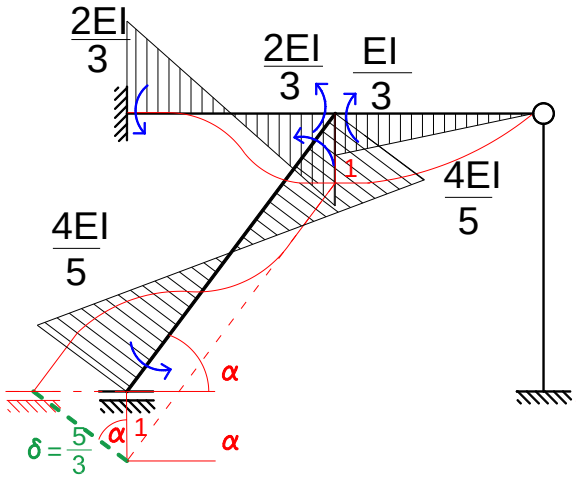
$$M_{Bp} = EI \cdot \frac{(-3{,}261)}{EI} + \frac{EI}{3} \cdot \frac{(-25{,}435)}{EI} = -11{,}74kNm$$

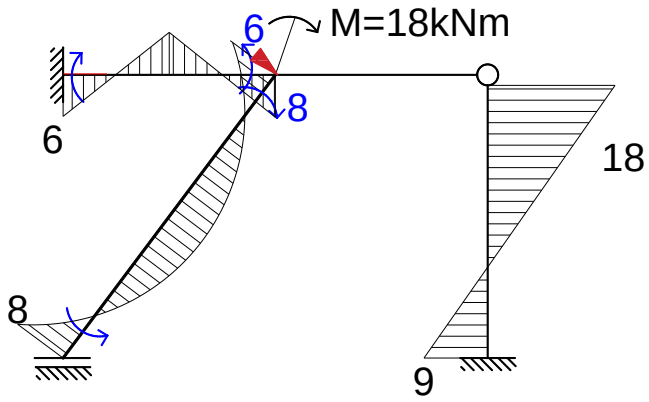
$$M_{BD} = \frac{8EI}{5} \cdot \frac{(-3{,}261)}{EI} - \frac{4EI}{5} \cdot \frac{(-25{,}435)}{EI} + 8 = 23{,}13kNm$$



Wyznaczenie momentów na końcach elementów: $\varphi_1 = \frac{-3{,}261}{EI}$ $\Delta_2 = \frac{-25{,}435}{EI}$







$$M_i = M_i^{\varphi 1=1} \cdot \varphi_1 + M_i^{\Delta 2=1} \cdot \Delta_2 + M_i^{obc.zewn.}$$

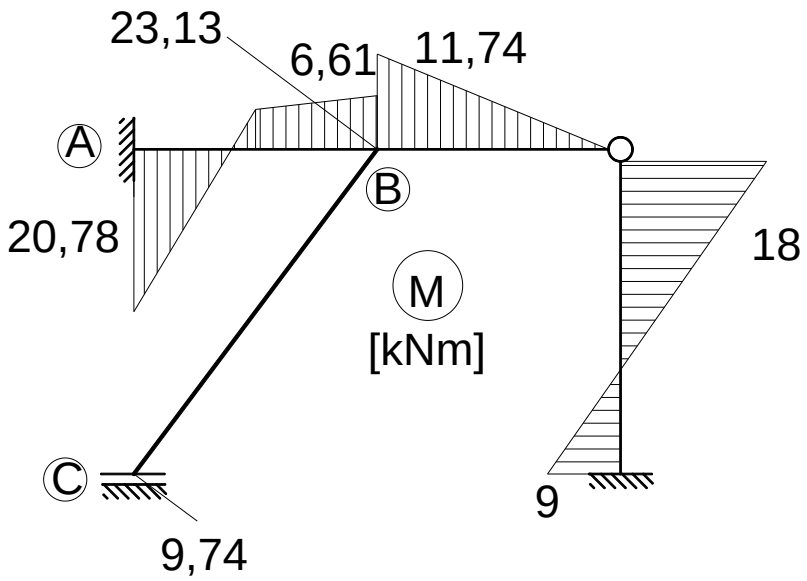
$$M_A = \frac{2EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} + 6 = 20{,}78kNm$$

$$M_{BL} = \frac{4EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} - 6 = 6{,}61kNm$$

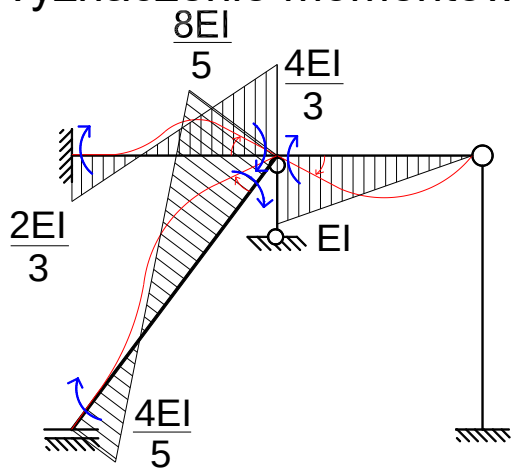
$$M_{Bp} = EI \cdot \frac{(-3{,}261)}{EI} + \frac{EI}{3} \cdot \frac{(-25{,}435)}{EI} = -11{,}74kNm$$

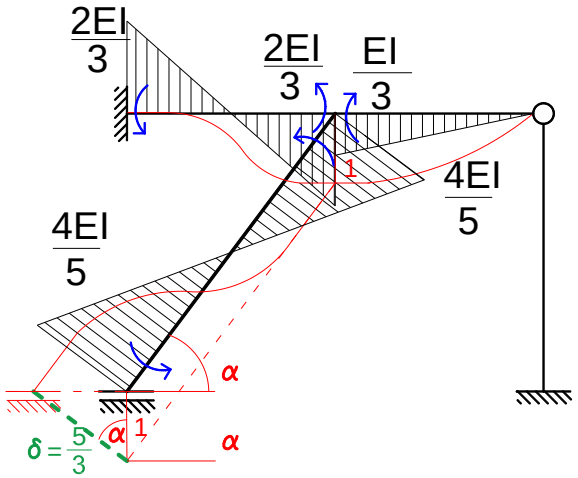
$$M_{BD} = \frac{8EI}{5} \cdot \frac{(-3{,}261)}{EI} - \frac{4EI}{5} \cdot \frac{(-25{,}435)}{EI} + 8 = 23{,}13kNm$$

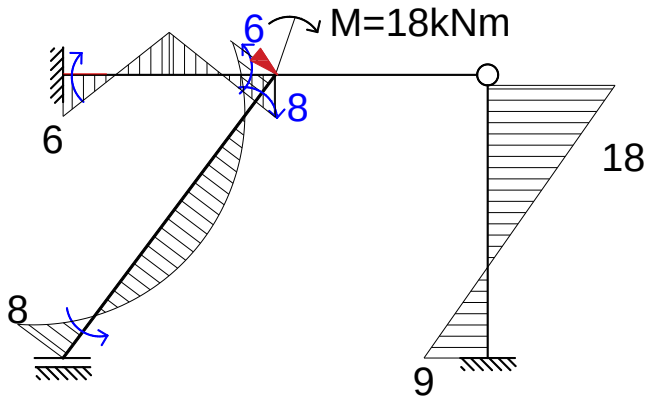
$$M_C = \frac{4EI}{5} \cdot \frac{(-3{,}261)}{EI} - \frac{4EI}{5} \cdot \frac{(-25{,}435)}{EI} - 8 = 9{,}74kNm$$



Wyznaczenie momentów na końcach elementów: $\varphi_1 = \frac{-3{,}261}{EI}$ $\Delta_2 = \frac{-25{,}435}{EI}$







$$M_i = M_i^{\varphi 1=1} \cdot \varphi_1 + M_i^{\Delta 2=1} \cdot \Delta_2 + M_i^{obc.zewn.}$$

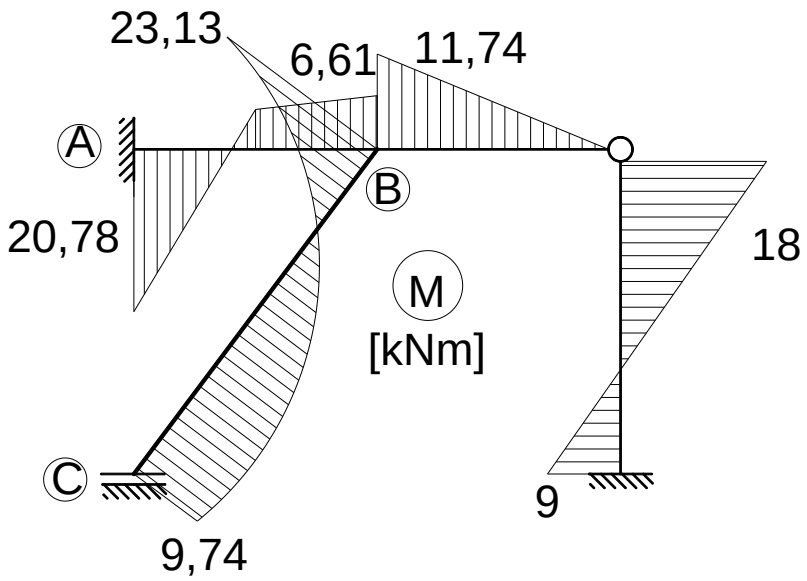
$$M_A = \frac{2EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} + 6 = 20{,}78kNm$$

$$M_{BL} = \frac{4EI}{3} \cdot \frac{(-3{,}261)}{EI} - \frac{2EI}{3} \cdot \frac{(-25{,}435)}{EI} - 6 = 6{,}61kNm$$

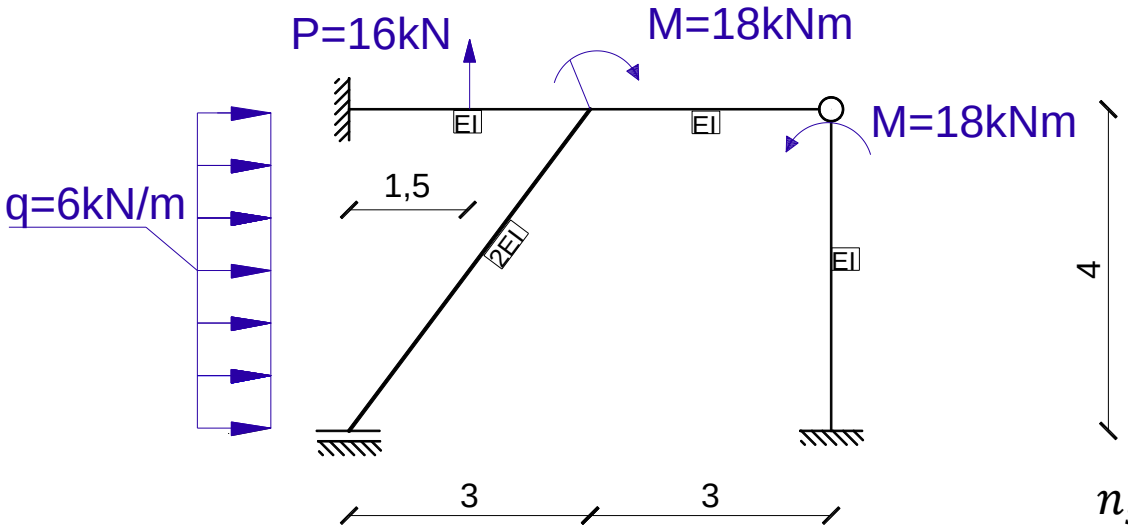
$$M_{Bp} = EI \cdot \frac{(-3{,}261)}{EI} + \frac{EI}{3} \cdot \frac{(-25{,}435)}{EI} = -11{,}74kNm$$

$$M_{BD} = \frac{8EI}{5} \cdot \frac{(-3{,}261)}{EI} - \frac{4EI}{5} \cdot \frac{(-25{,}435)}{EI} + 8 = 23{,}13kNm$$

$$M_C = \frac{4EI}{5} \cdot \frac{(-3{,}261)}{EI} - \frac{4EI}{5} \cdot \frac{(-25{,}435)}{EI} - 8 = 9{,}74kNm$$



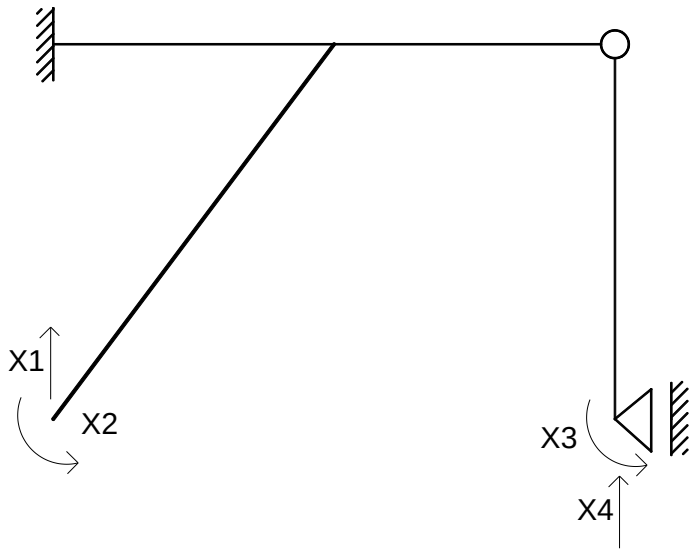
Określenie stopnia statycznej niewyznaczalności ramy:



$$n_s = lr - lp - 3 = 8 - 1 - 3 = 4$$

Rama czterokrotnie statycznie niewyznaczalna.

Schemat podstawowy statycznie wyznaczalny:



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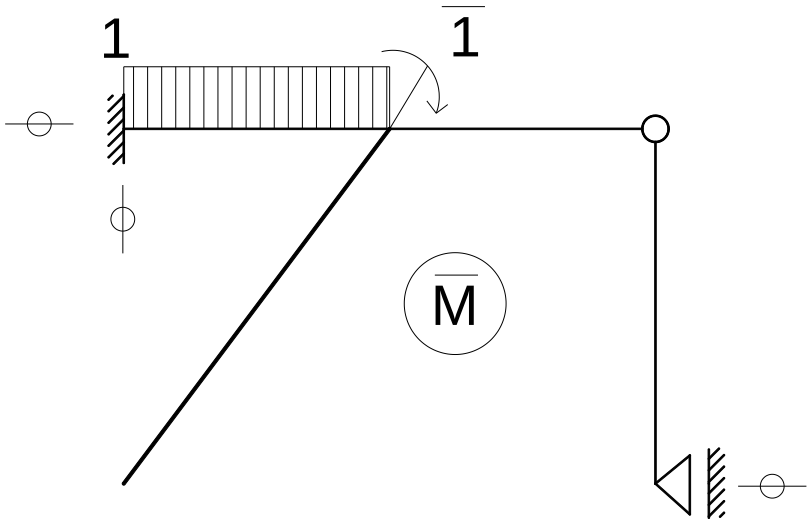
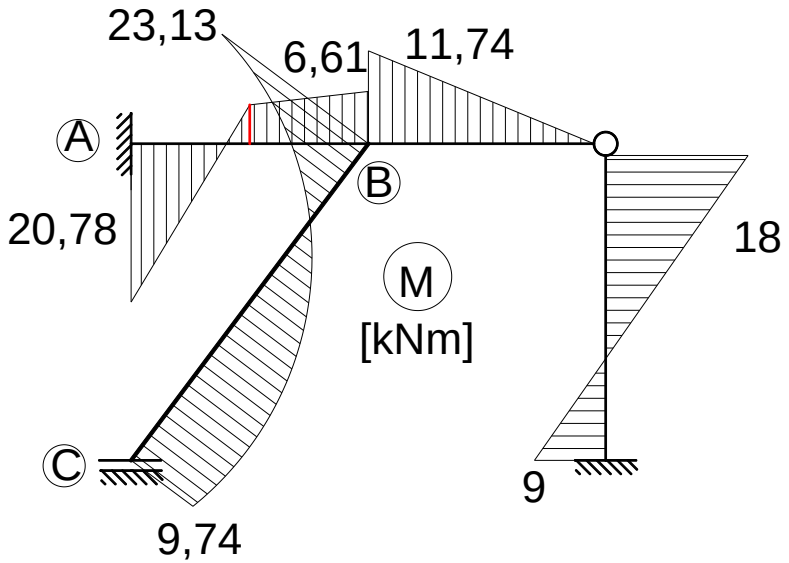
Sprawdzenie z twierdzenia redukcyjnego:





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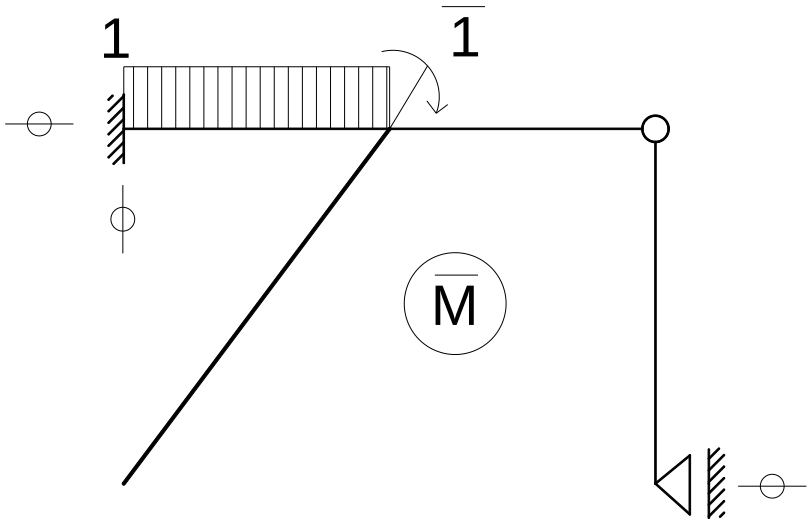
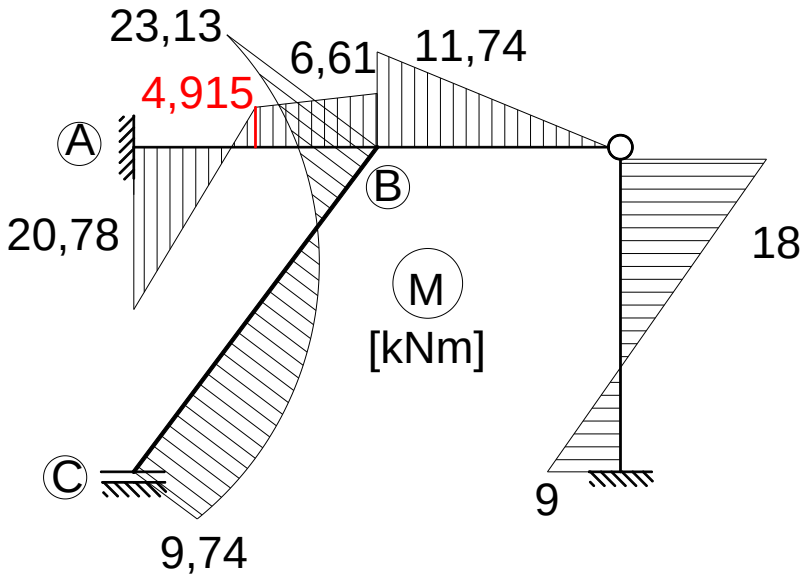
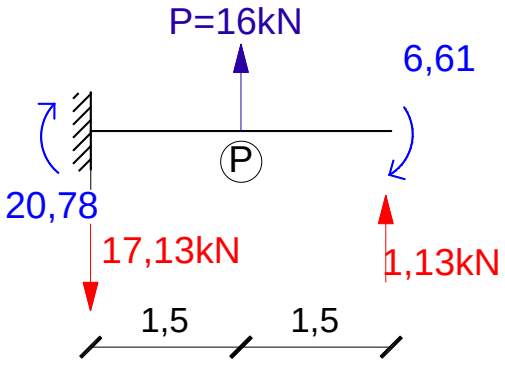
Sprawdzenie z twierdzenia redukcyjnego:

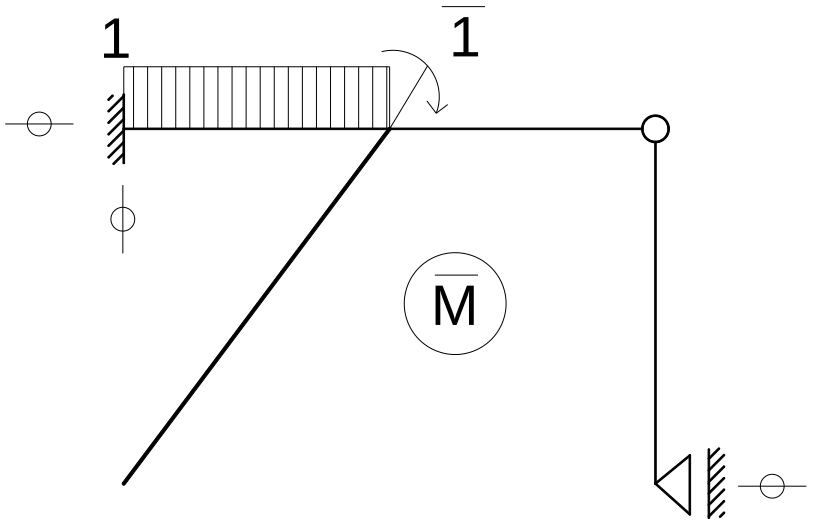


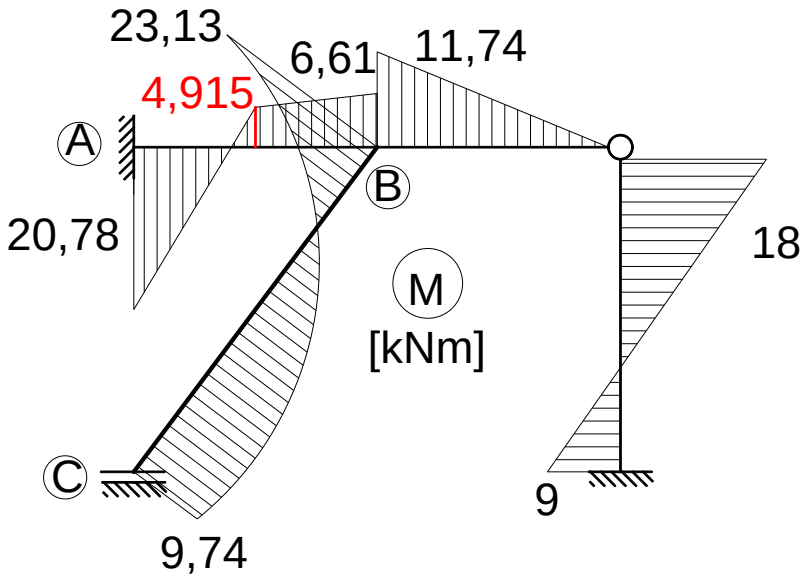


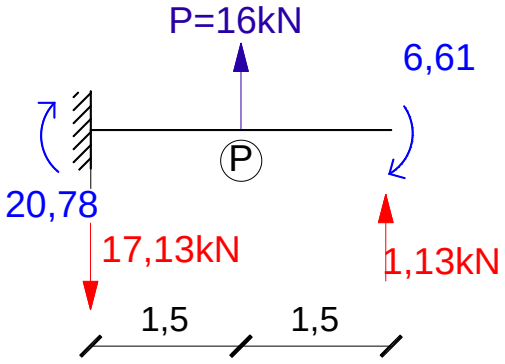


$$M_P = 20{,}78 - 17{,}13 \cdot 1{,}5 = -4{,}915 kNm$$

Sprawdzenie z twierdzenia redukcyjnego:







$$\varphi_B = \frac{1}{EI}\left(-\frac{1}{2}\cdot 20{,}78\cdot 1{,}5\cdot 1+\frac{1}{2}\cdot 4{,}915\cdot 1{,}5\cdot 1+\frac{1}{2}\cdot 4{,}915\cdot 1{,}5\cdot 1+\frac{1}{2}\cdot 6{,}61\cdot 1{,}5\cdot 1\right)$$

$$M_P = 20{,}78 - 17{,}13\cdot 1{,}5 = -4{,}915 kNm$$

$$\varphi_B = \frac{-3{,}26}{EI} = \varphi_1$$

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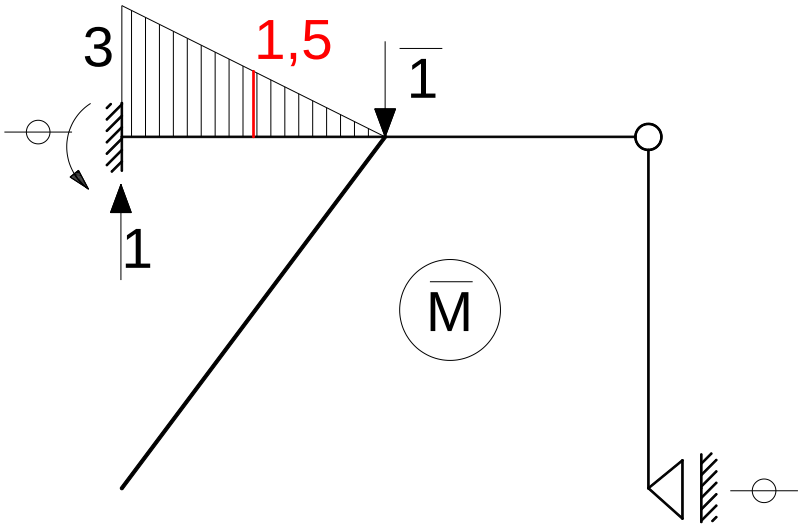
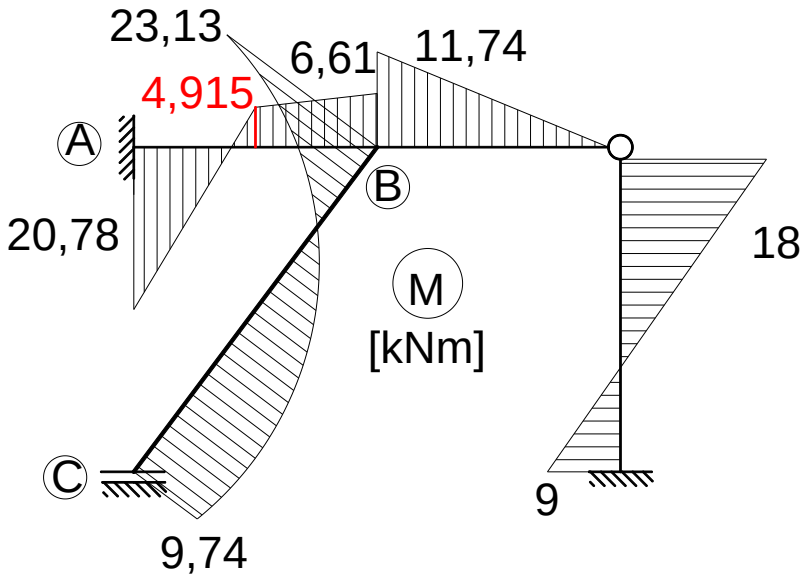
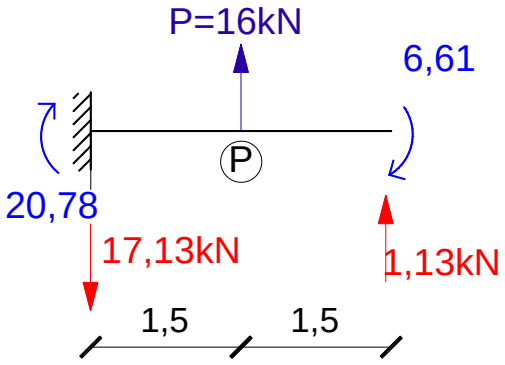
Sprawdzenie z twierdzenia redukcyjnego:

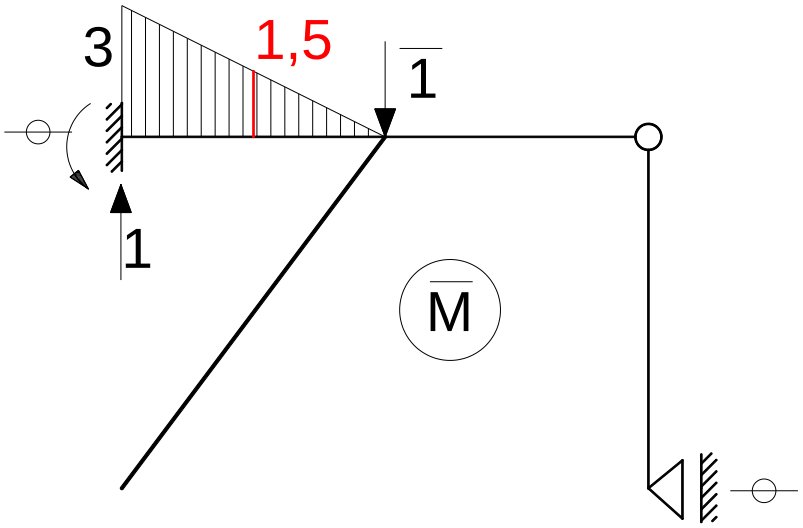


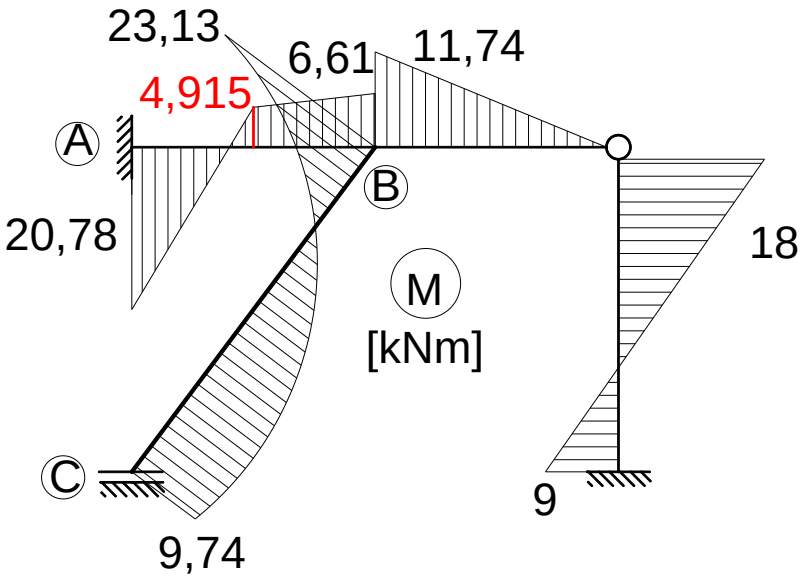


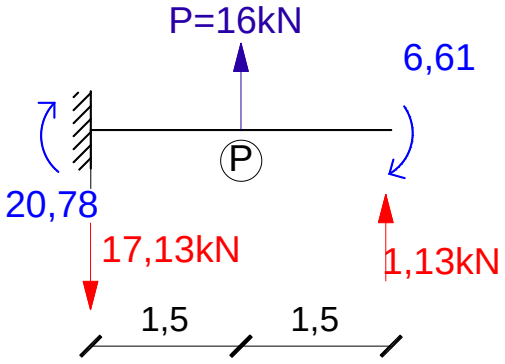


$$M_P = 20{,}78 - 17{,}13 \cdot 1{,}5 = -4{,}915 kNm$$

Sprawdzenie z twierdzenia redukcyjnego:







$$\Delta_B = \frac{1}{EI}\left(\begin{array}{c} -\frac{1}{2}\cdot 20{,}78\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 3+\frac{1}{3}\cdot 1{,}5\right)+ \\ \frac{1}{2}\cdot 4{,}915\cdot 1{,}5\left(\frac{2}{3}\cdot 1{,}5+\frac{1}{3}\cdot 3\right)+ \\ \frac{1}{2}\cdot 4{,}915\cdot 1{,}5\cdot\frac{2}{3}\cdot 1{,}5+\frac{1}{2}\cdot 6{,}61\cdot 1{,}5\cdot\frac{1}{3}\cdot 1{,}5 \end{array}\right)$$

$$M_P = 20{,}78 - 17{,}13\cdot 1{,}5 = -4{,}915 kNm$$

$$\Delta_B = \frac{-25{,}43}{EI} \sim \Delta_2$$

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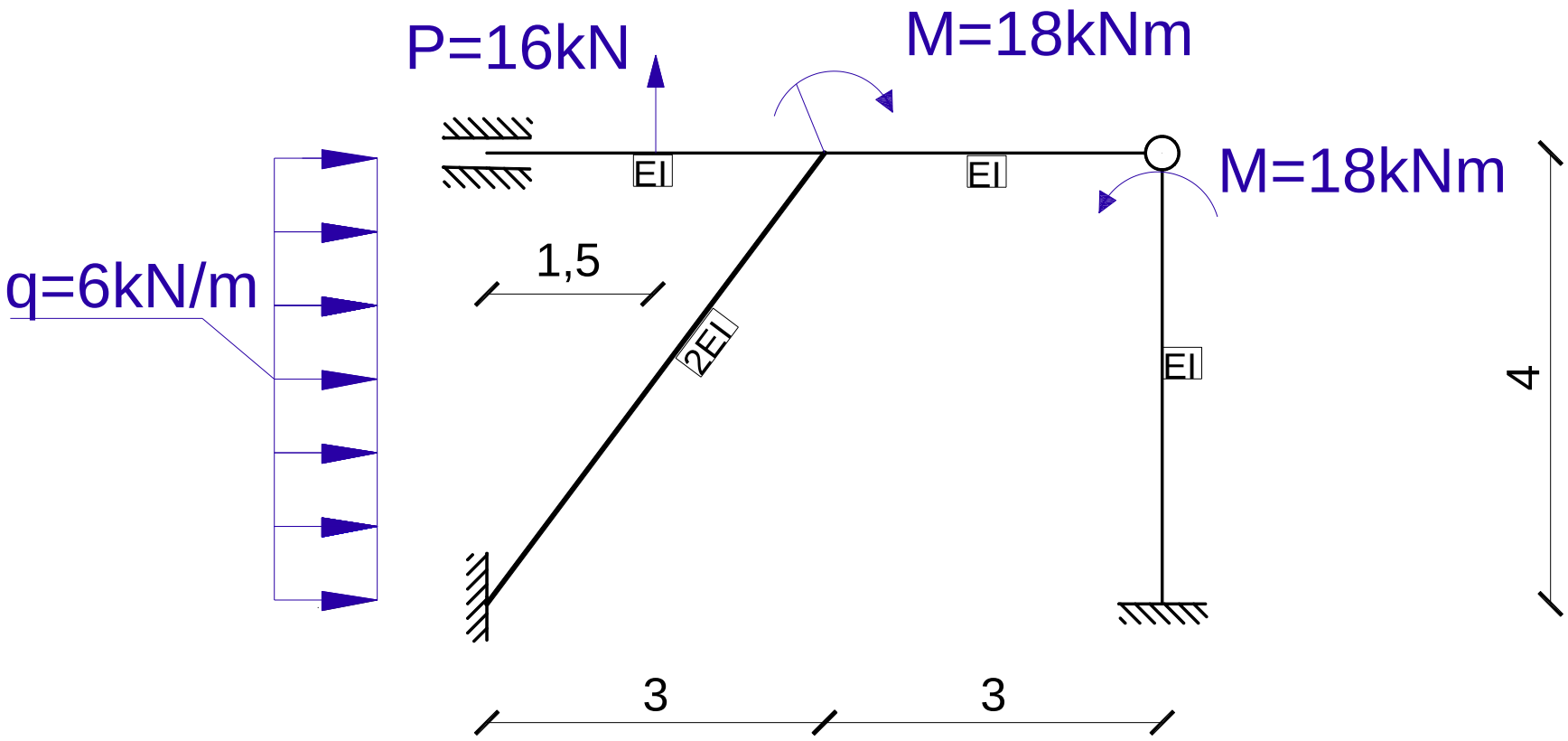
Zadanie 3: Wyznaczyć współczynniki układu równań metody przemieszczeń. Zadanie rozwiązać w minimalnej bazie niewiadomych.



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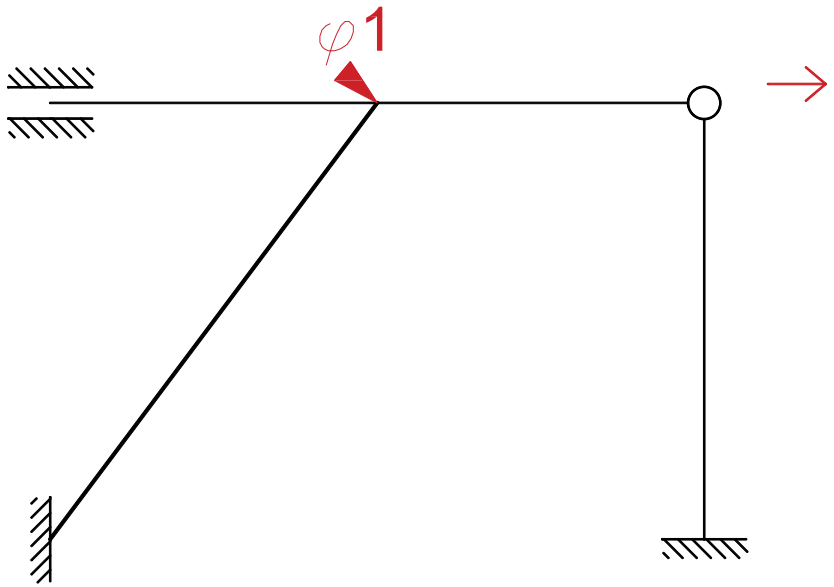
Schemat podstawowy geometrycznie wyznaczalny



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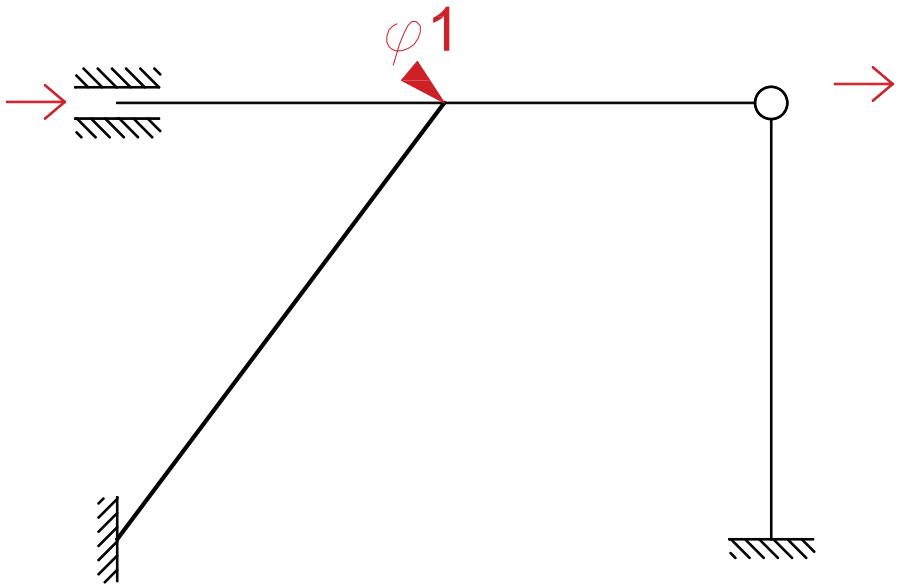
Schemat podstawowy geometrycznie wyznaczalny



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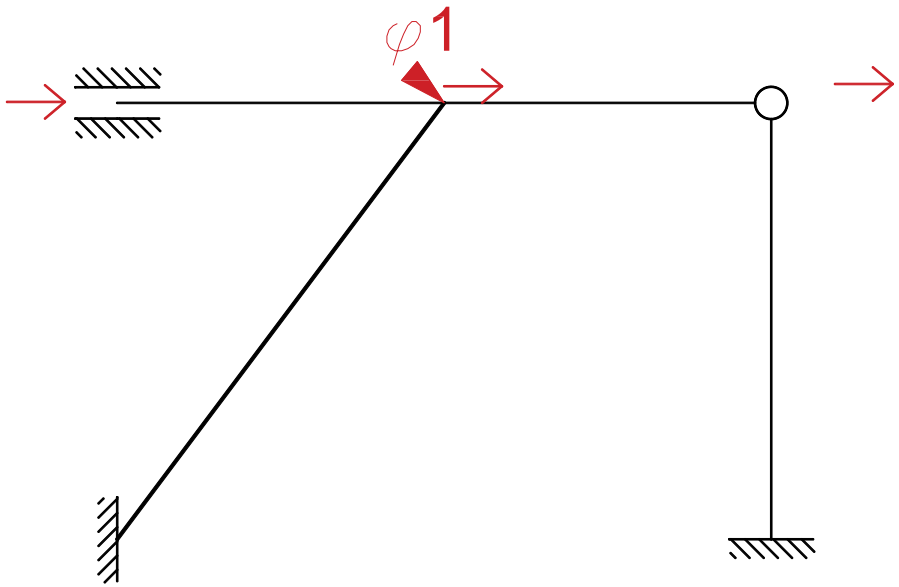
Schemat podstawowy geometrycznie wyznaczalny



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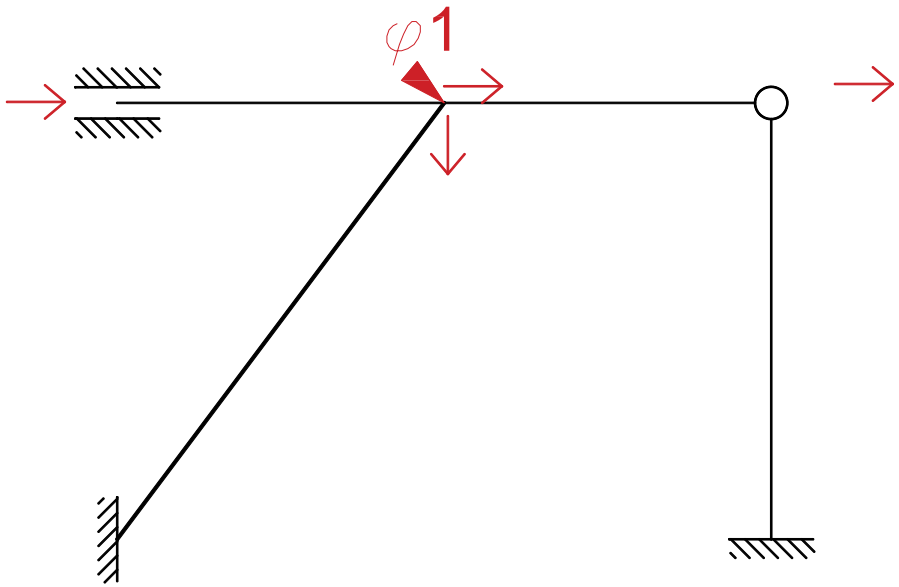
Schemat podstawowy geometrycznie wyznaczalny



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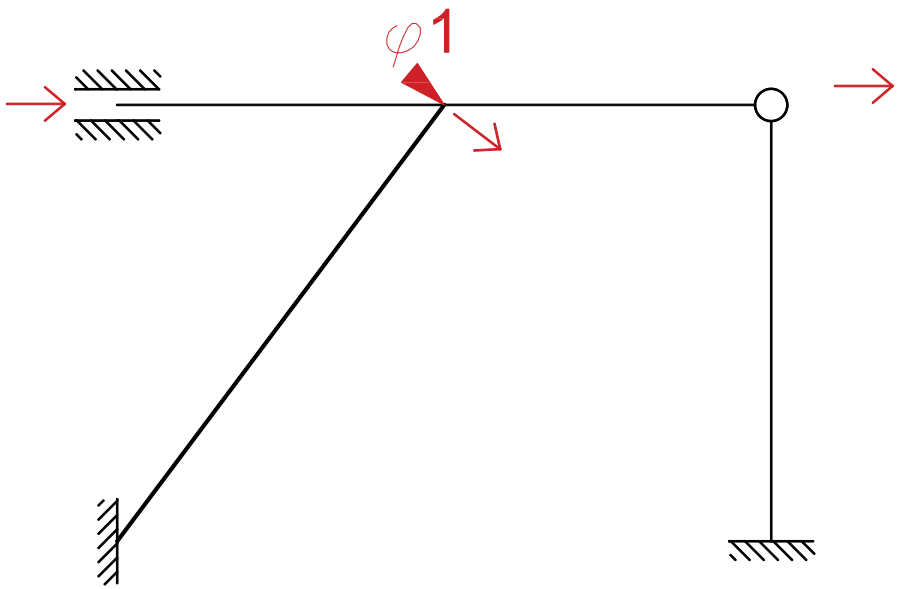
Schemat podstawowy geometrycznie wyznaczalny



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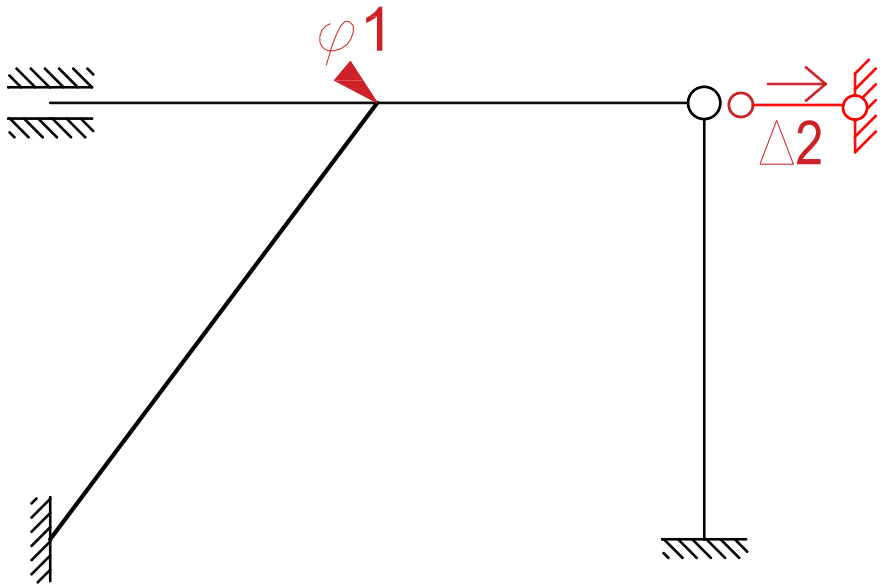
Schemat podstawowy geometrycznie wyznaczalny



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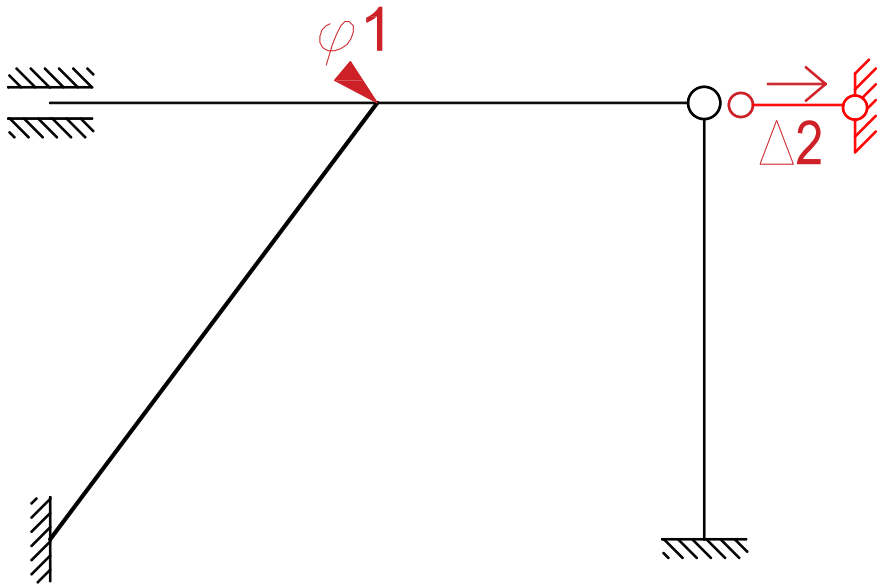
Schemat podstawowy geometrycznie wyznaczalny



Układ dwukrotnie geometrycznie niewyznaczalny

$n_g = 2(\varphi 1, \triangle 2)$

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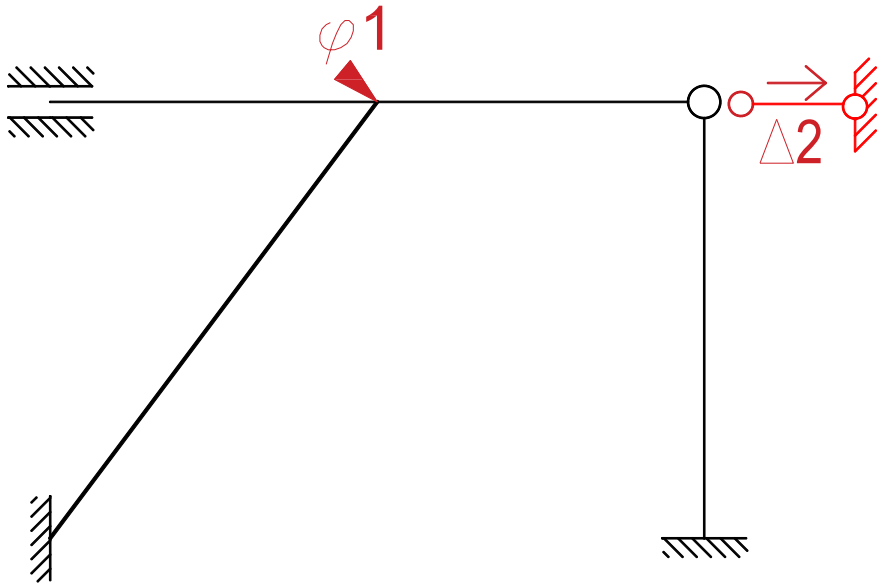
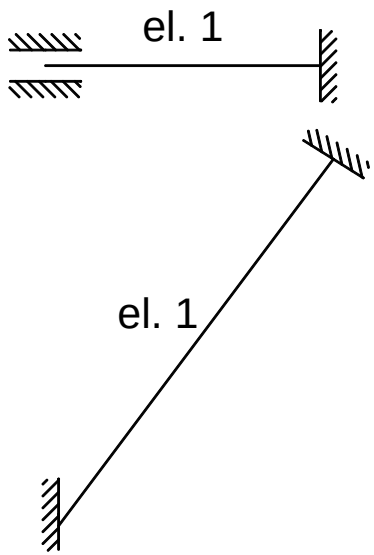
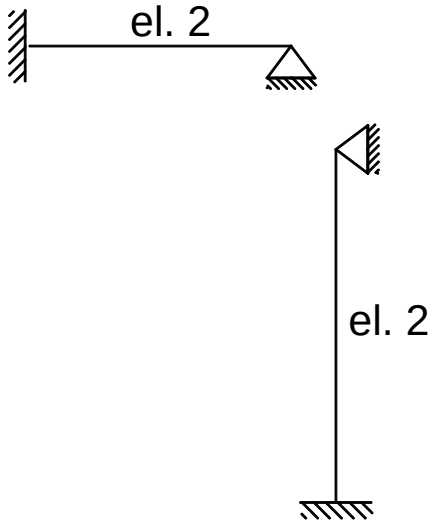
Schemat podstawowy geometrycznie wyznaczalny



Podział na elementy:





Układ dwukrotnie geometrycznie niewyznaczalny

n_g=2(φ1, $\triangle$2)

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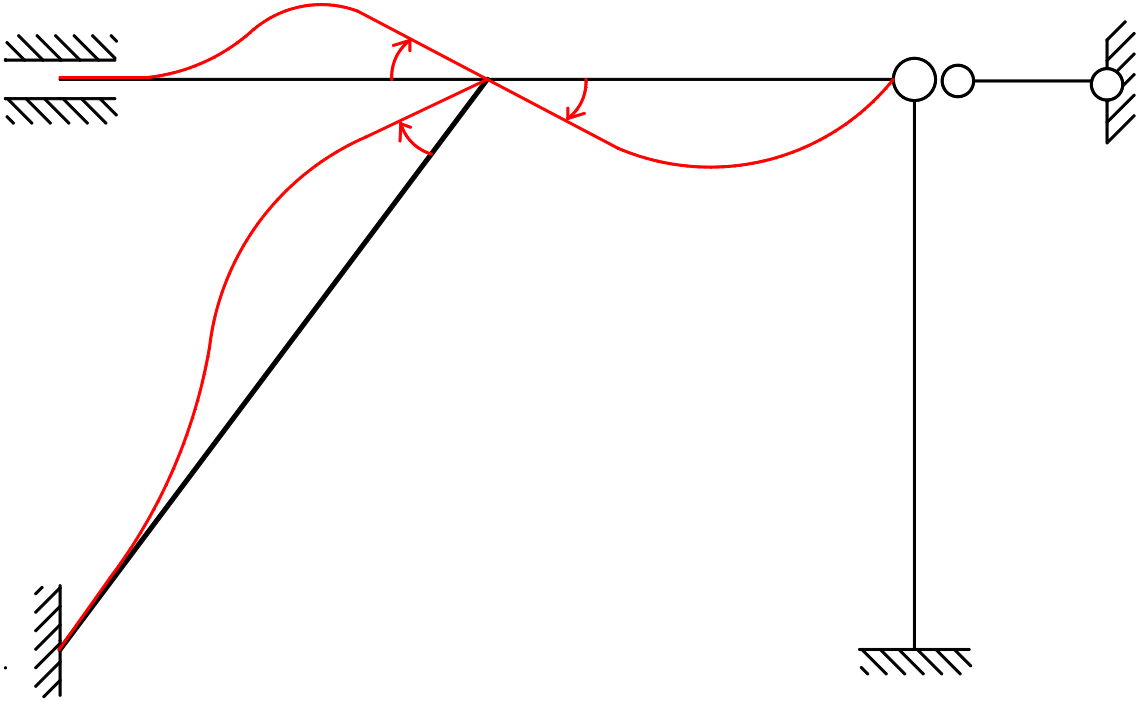
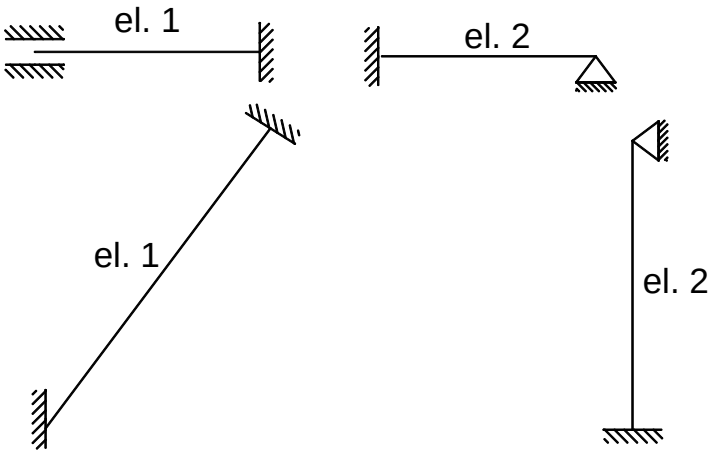
Stan $\varphi_1=1$





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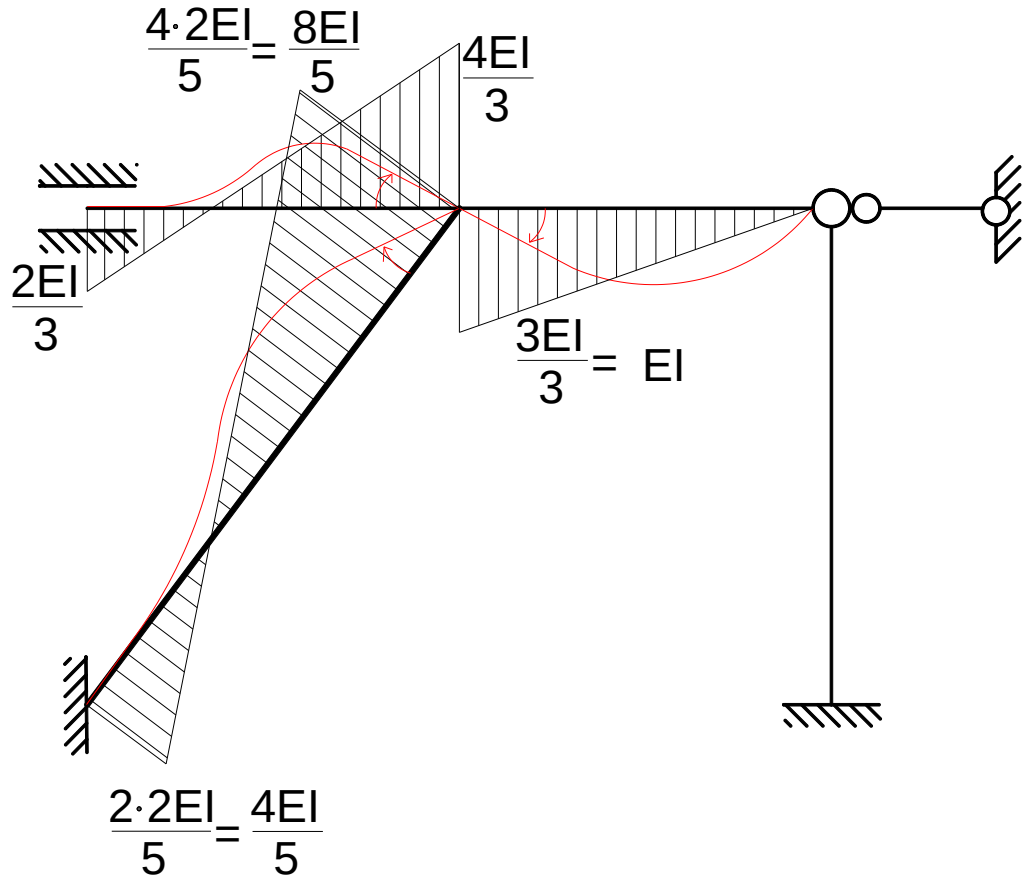
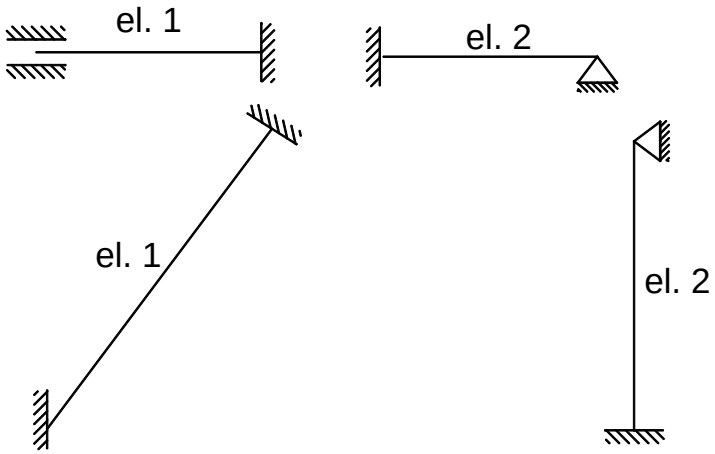


Stan $\varphi_1 = 1$



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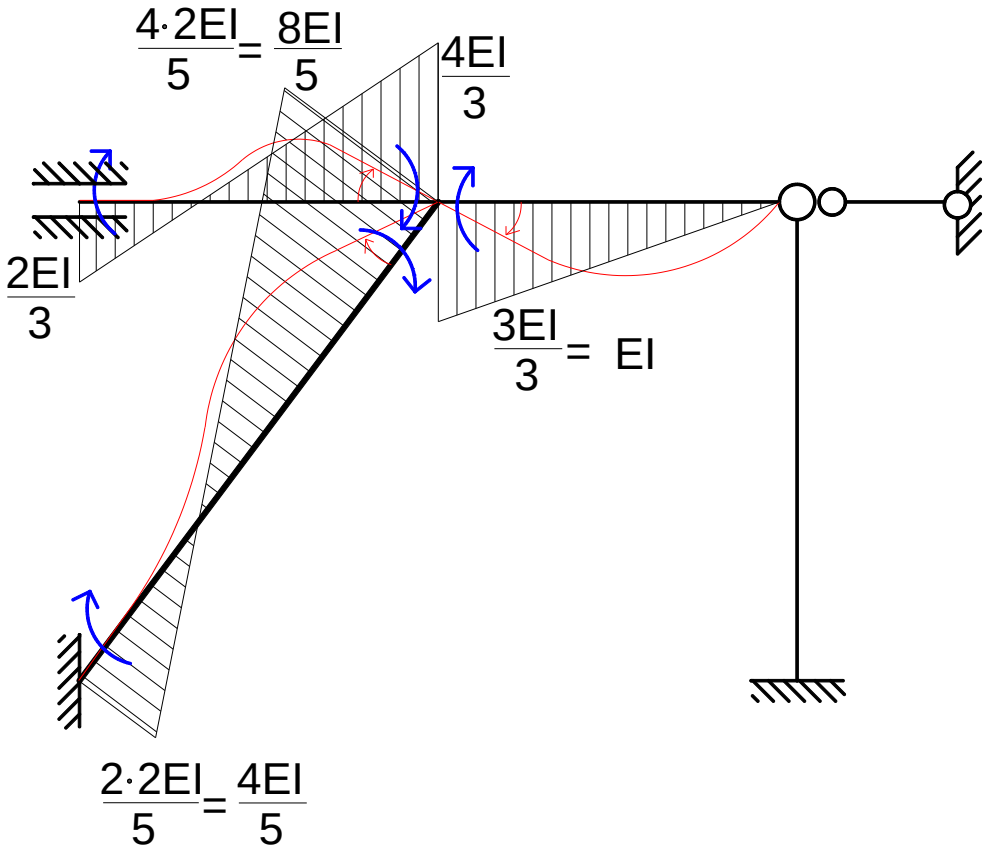
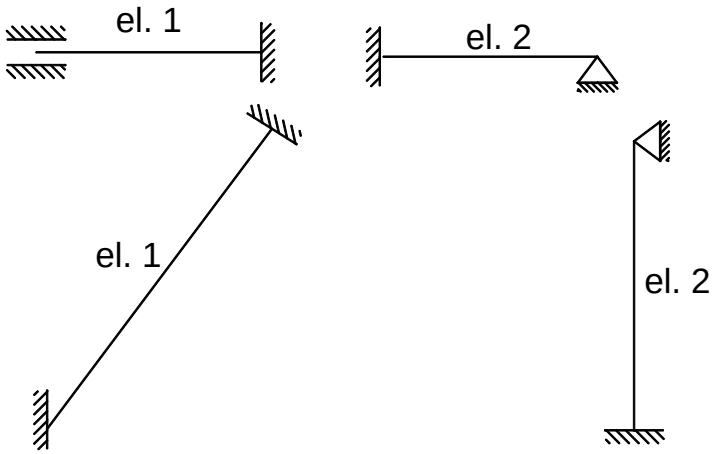
Stan $\varphi_1=1$





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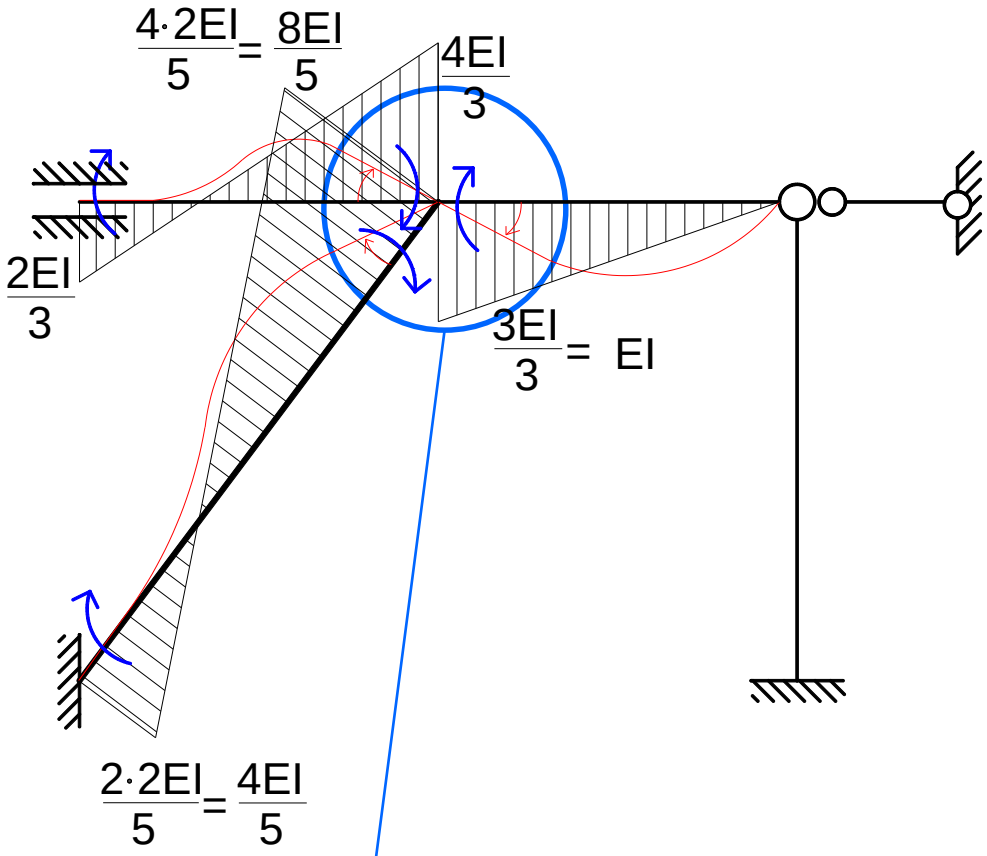
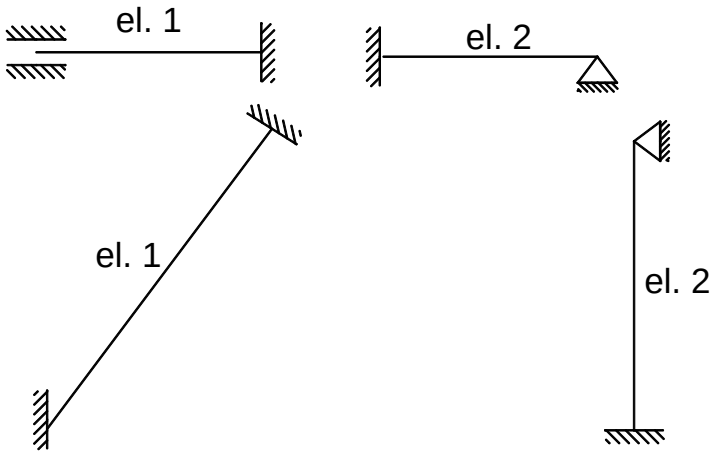
Stan $\varphi_1=1$



$$k_{11} = \frac{4EI}{3} + \frac{8EI}{5} + EI = \frac{59EI}{15}$$



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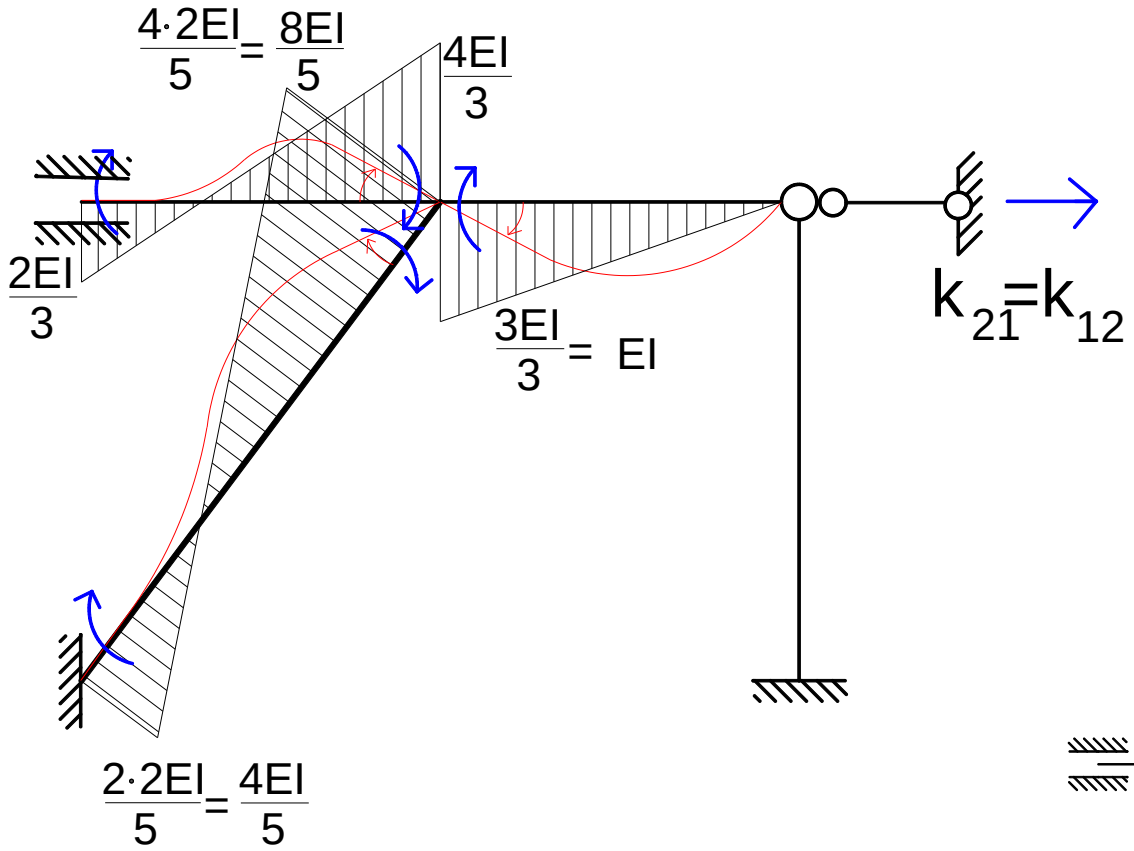
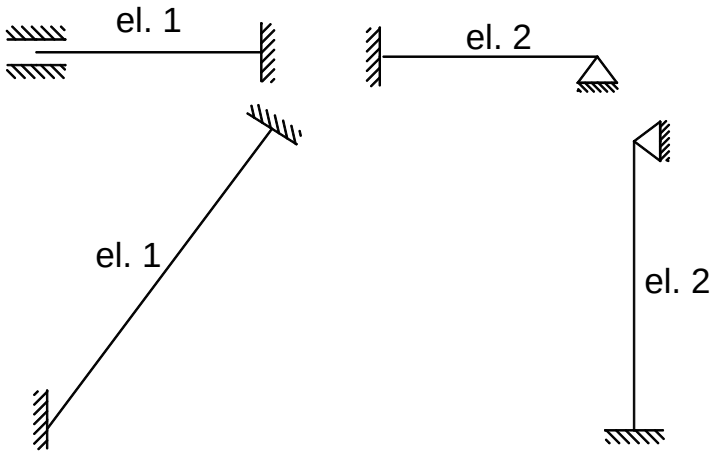
Stan $\varphi_1=1$



$k_{21}=k_{12}$

$$k_{11} = \frac{4EI}{3} + \frac{8EI}{5} + EI = \frac{59EI}{15}$$



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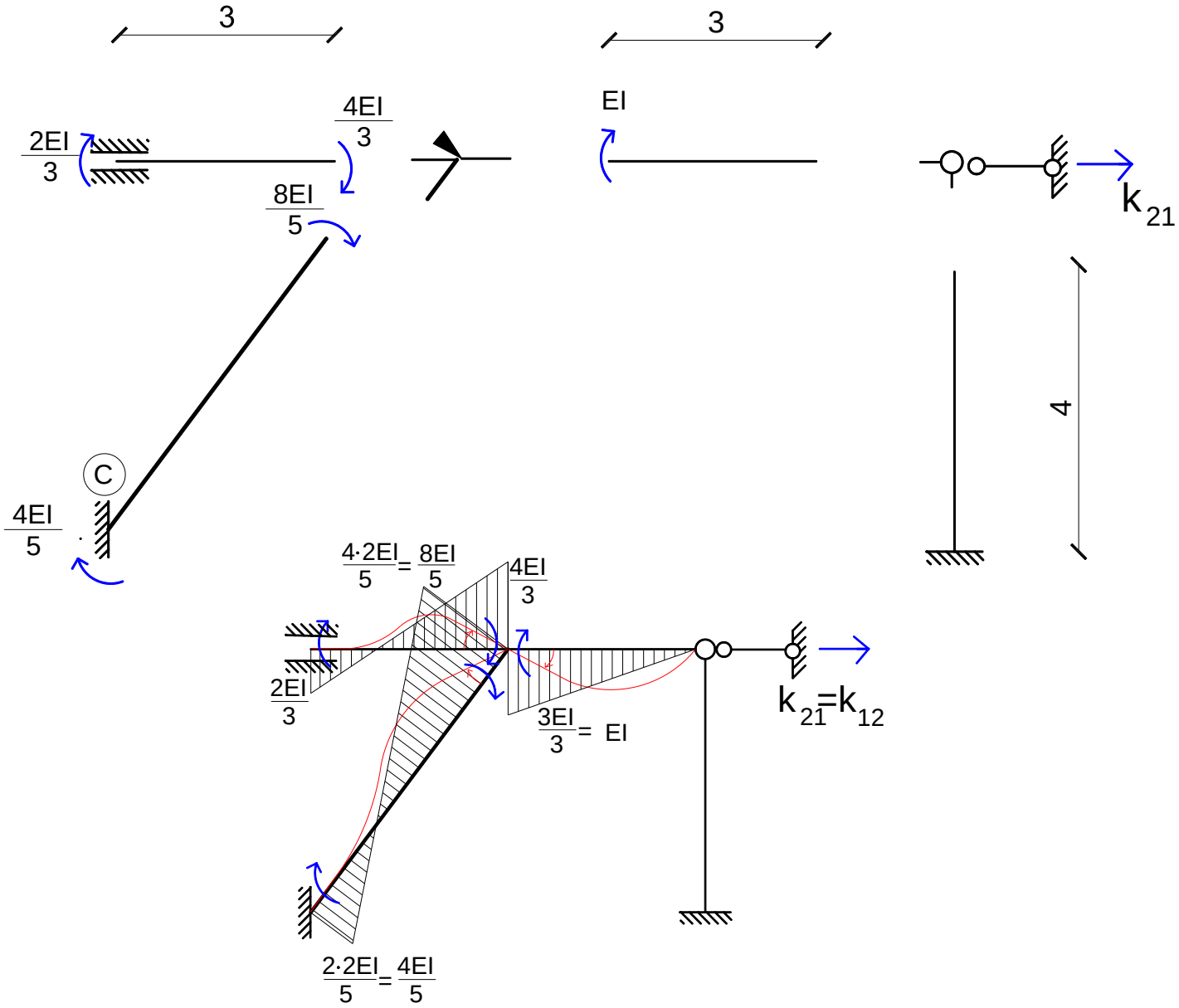
Wyznaczenie reakcji od przemieszczeń k_{21}



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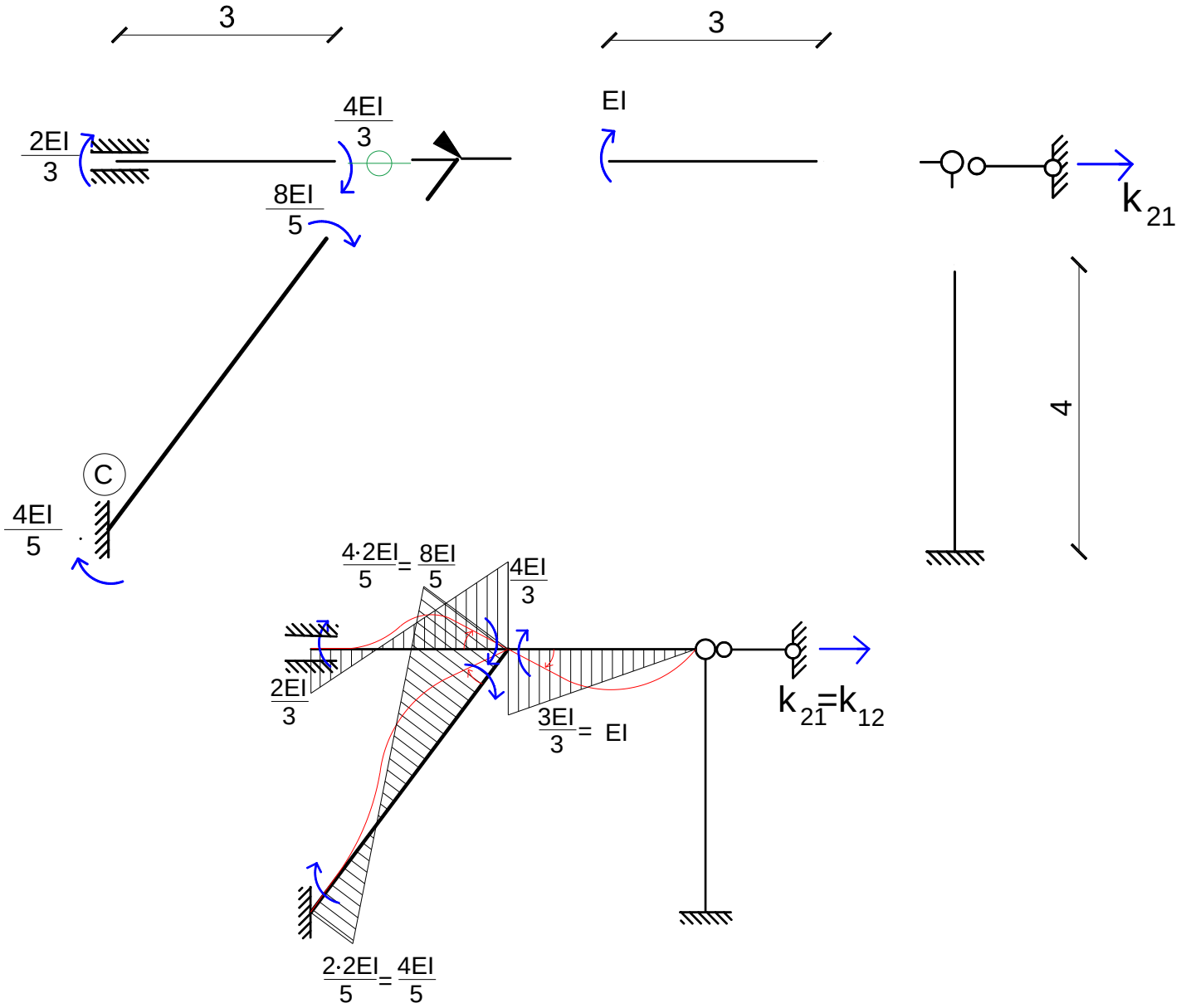
Wyznaczenie reakcji od przemieszczeń k_{21}



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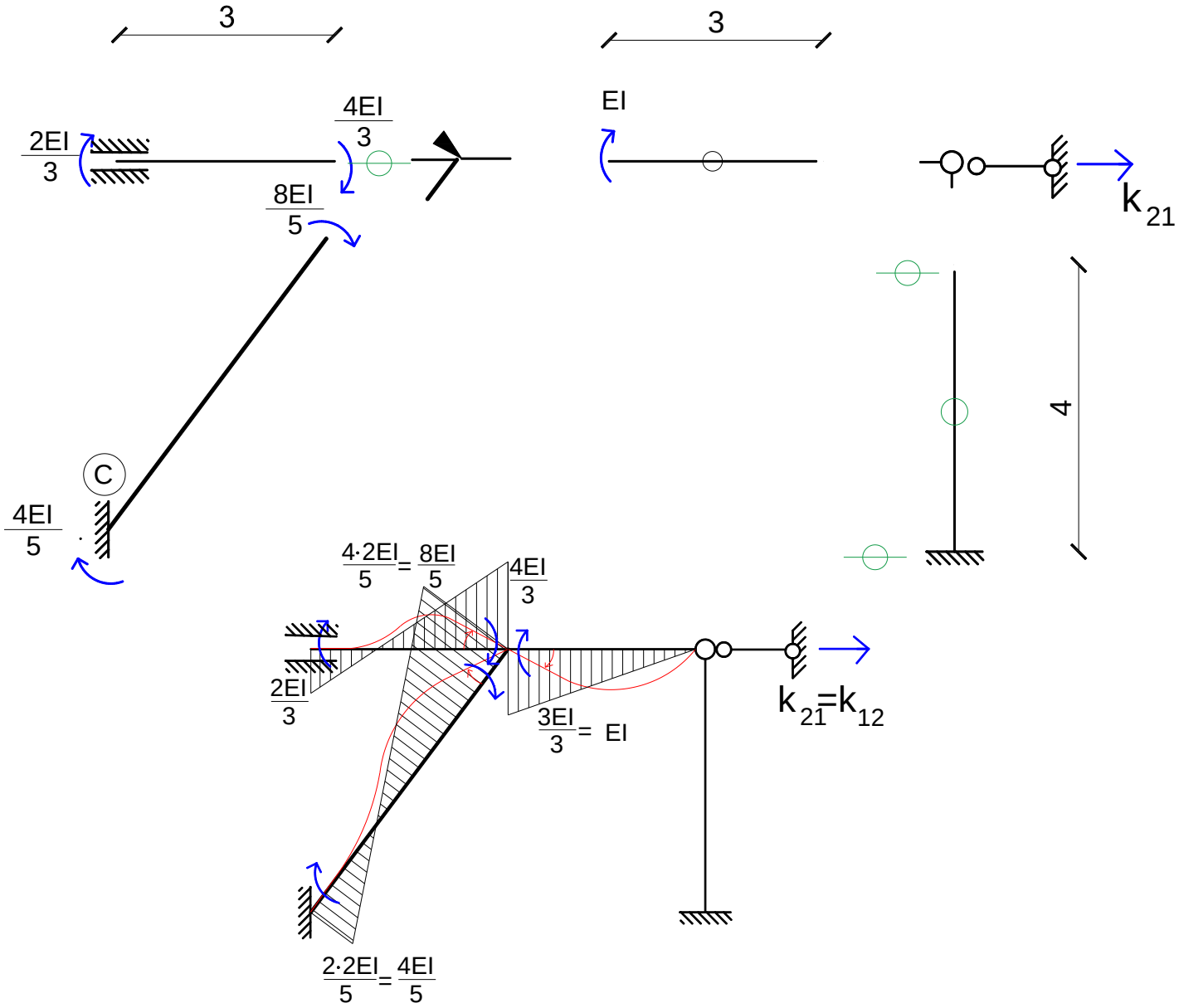
Wyznaczenie reakcji od przemieszczeń k_{21}



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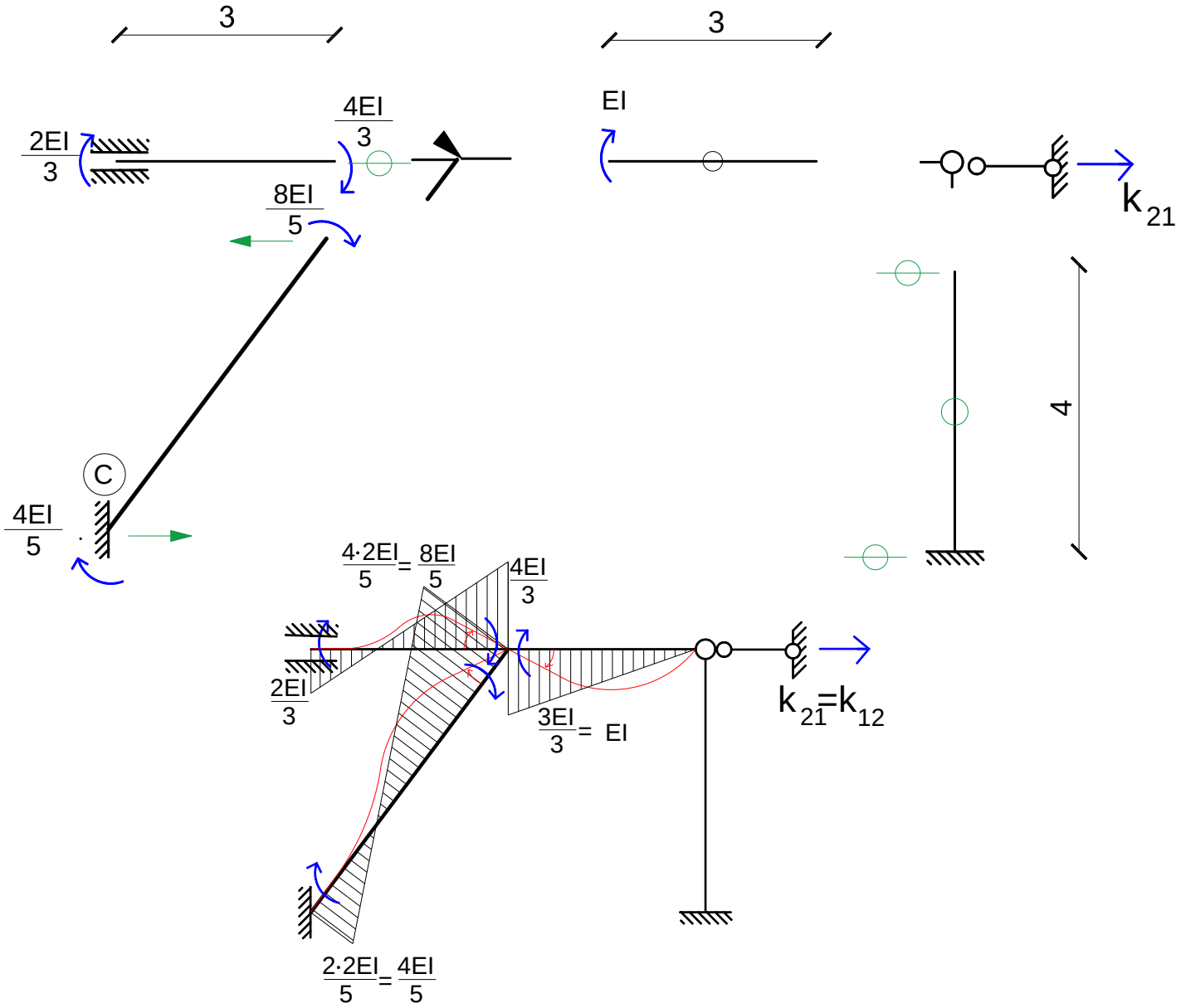
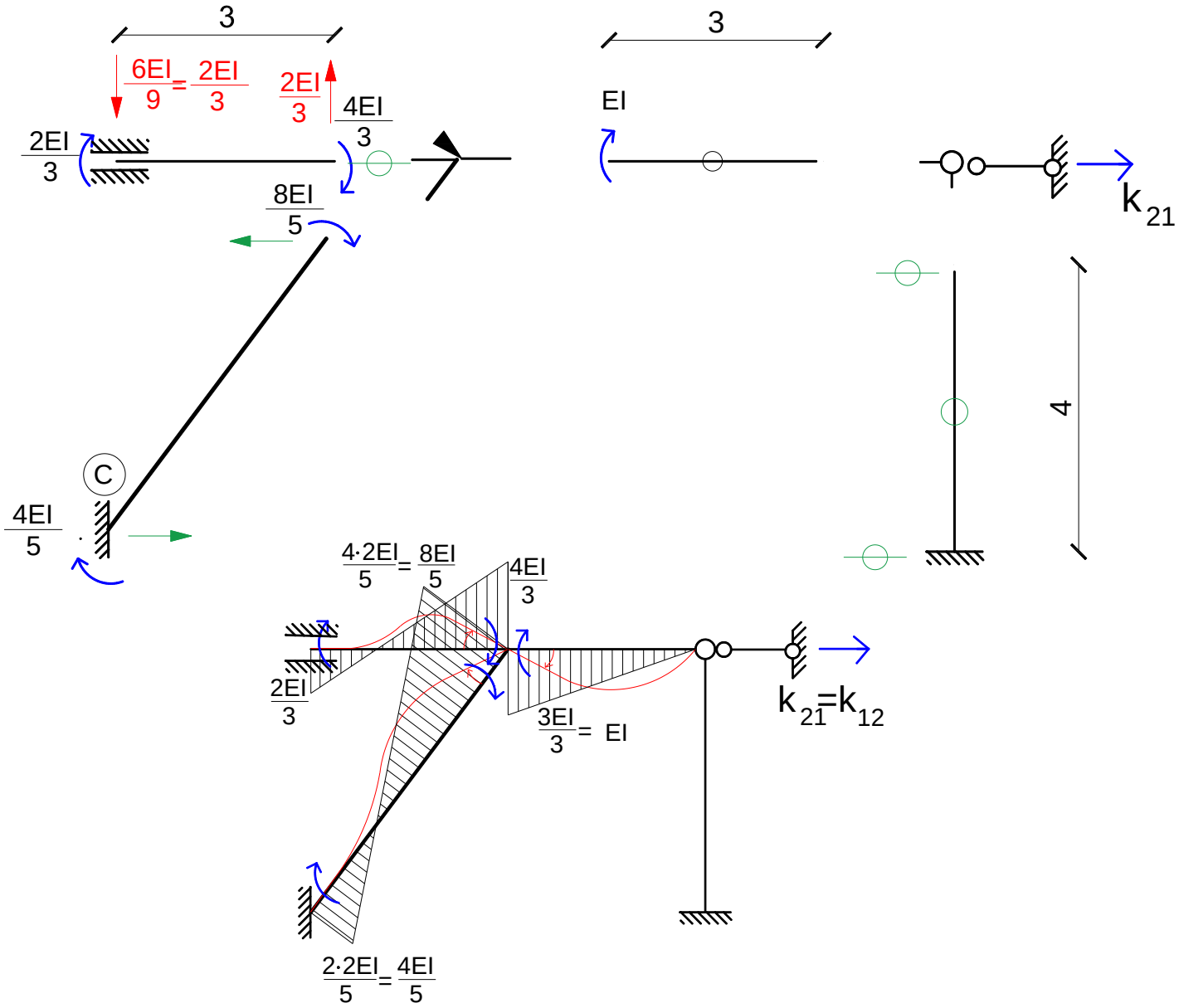
Wyznaczenie reakcji od przemieszczeń k_{21}



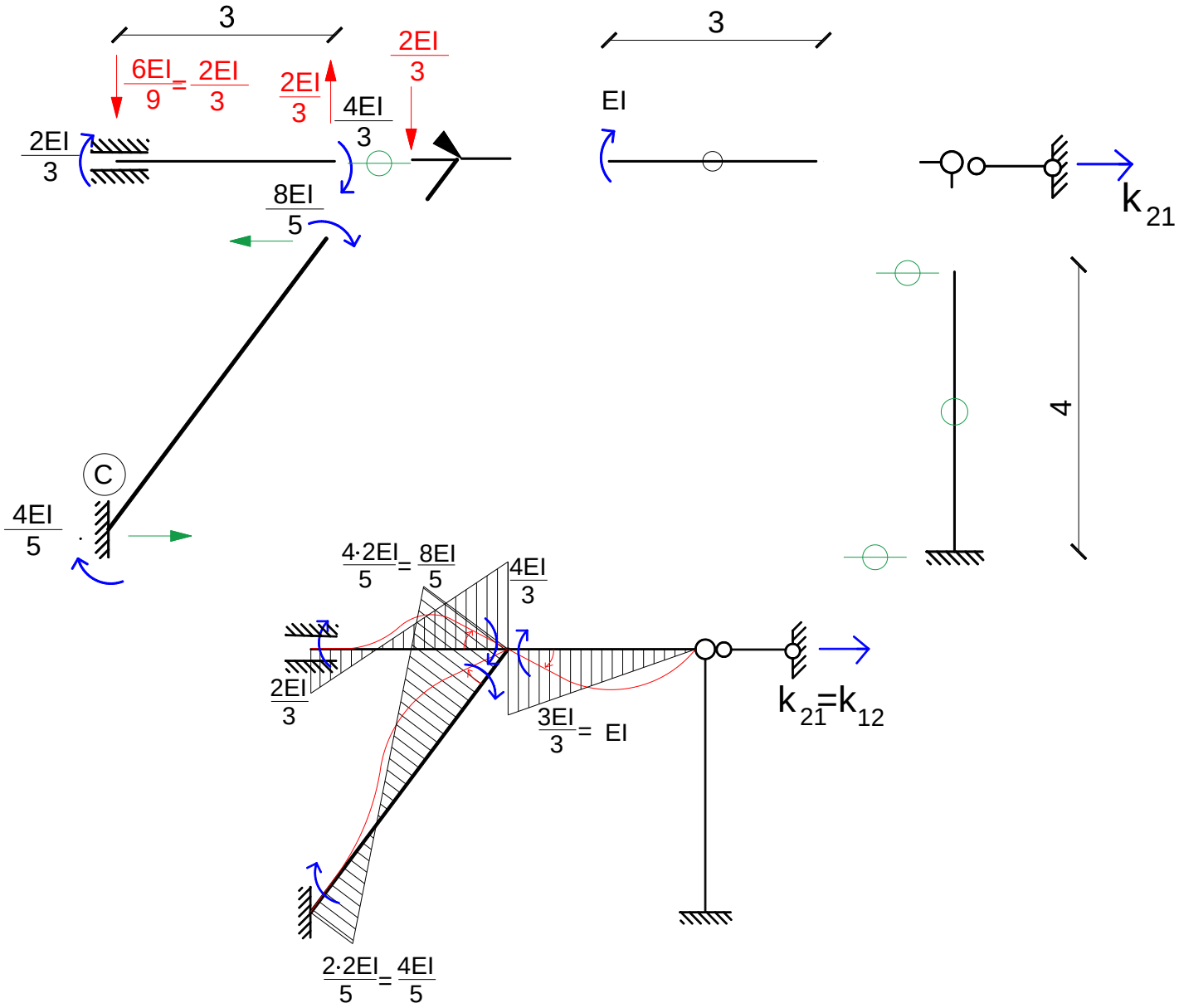
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



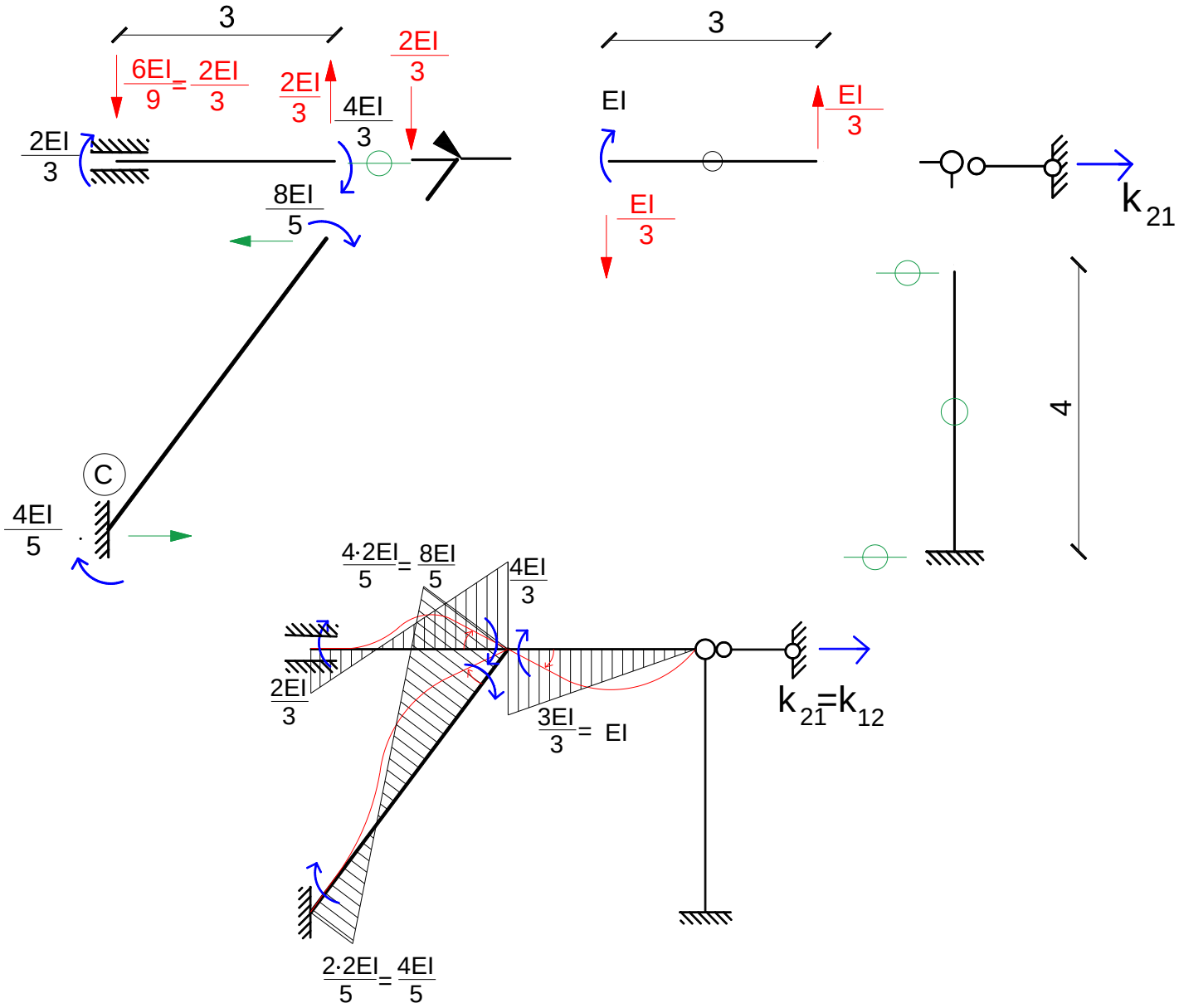
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



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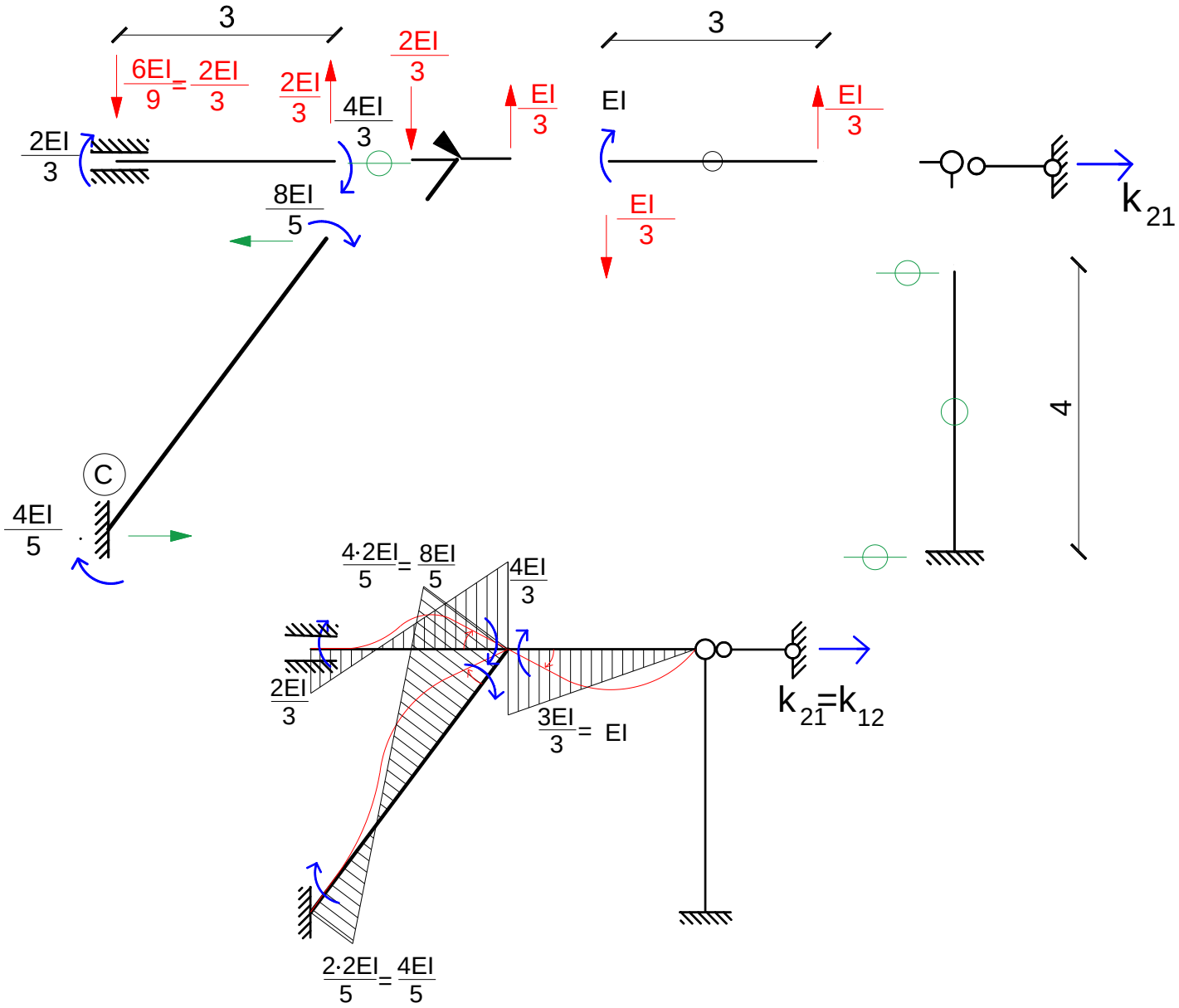
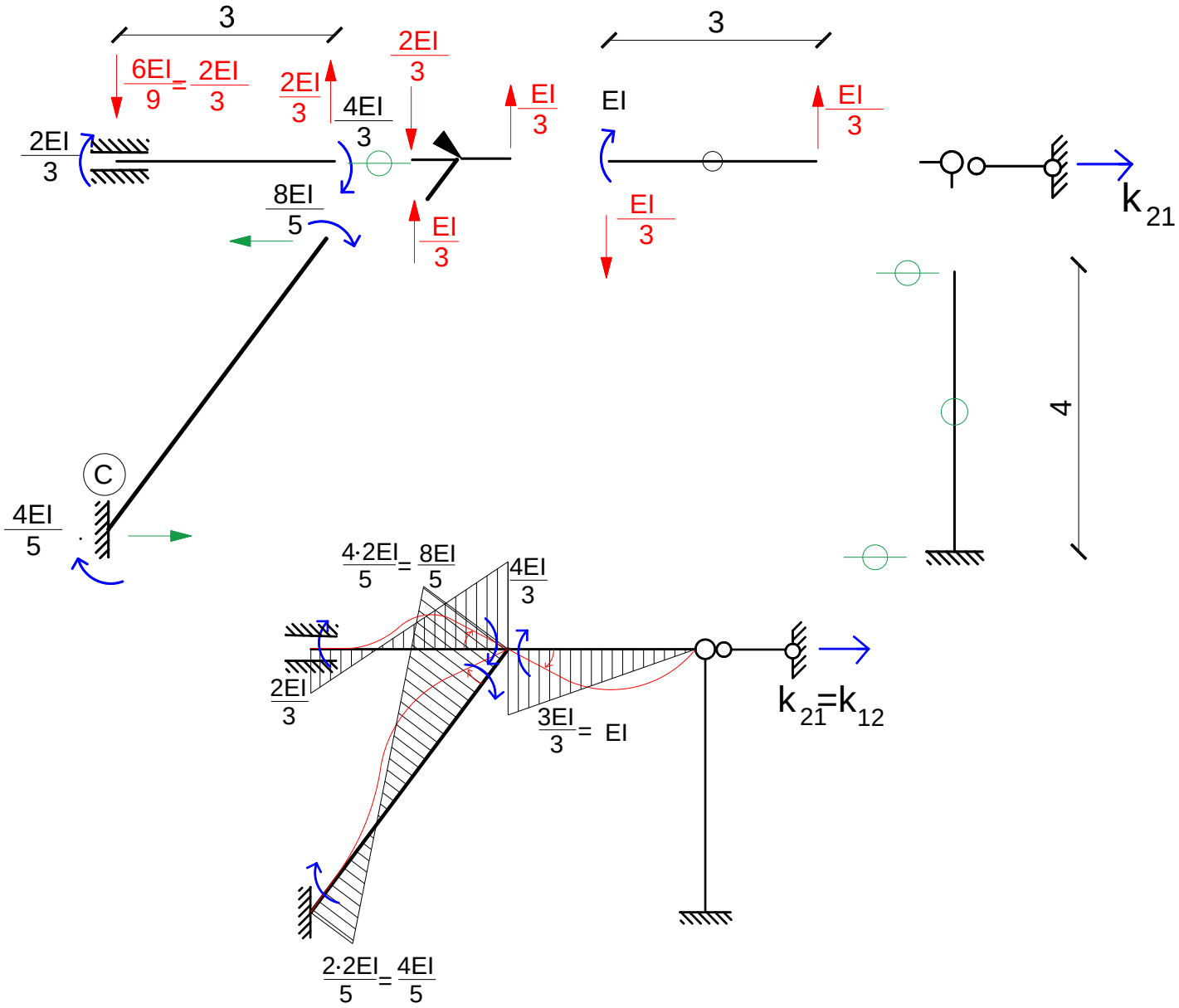
Wyznaczenie reakcji od przemieszczeń k_{21}



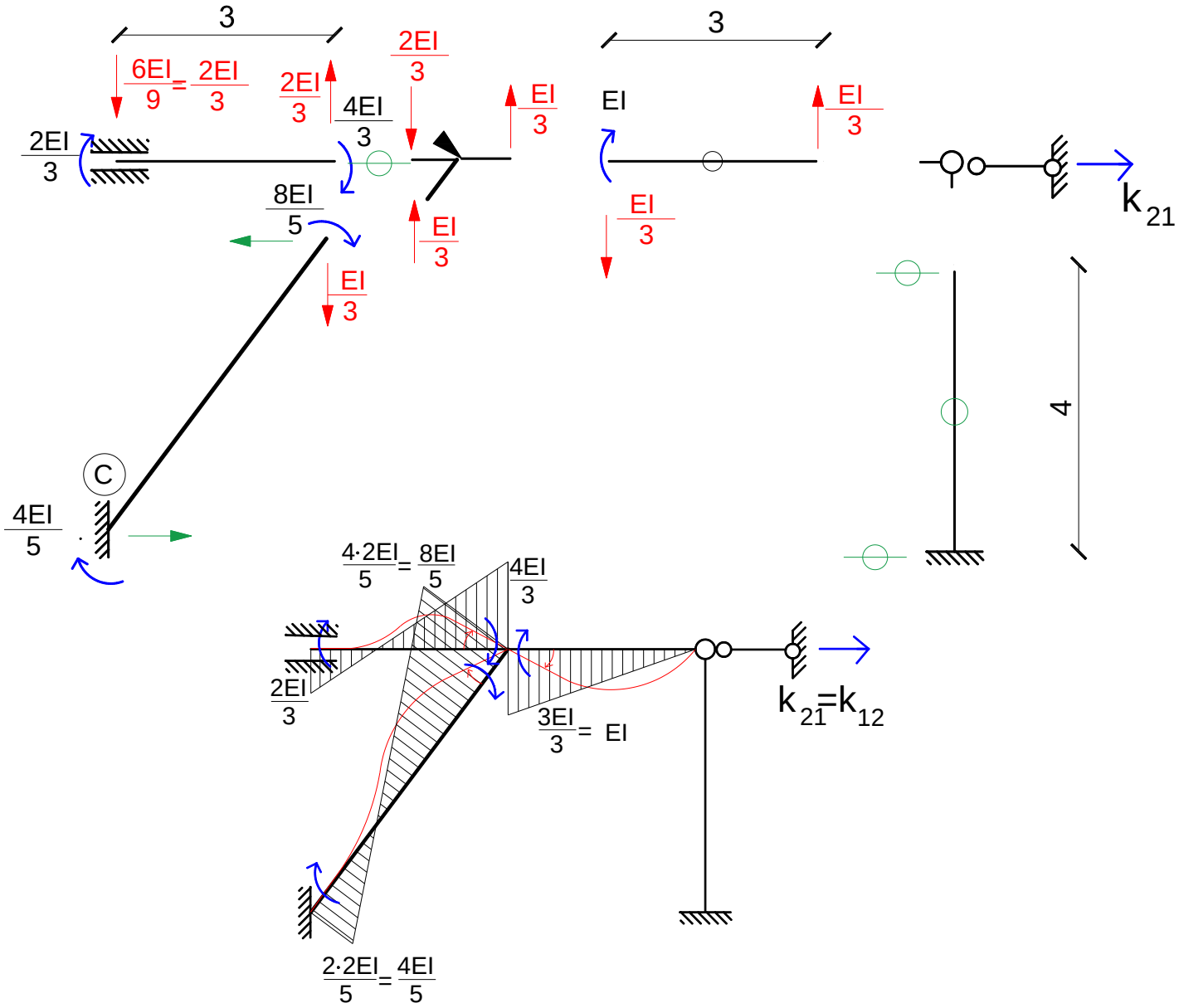
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



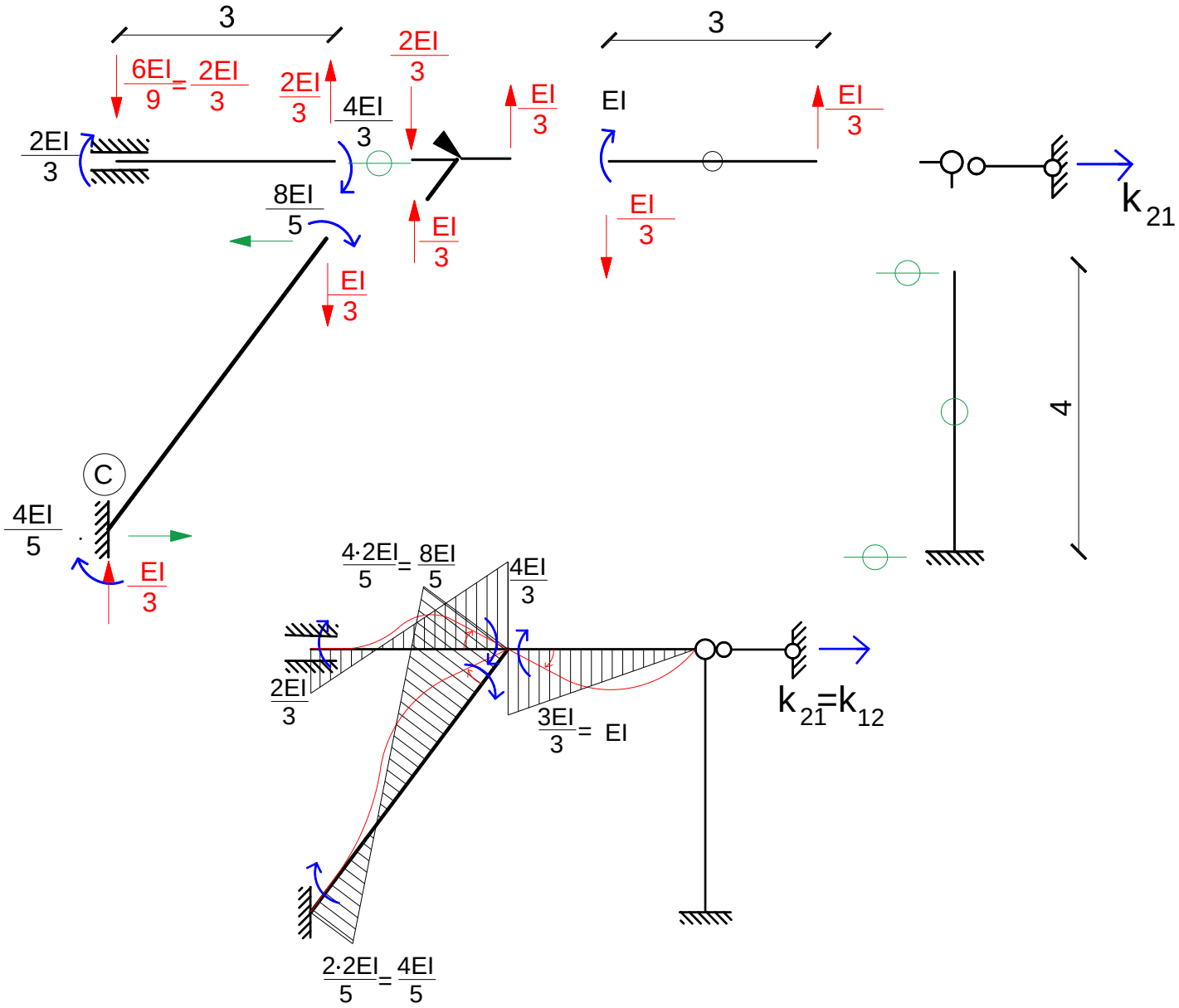
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



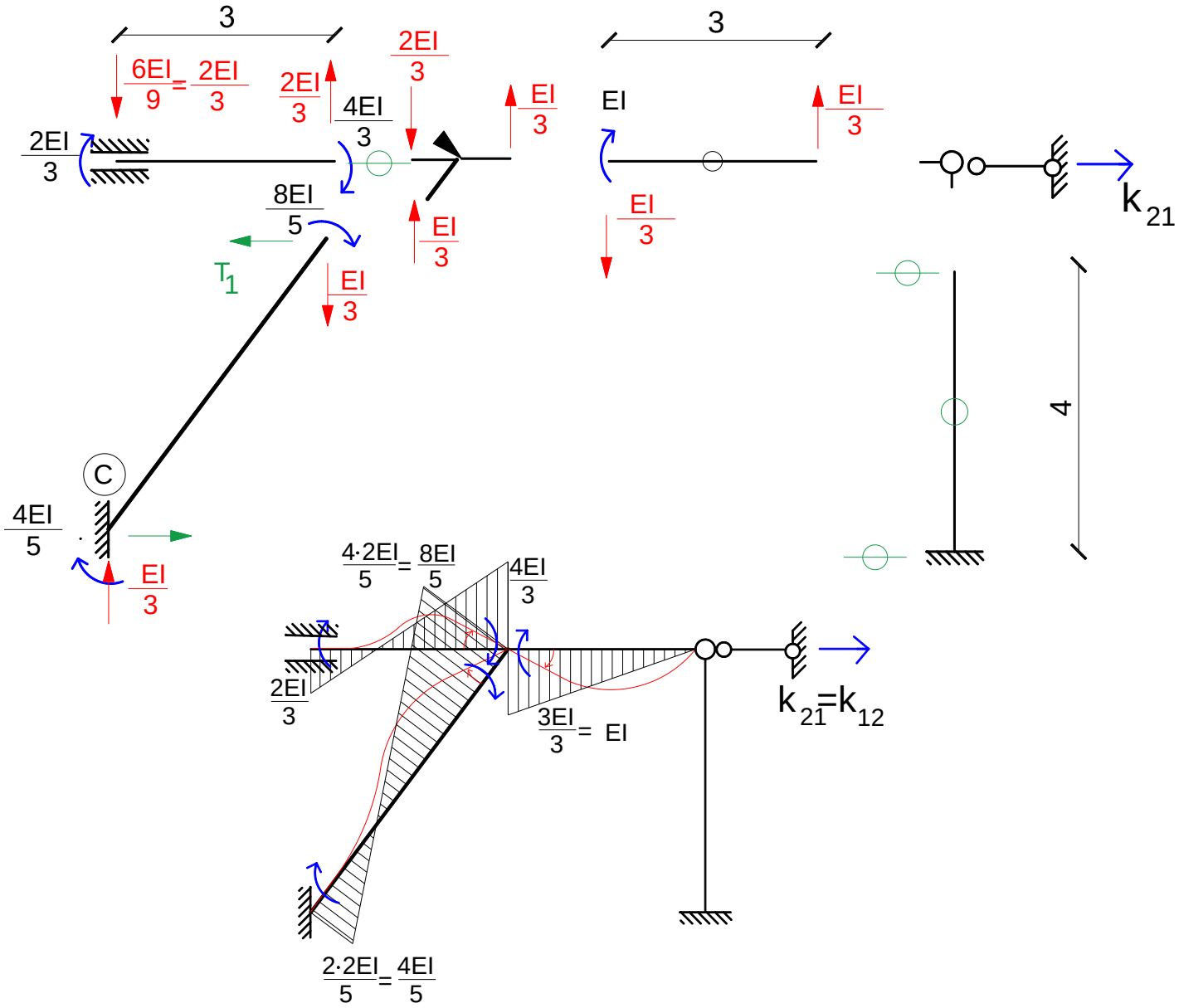
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



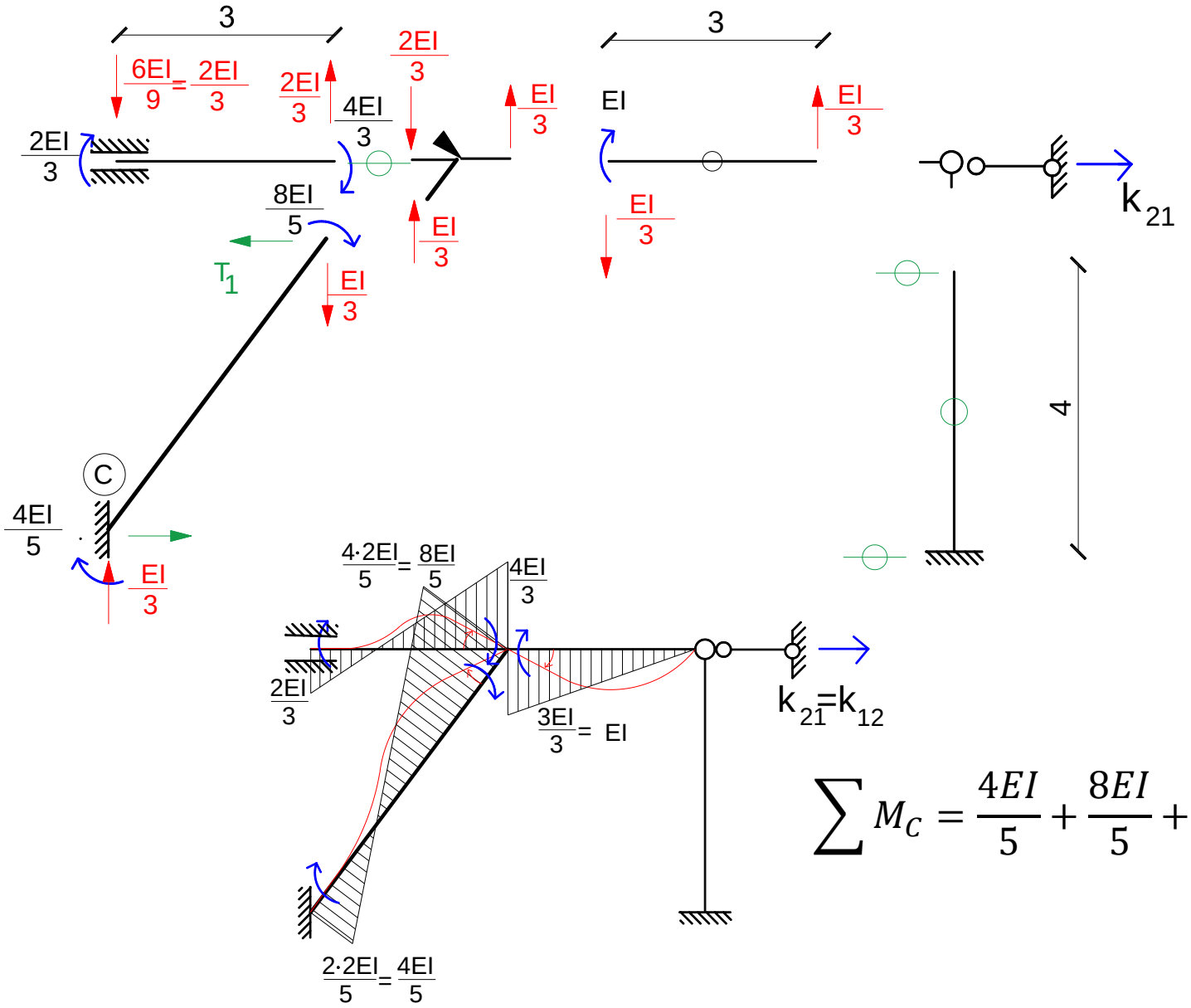
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



$$\sum M_C = \frac{4EI}{5} + \frac{8EI}{5} + \frac{EI}{3} \cdot 3 - T_1 \cdot 4 = 0$$

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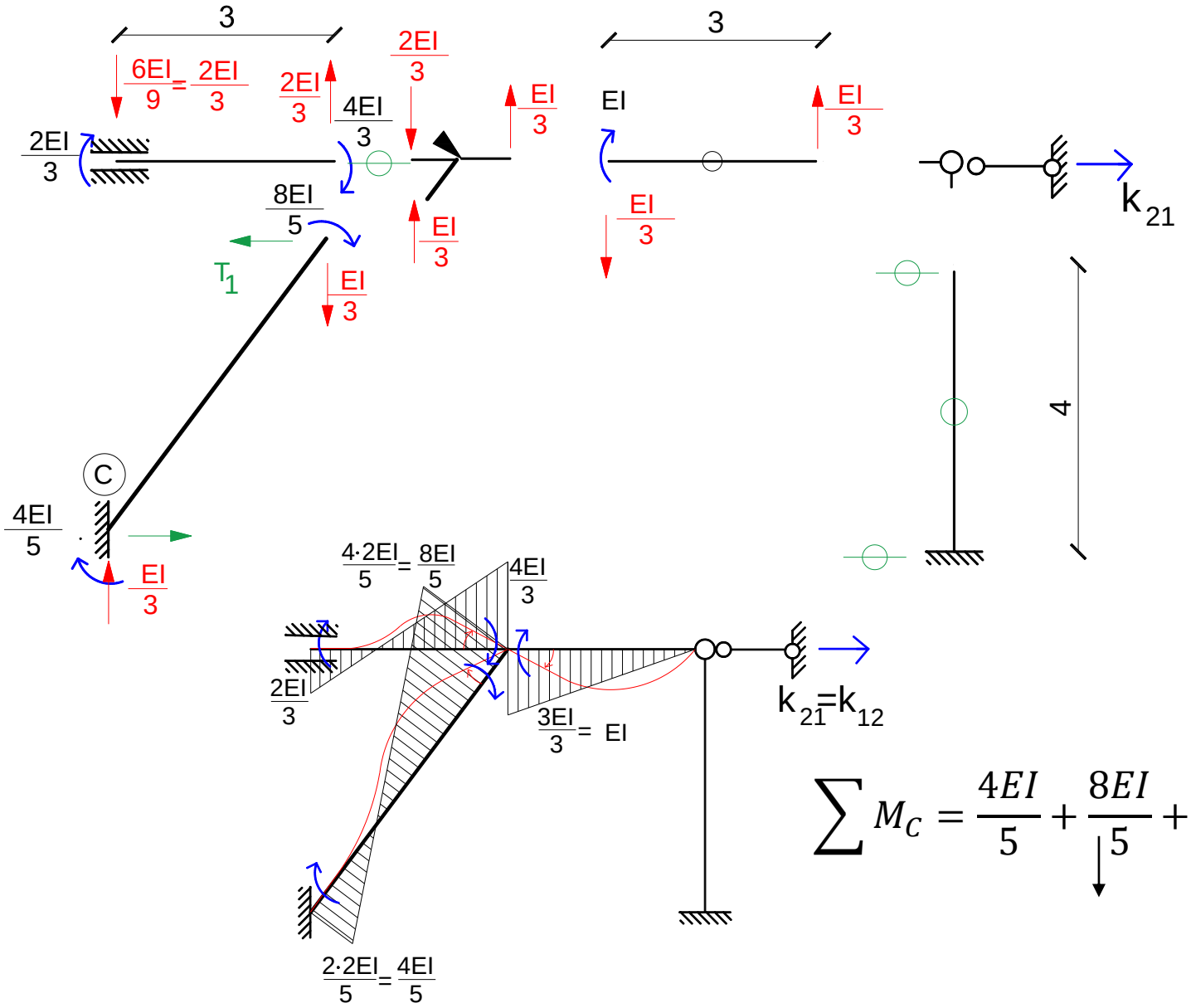
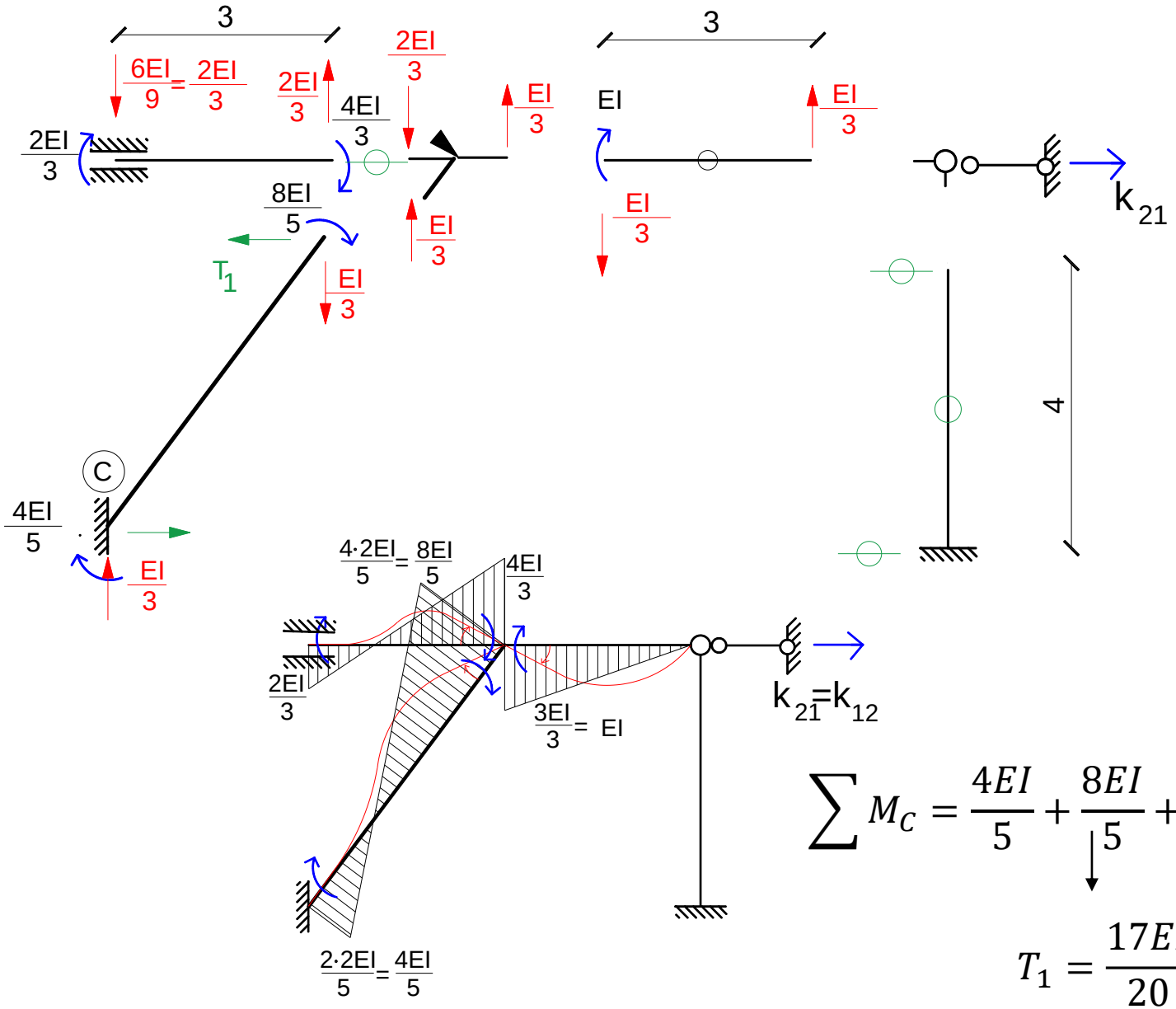
Wyznaczenie reakcji od przemieszczeń k_{21}



$$\sum M_C = \frac{4EI}{5} + \frac{8EI}{5} + \frac{EI}{3} \cdot 3 - T_1 \cdot 4 = 0$$

dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}

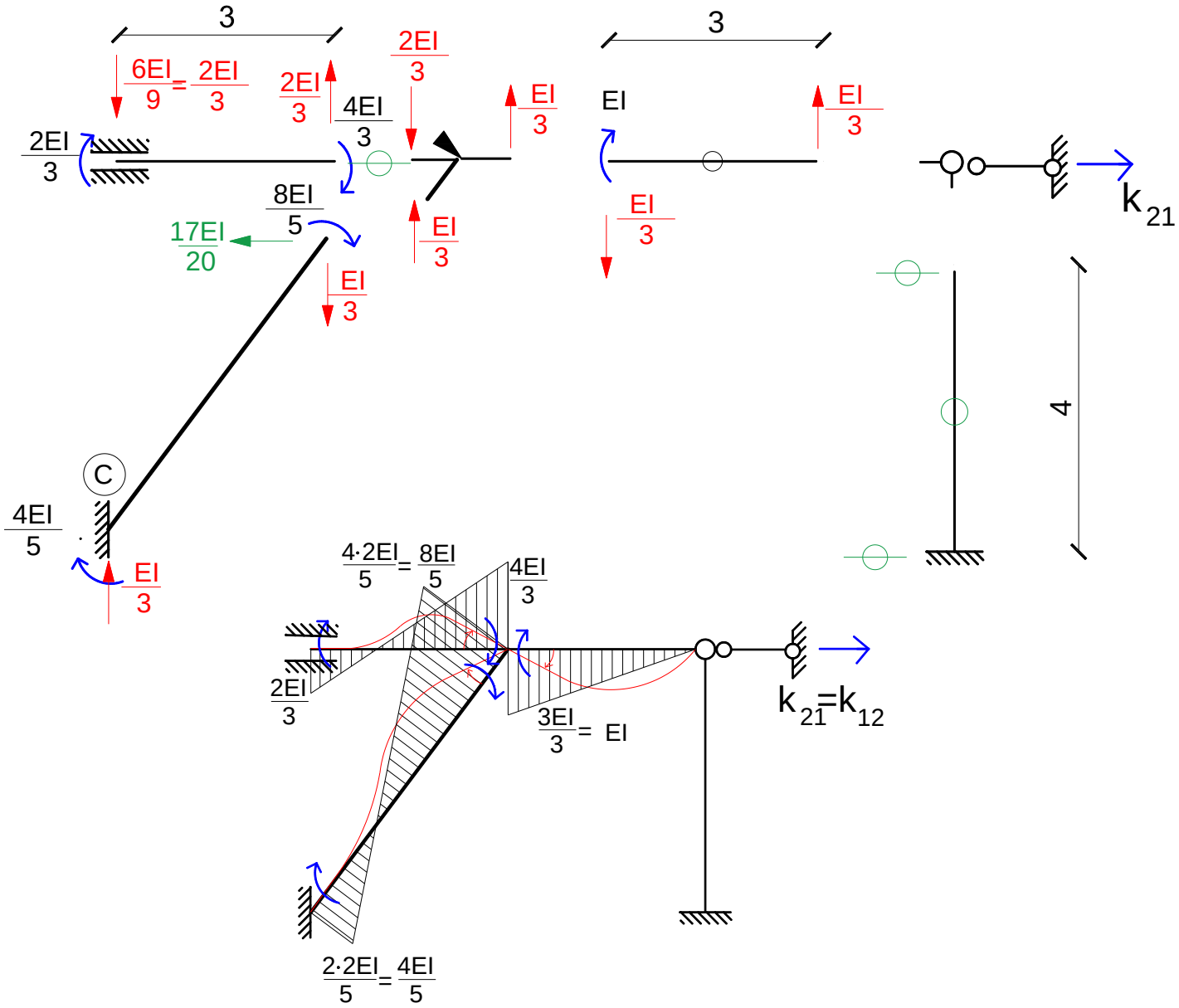


$$\sum M_C = \frac{4EI}{5} + \frac{8EI}{5} + \frac{EI}{3} \cdot 3 - T_1 \cdot 4 = 0$$

$$T_1 = \frac{17EI}{20}$$

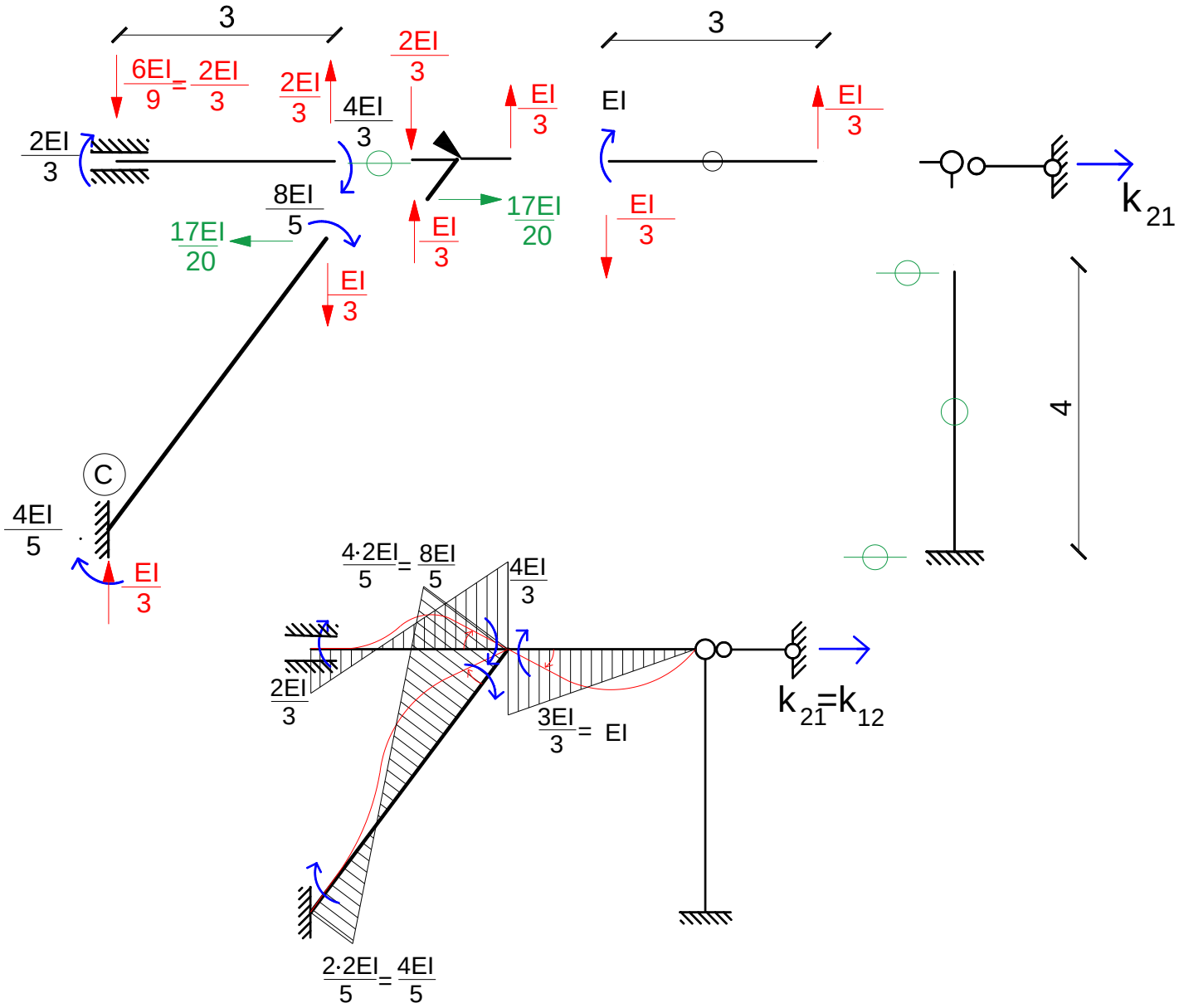
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



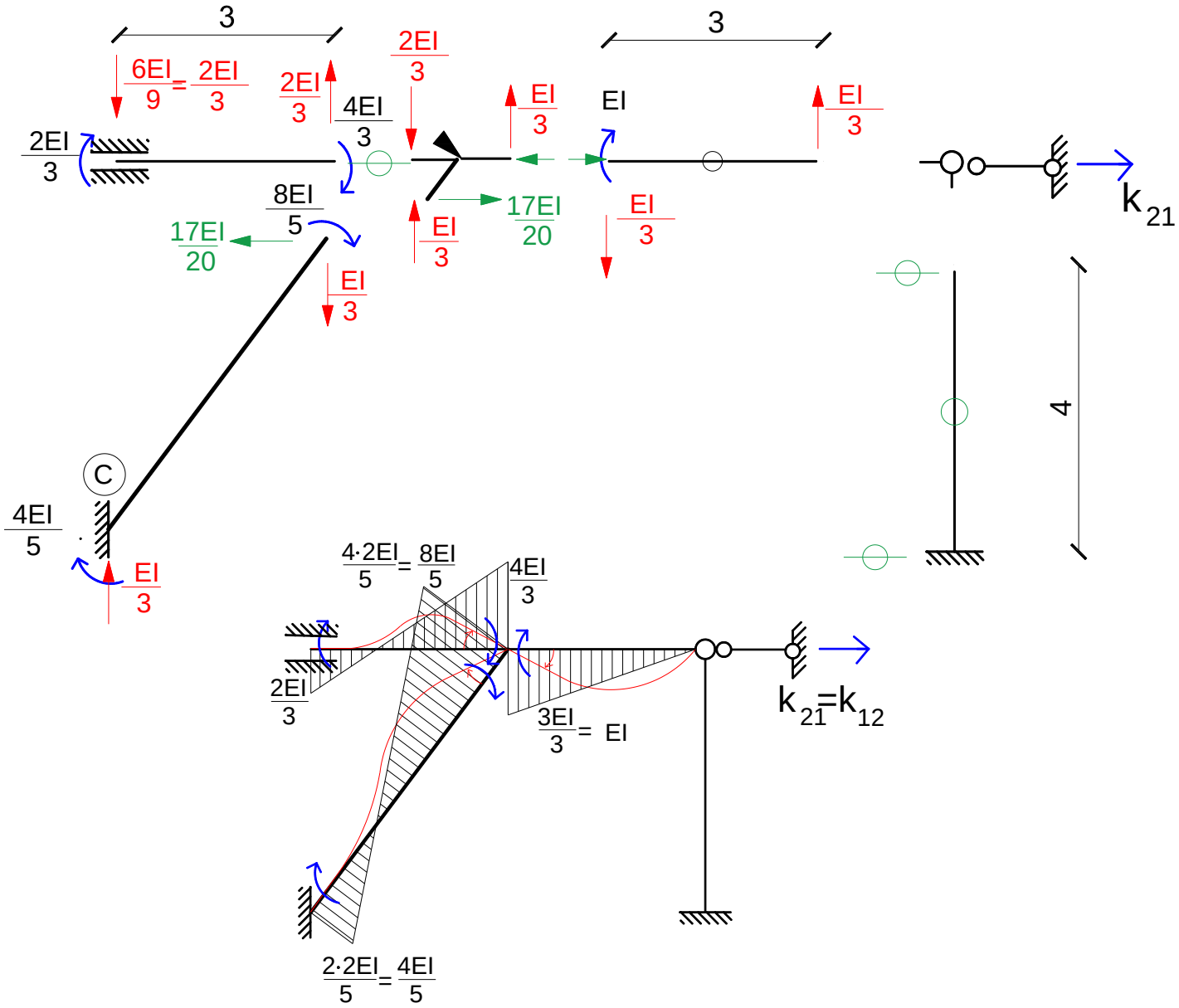
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



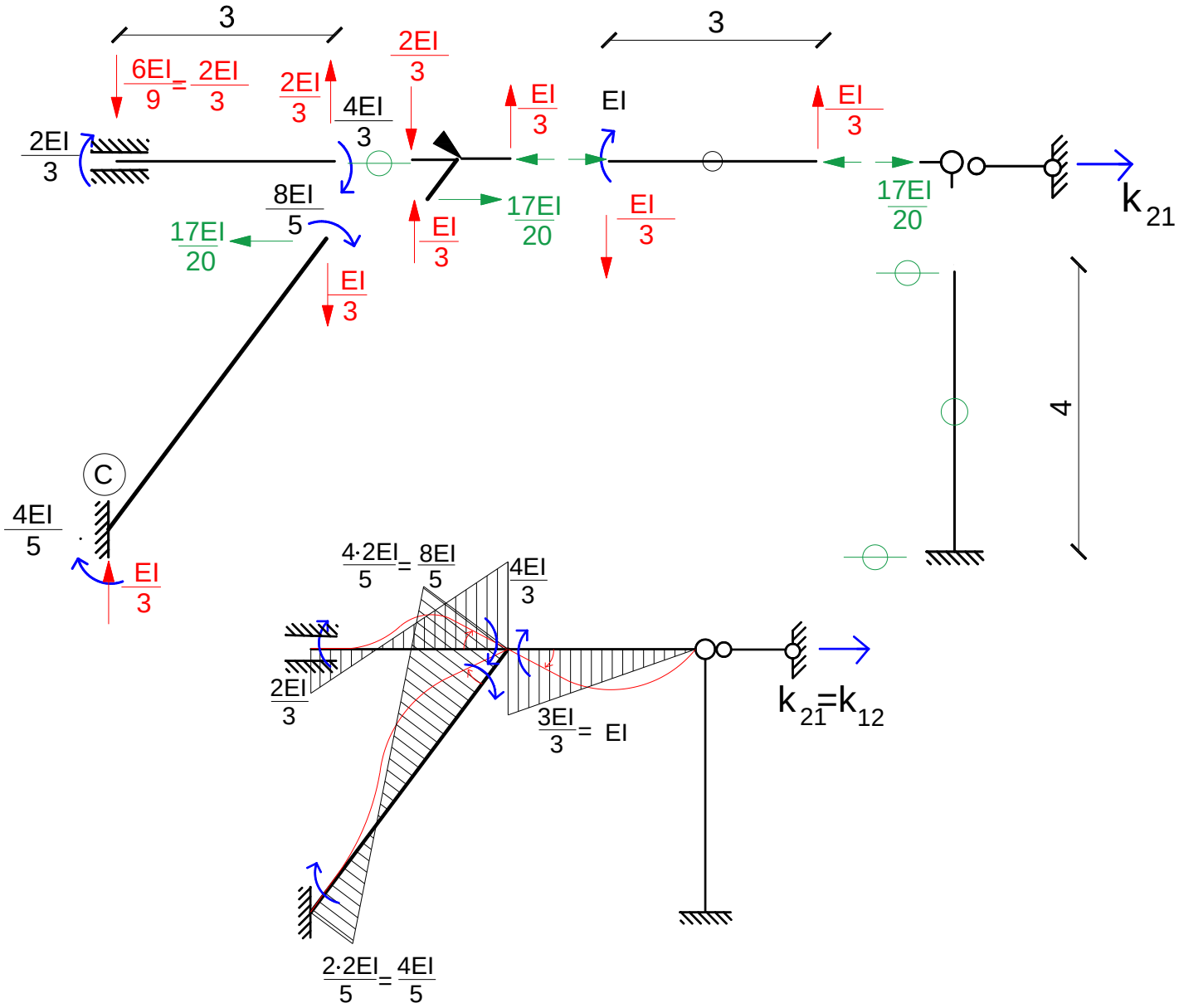
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



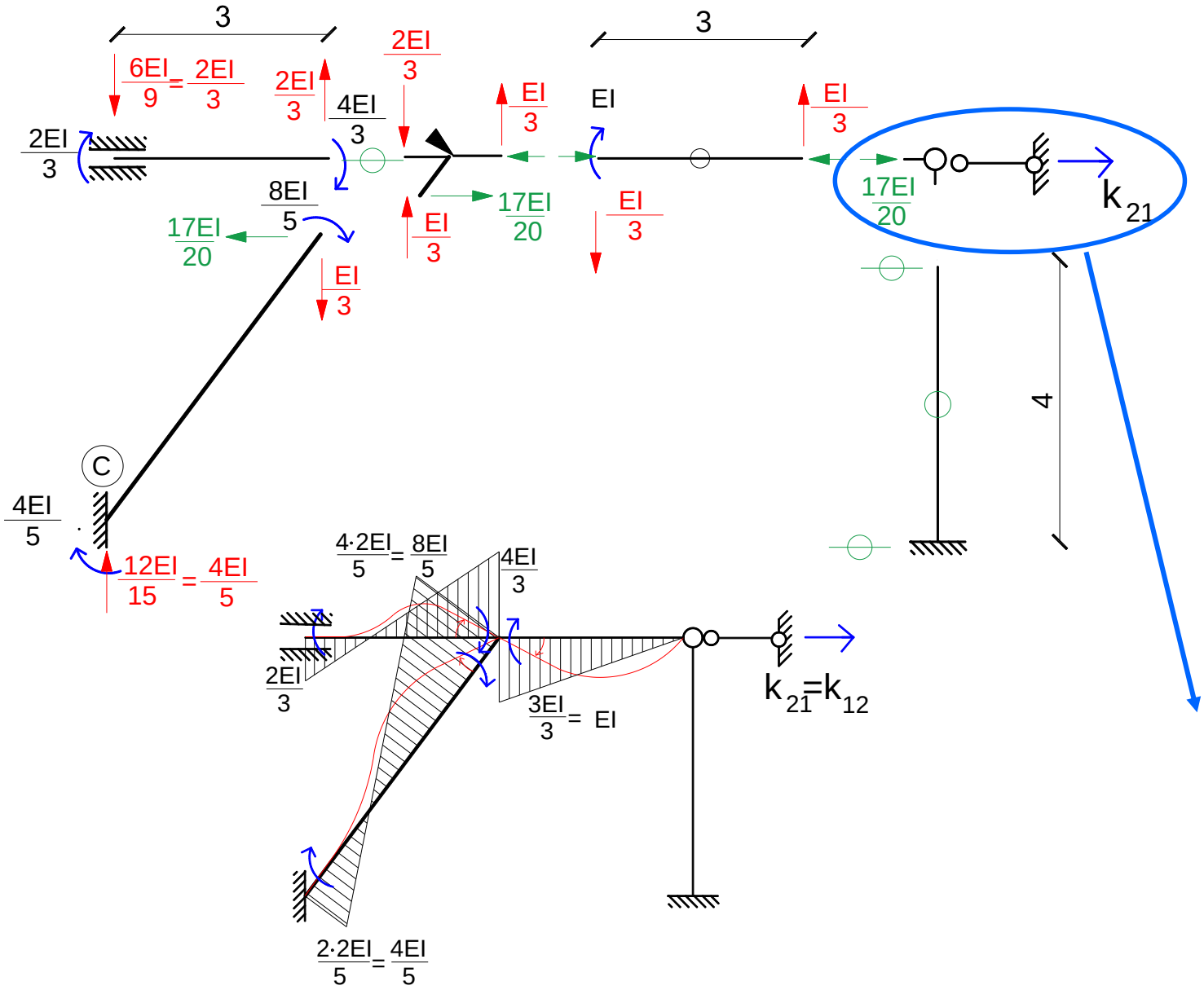
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



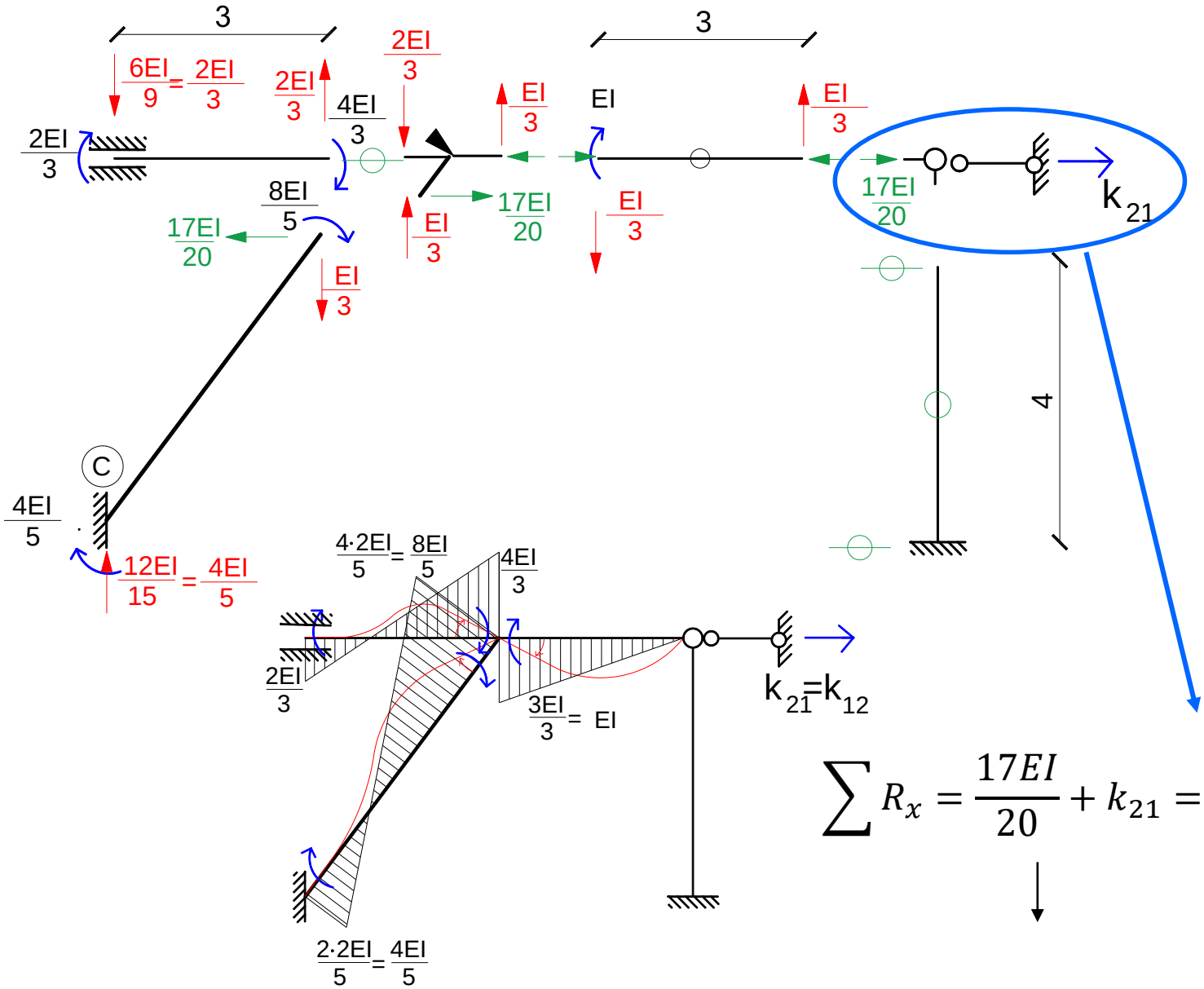
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{21}



$$\sum R_x = \frac{17EI}{20} + k_{21} = 0$$

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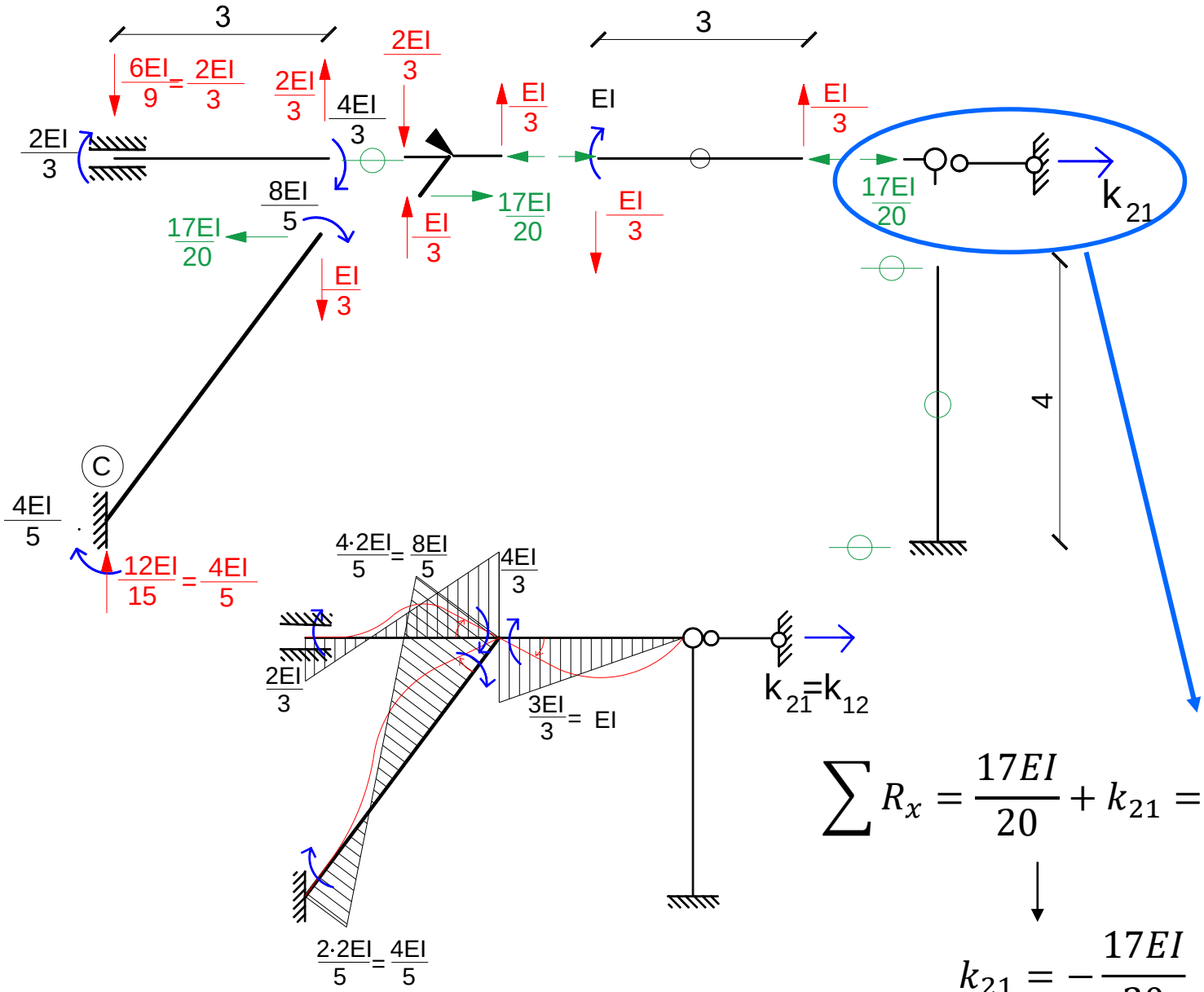
Wyznaczenie reakcji od przemieszczeń k_{21}



$$\sum R_x = \frac{17EI}{20} + k_{21} = 0$$

$$\downarrow$$

$$k_{21} = -\frac{17EI}{20}$$

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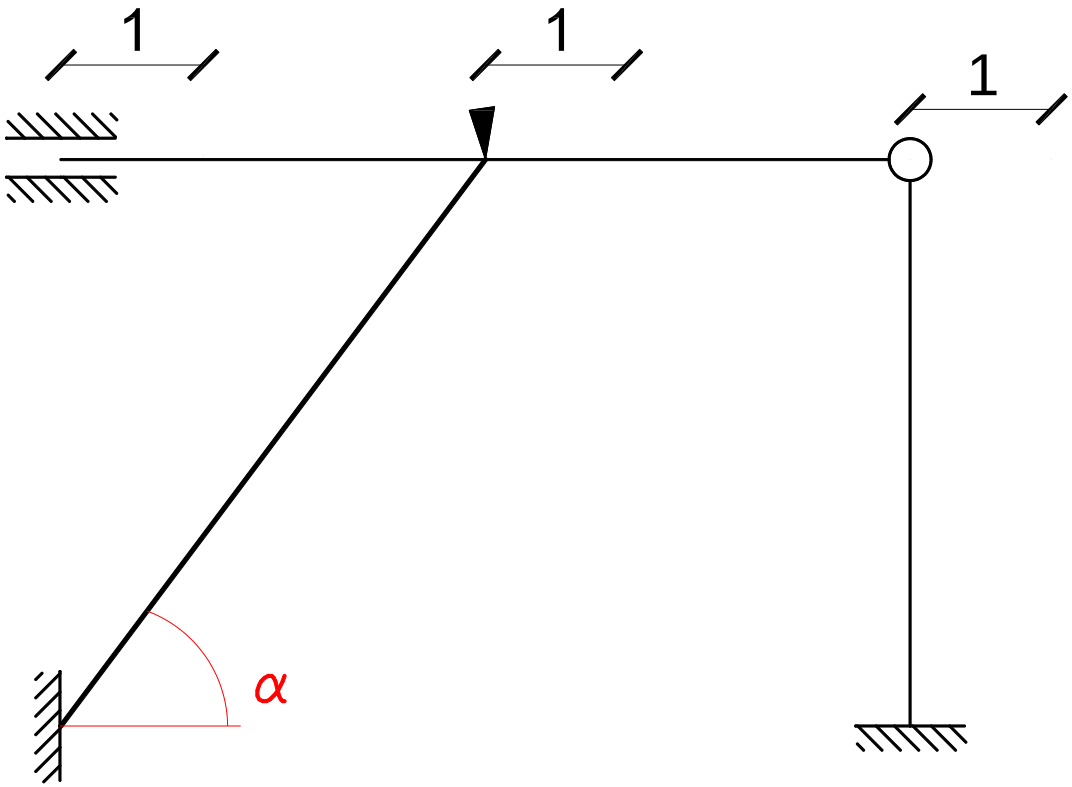
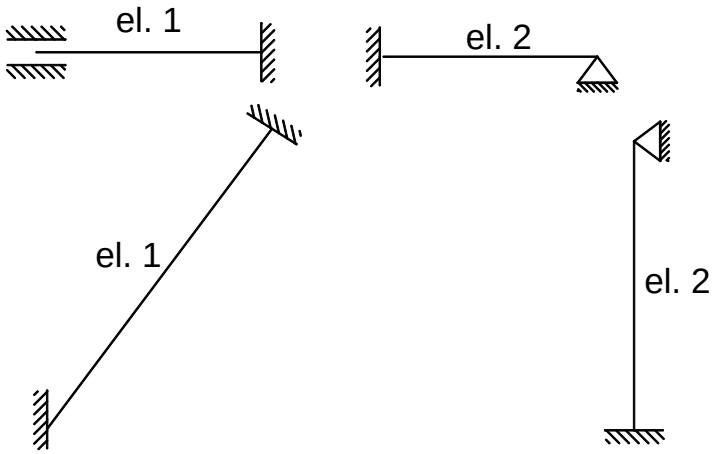
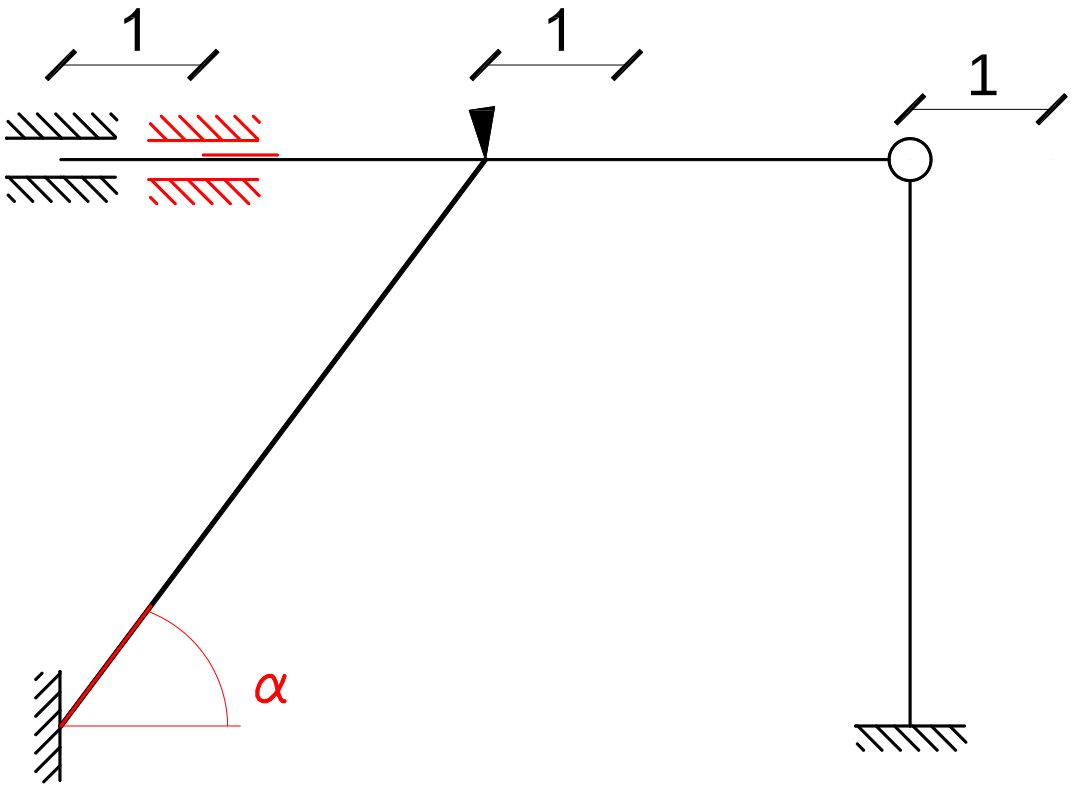


Stan $\Delta_2=1$

$\rightarrow k_{22}$

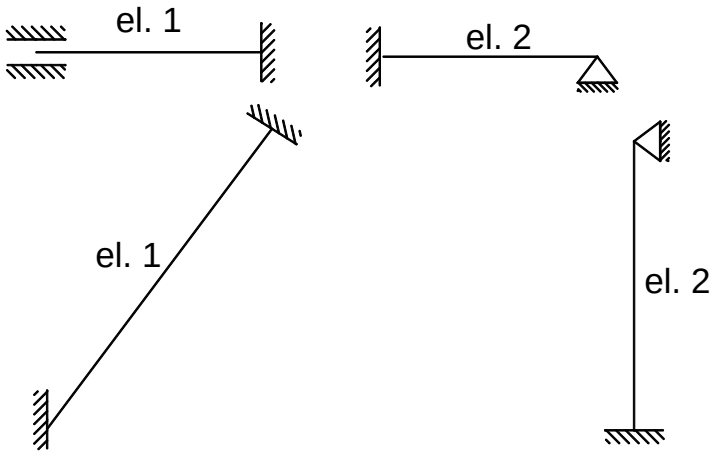


dr inż. Hanna Weber

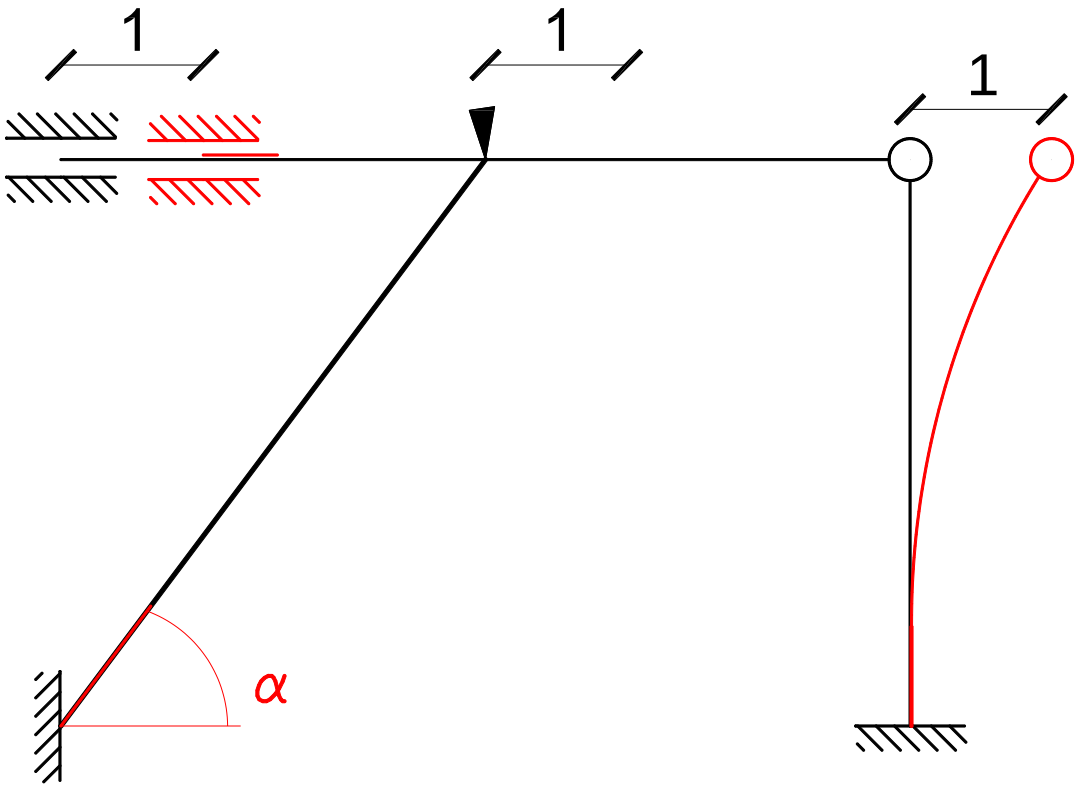


Stan $\Delta_2=1$

k_{22}

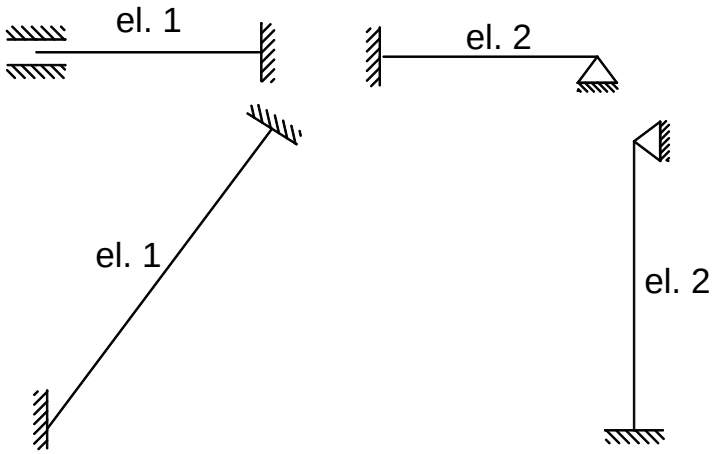


dr inż. Hanna Weber

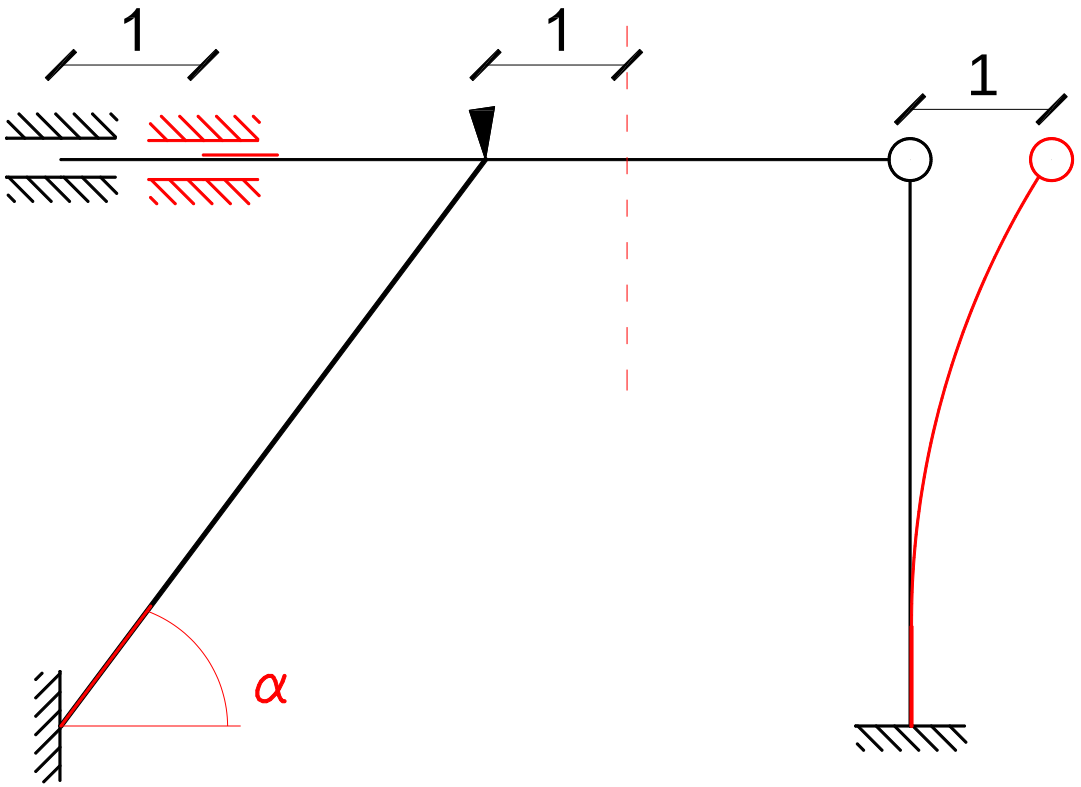


Stan $\Delta_2=1$

k_{22}

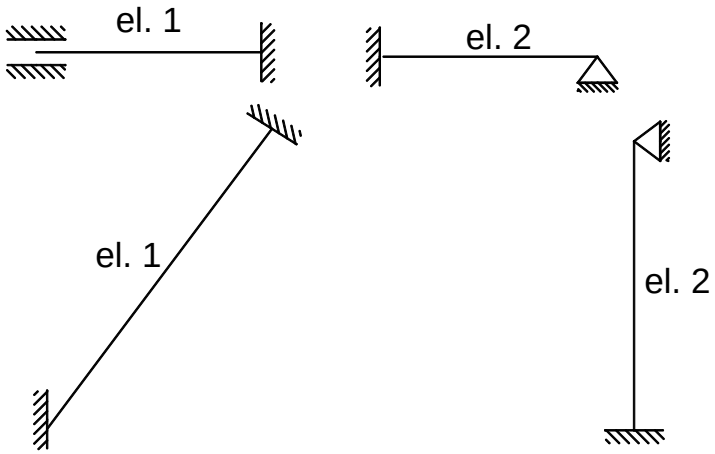


dr inż. Hanna Weber

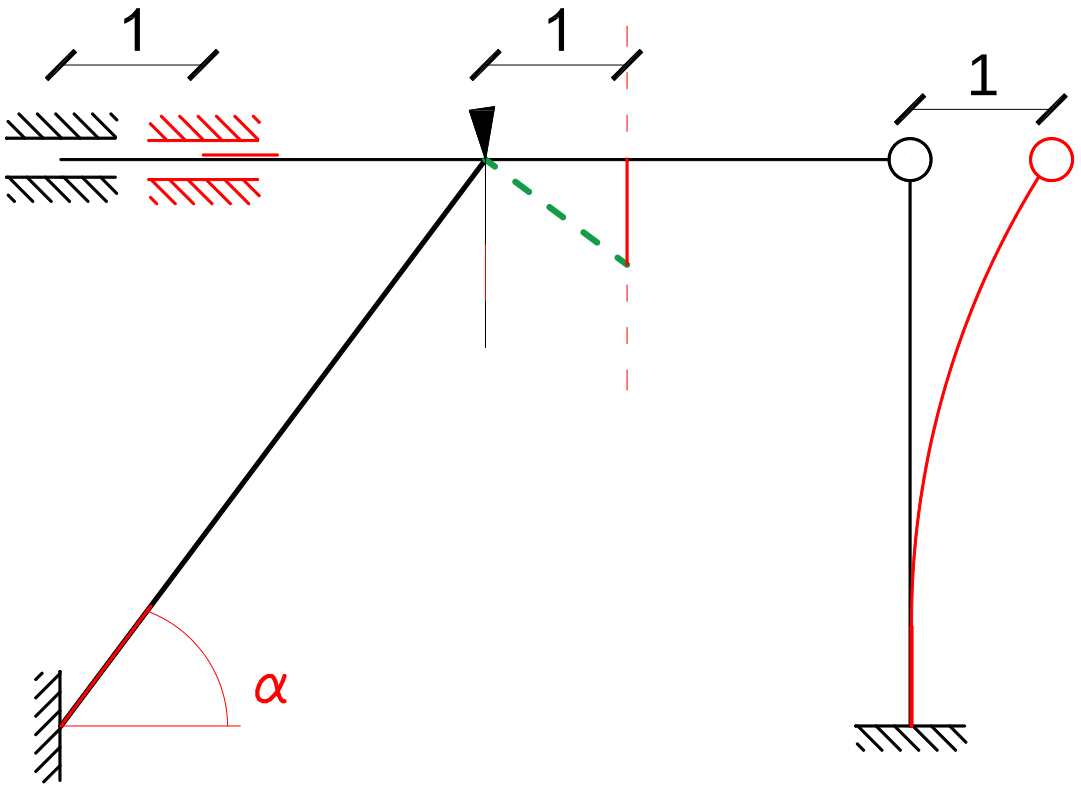


Stan $\Delta_2=1$

k_{22}

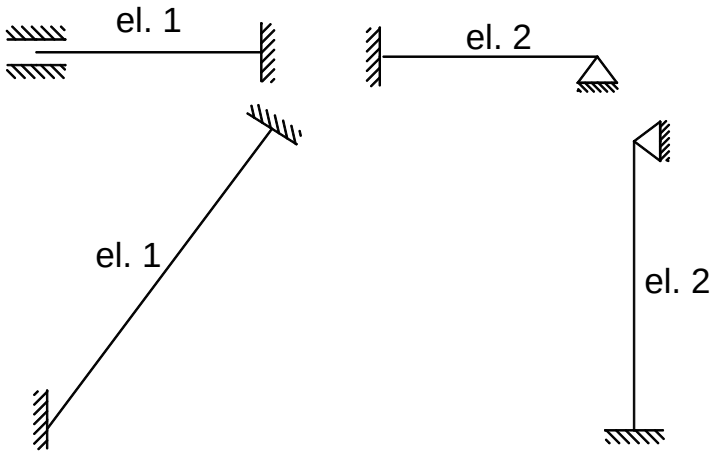


dr inż. Hanna Weber

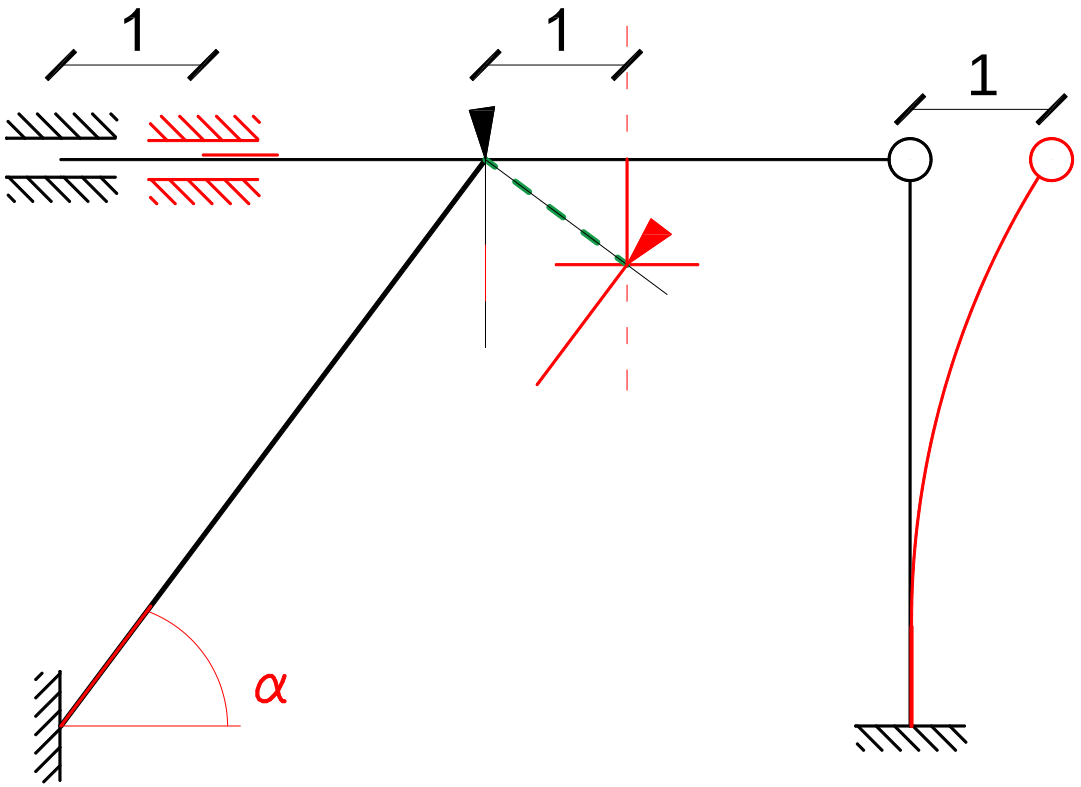


Stan $\Delta_2=1$

k_{22}

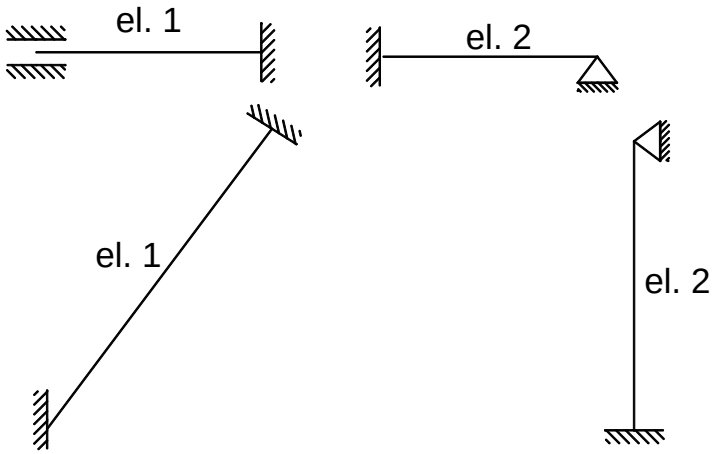


dr inż. Hanna Weber



Stan $\Delta_2=1$

k_{22}



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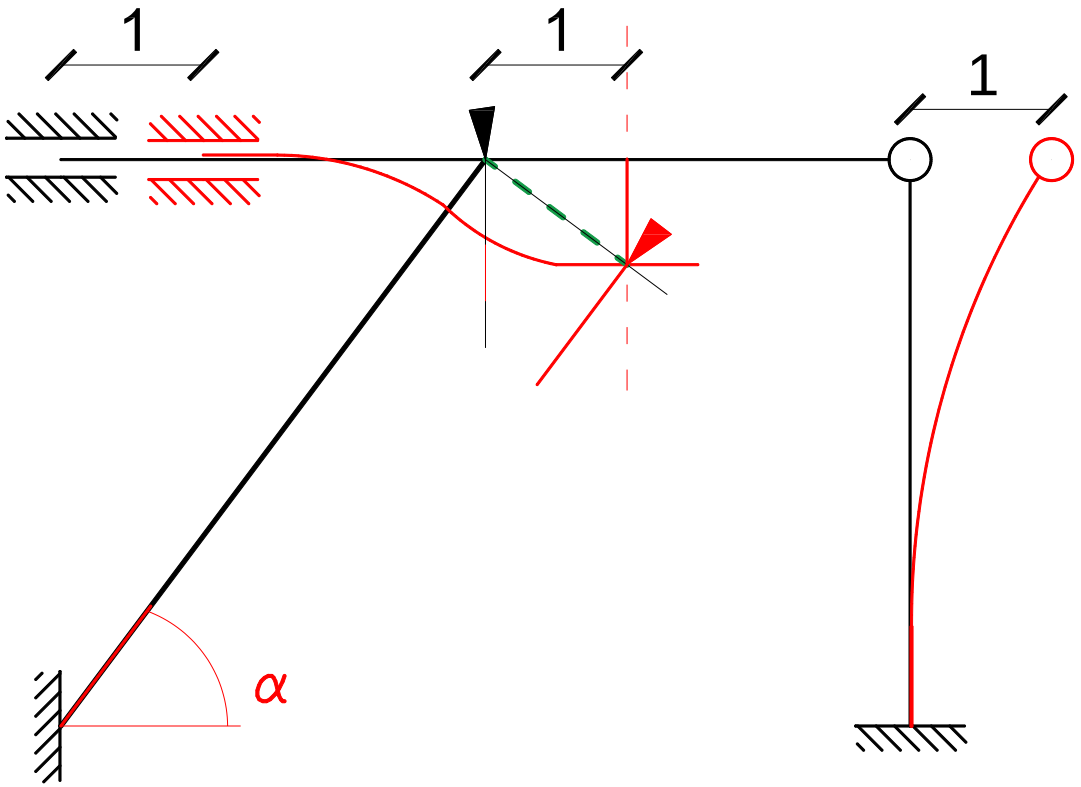
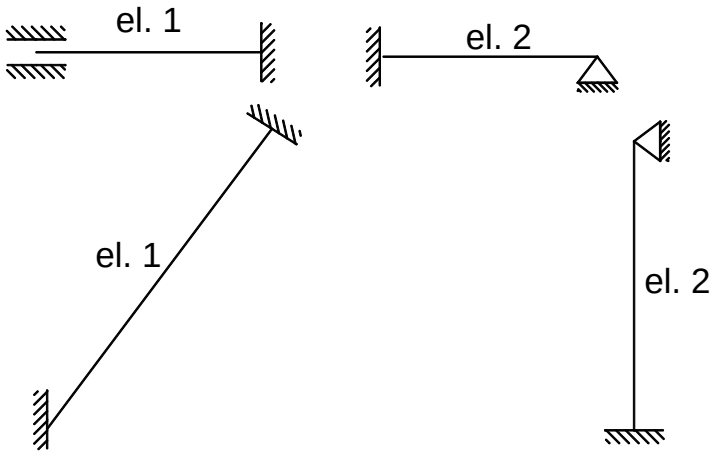
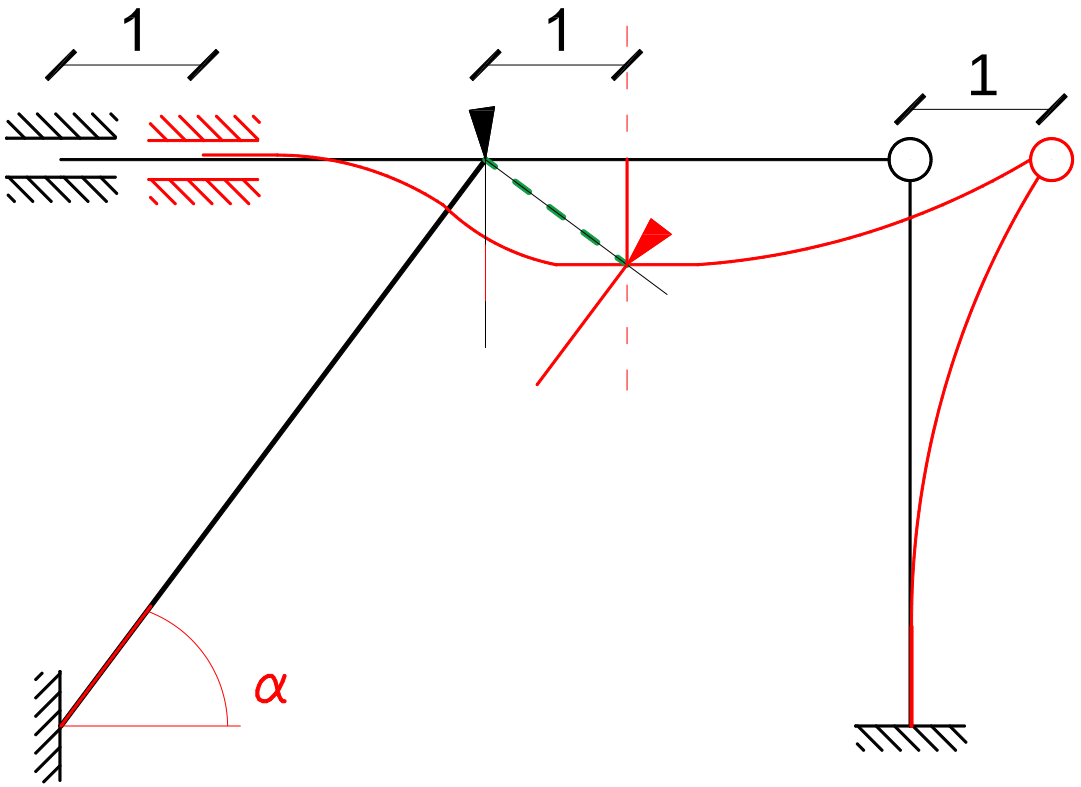


Stan $\Delta_2=1$

k_{22}

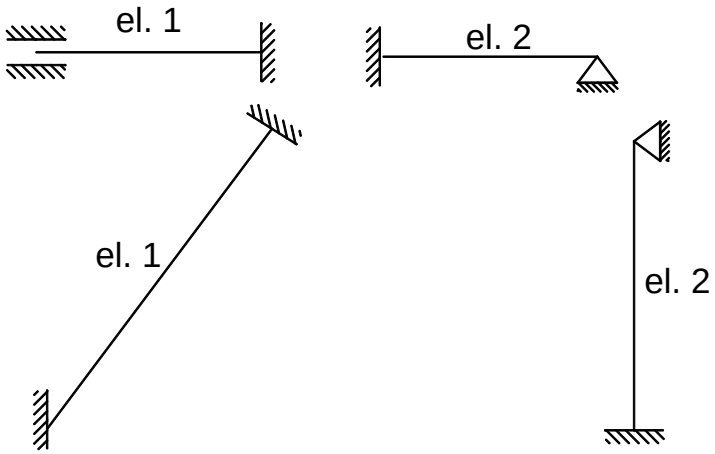


dr inż. Hanna Weber



Stan $\Delta_2=1$

k_{22}



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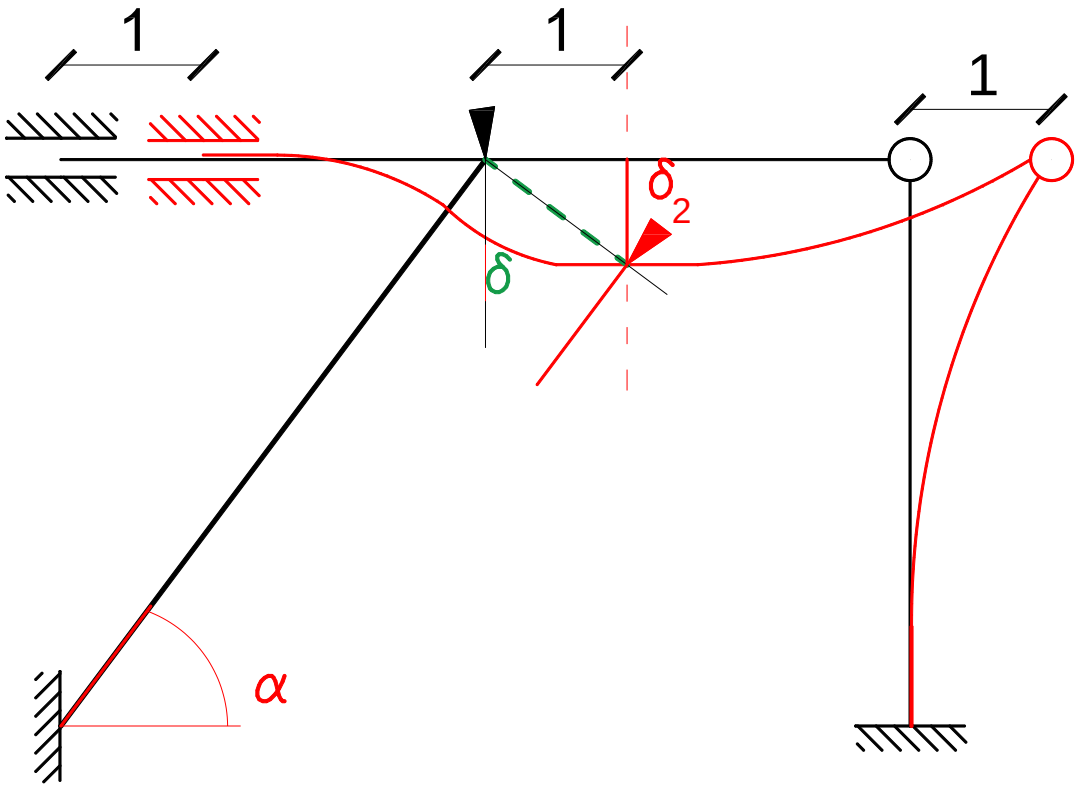

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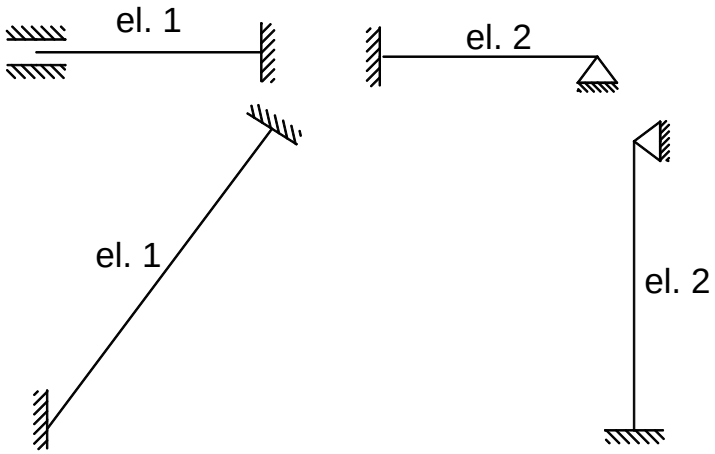
k_{22}



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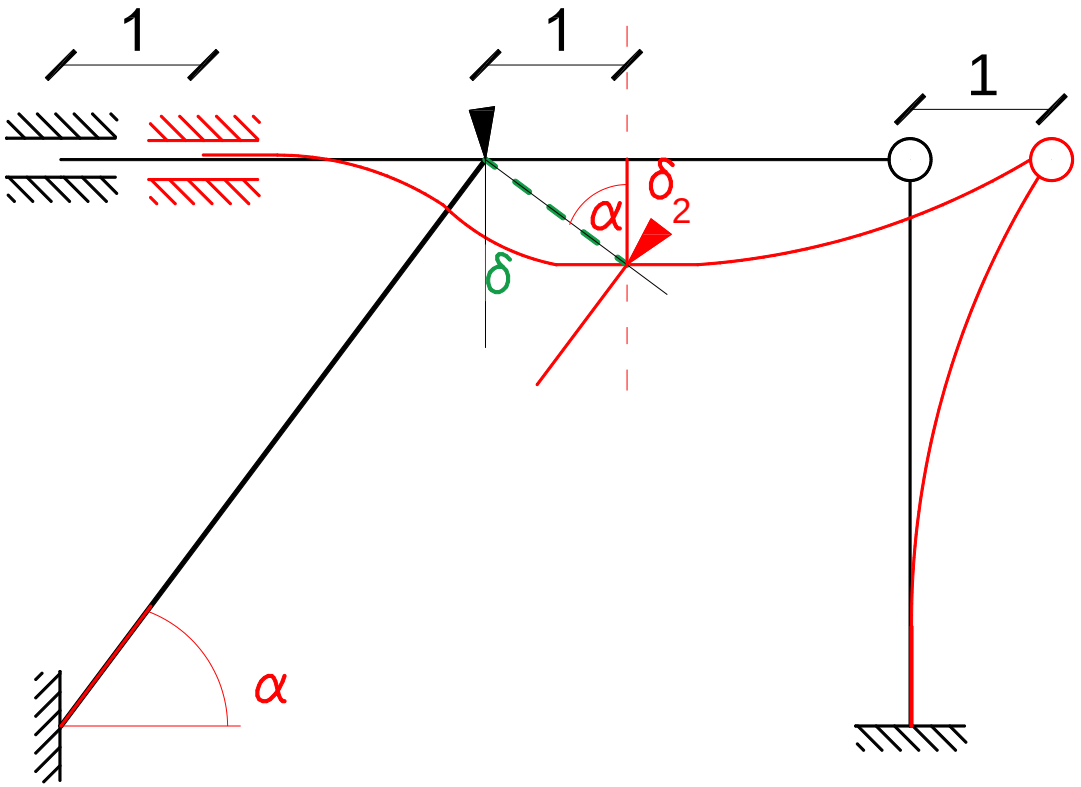
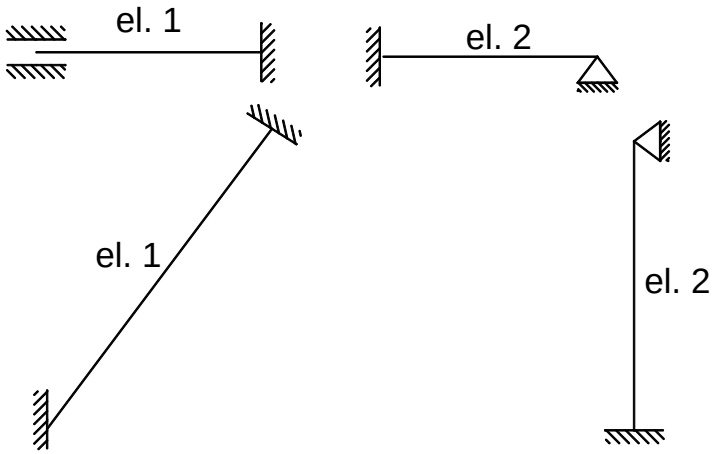


Stan $\Delta_2=1$

k_{22}



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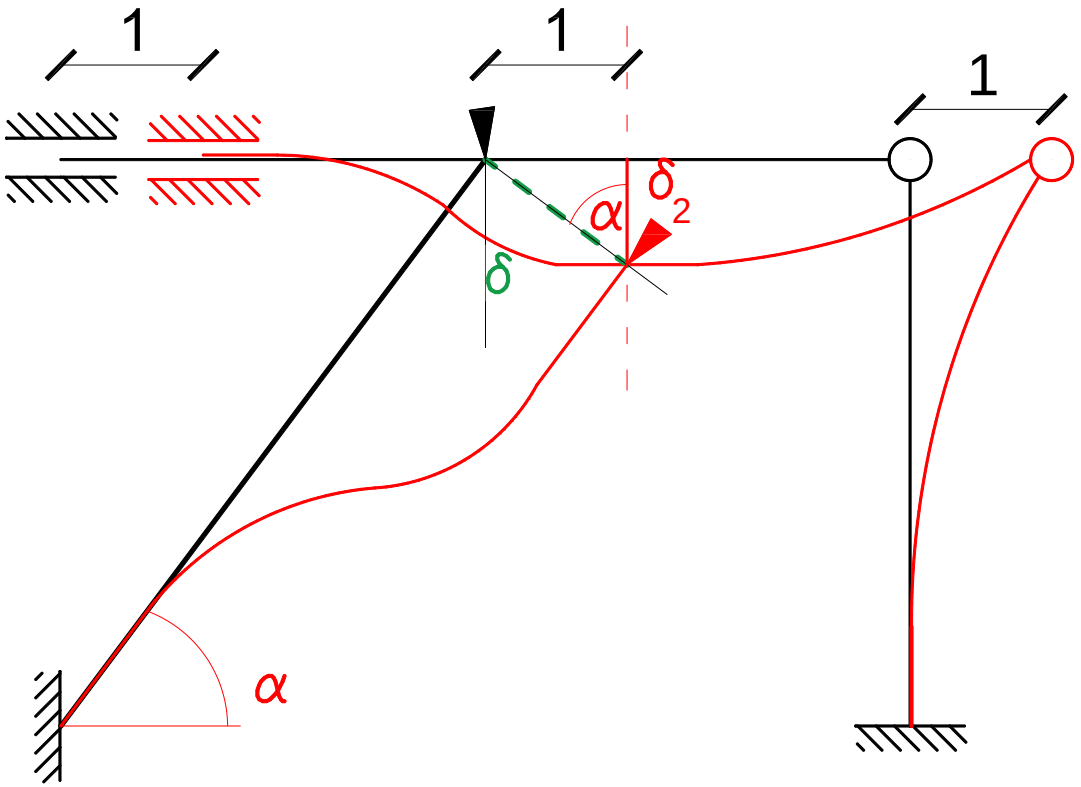

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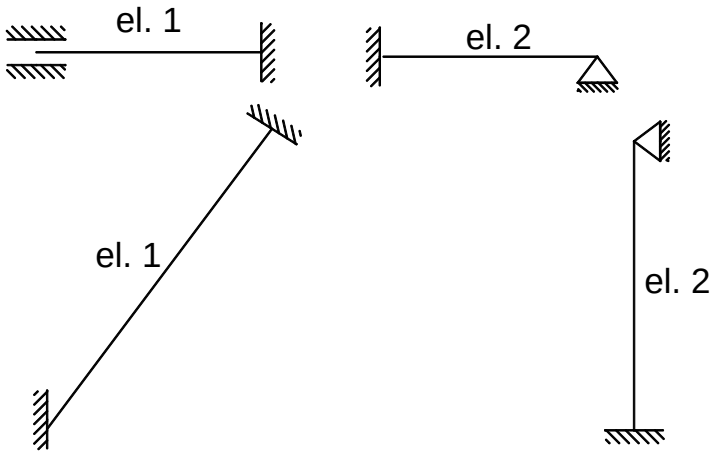
k_{22}



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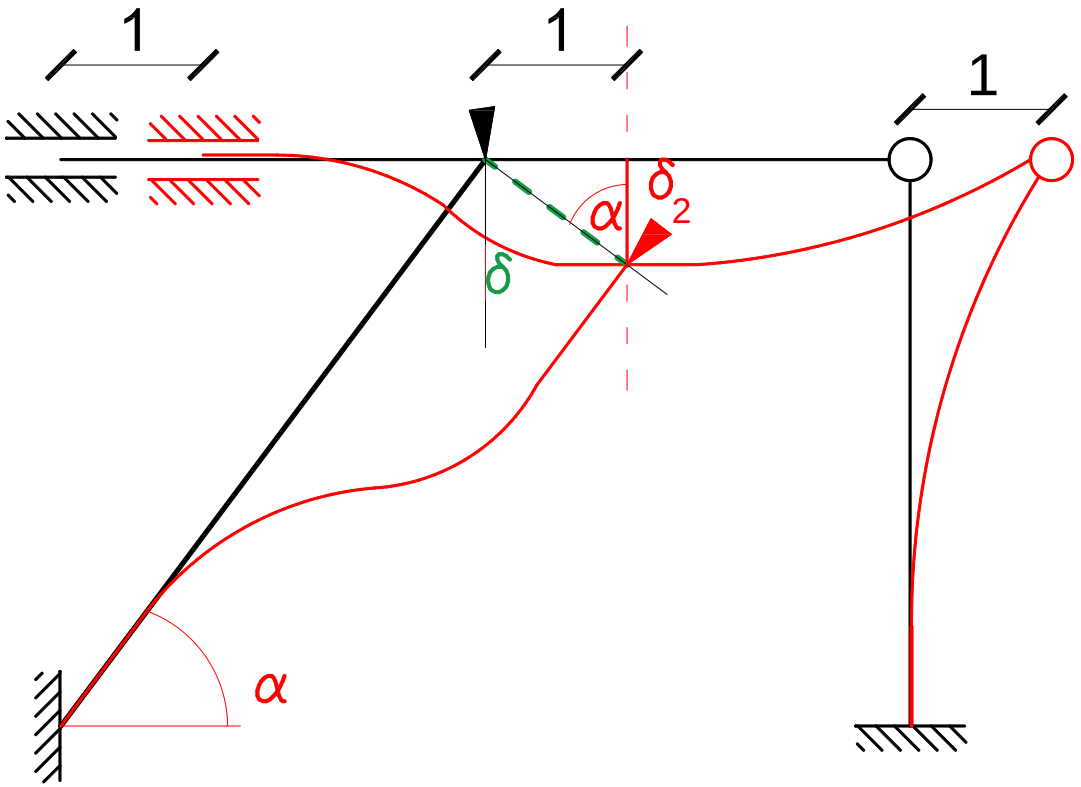
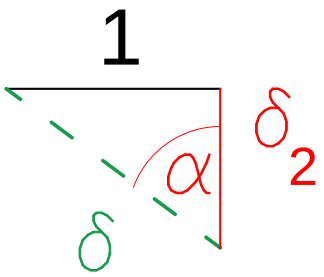
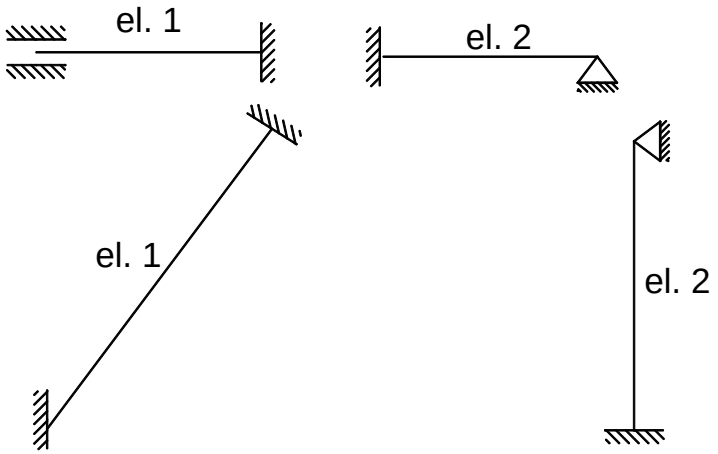


Stan $\Delta_2=1$

k_{22}





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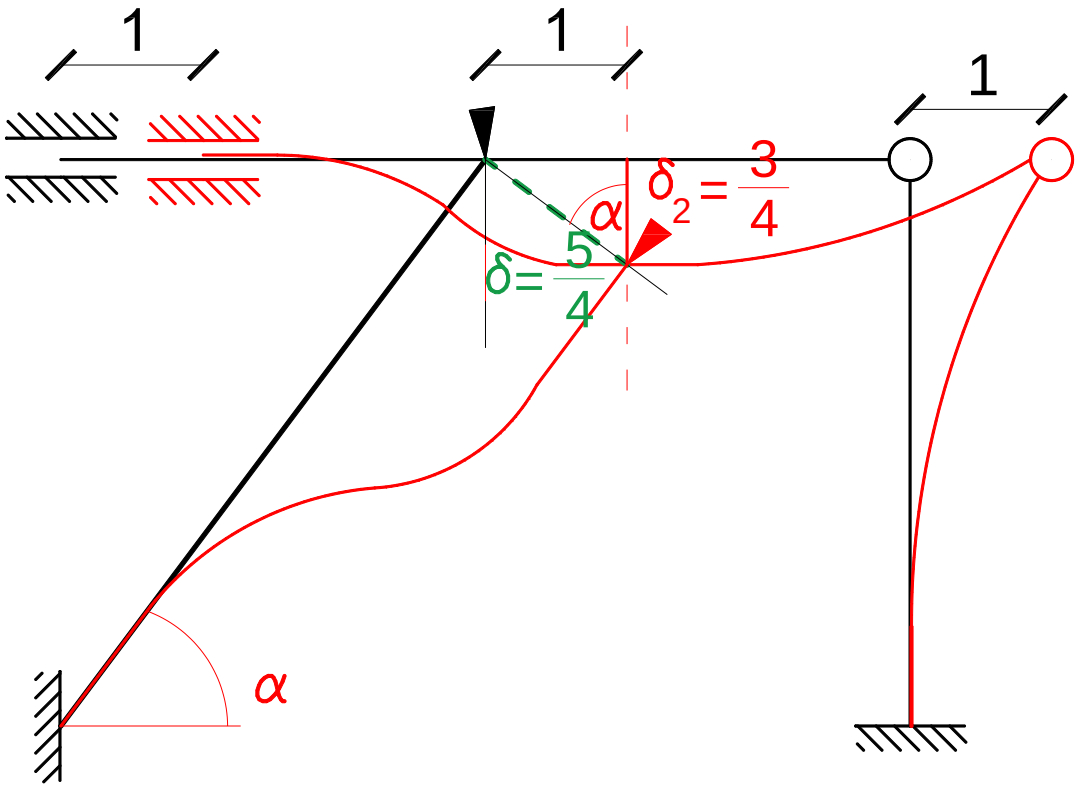
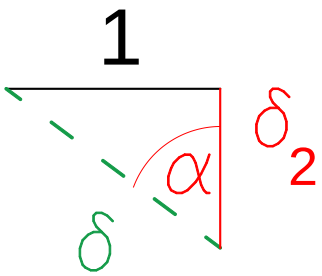


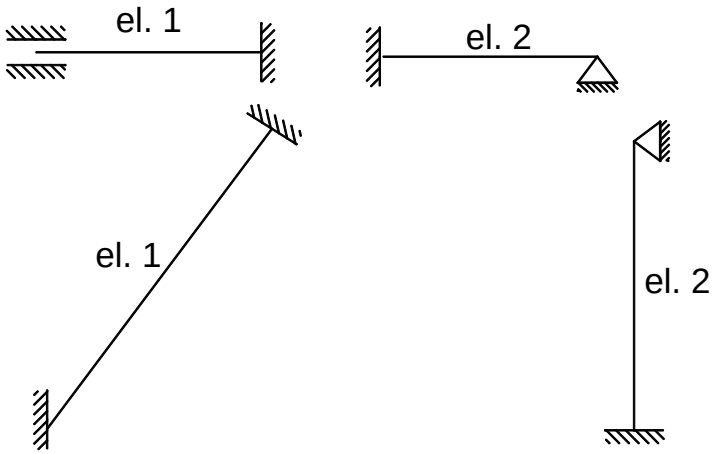
Stan $\Delta_2=1$

k_{22}



$$\frac{\delta_2}{1} = cot\alpha = \frac{3}{4} \rightarrow \delta_2 = \frac{3}{4}$$

$$\frac{1}{\delta} = sin\alpha = \frac{4}{5} \rightarrow \delta = \frac{1}{sin\alpha} = \frac{5}{4}$$



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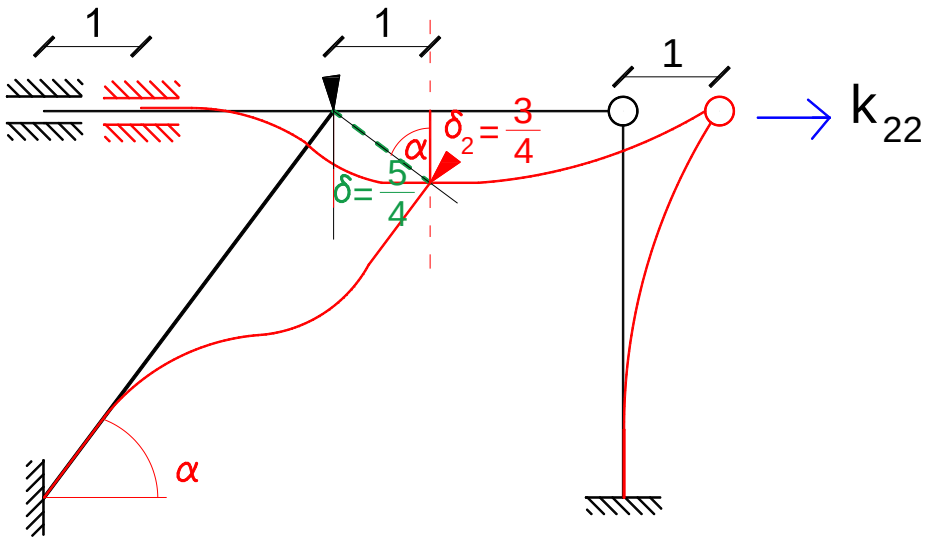
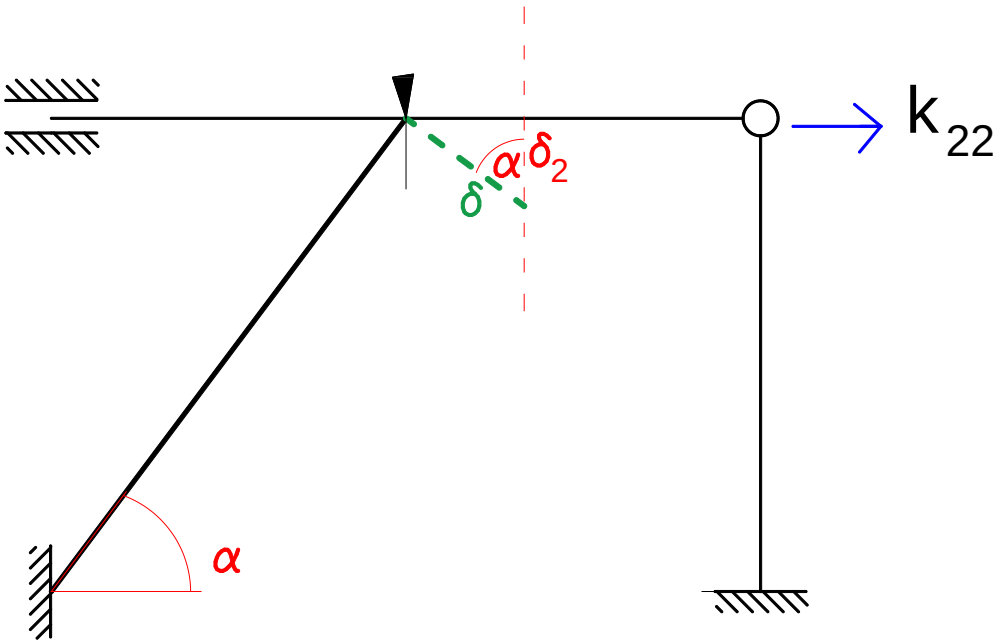
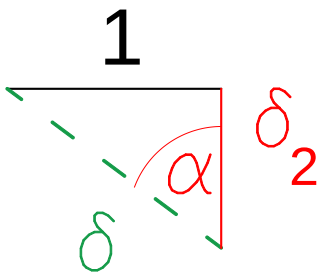


Stan $\Delta_2=1$





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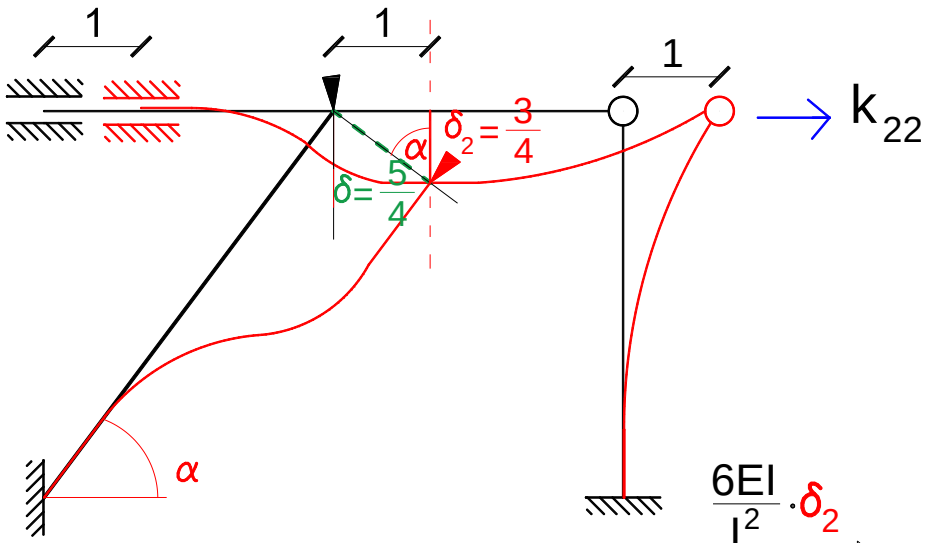

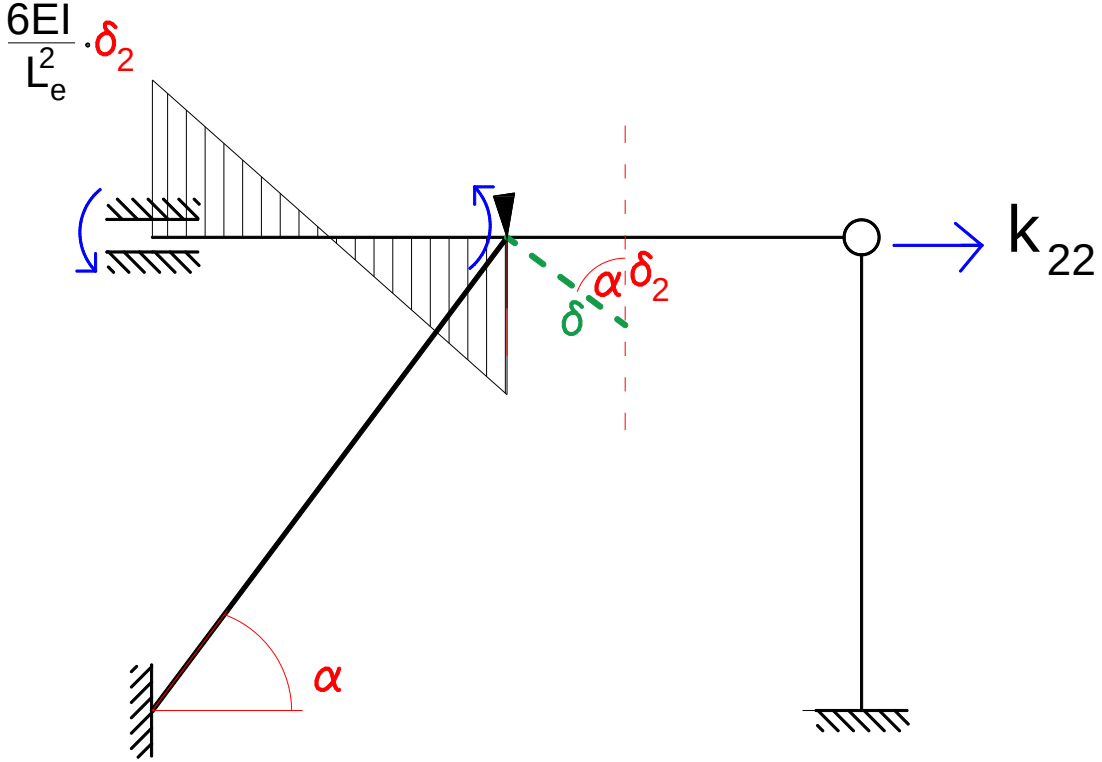

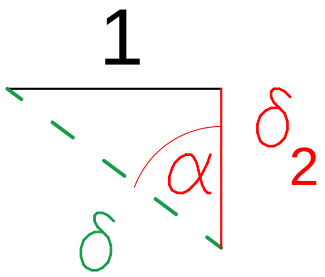

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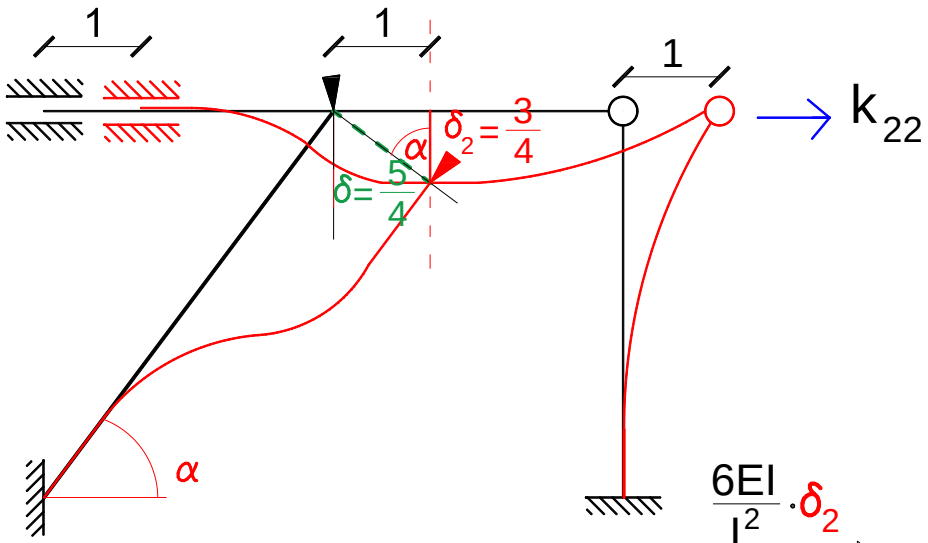

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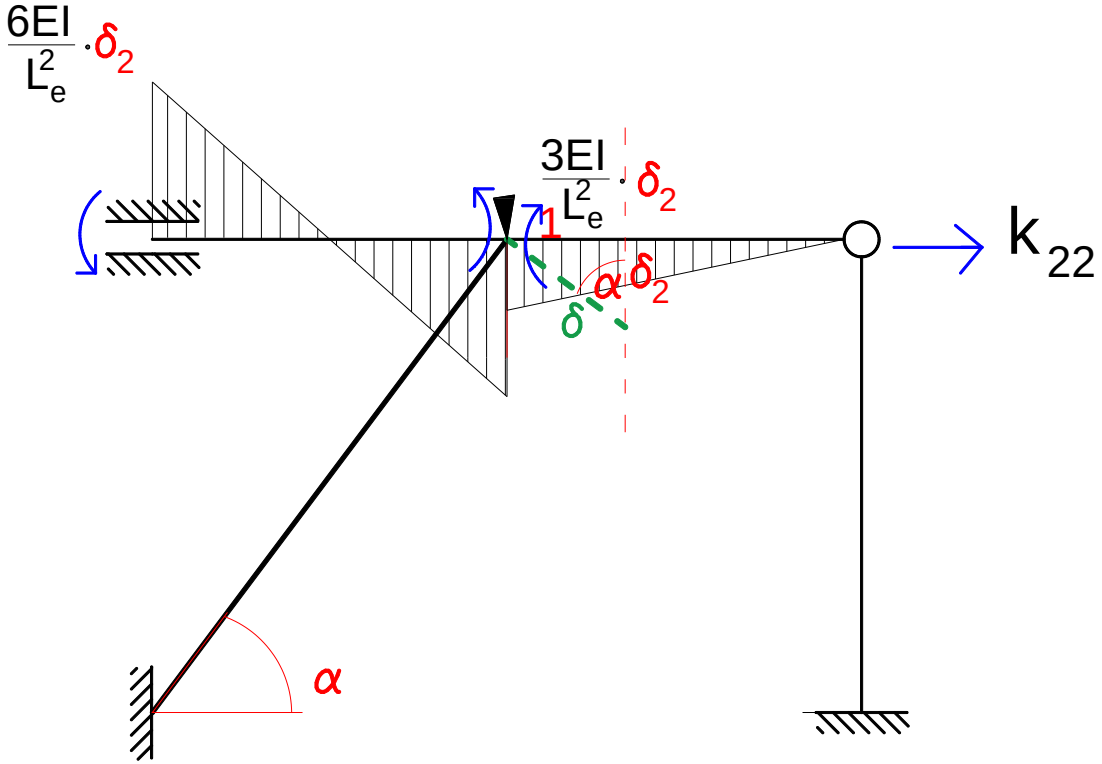

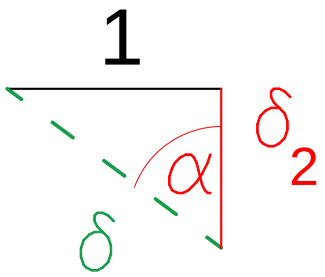

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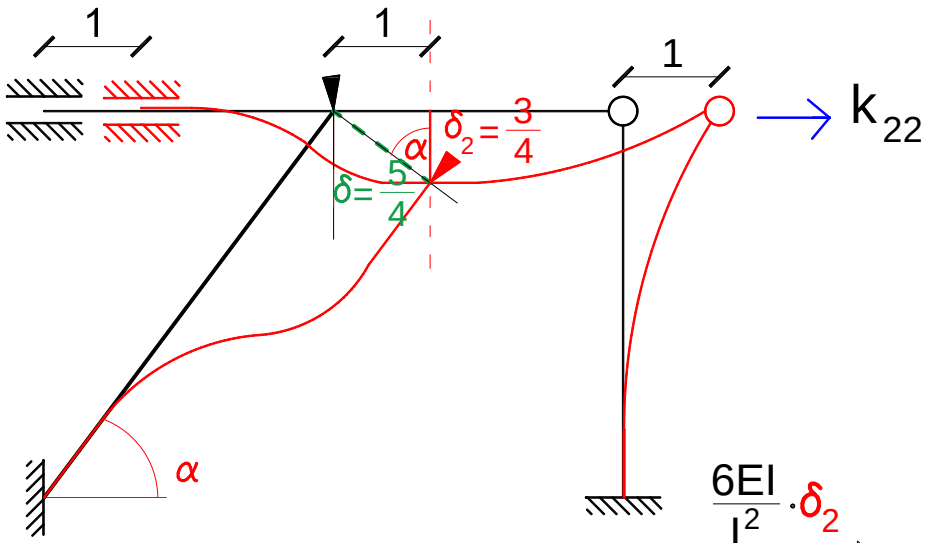

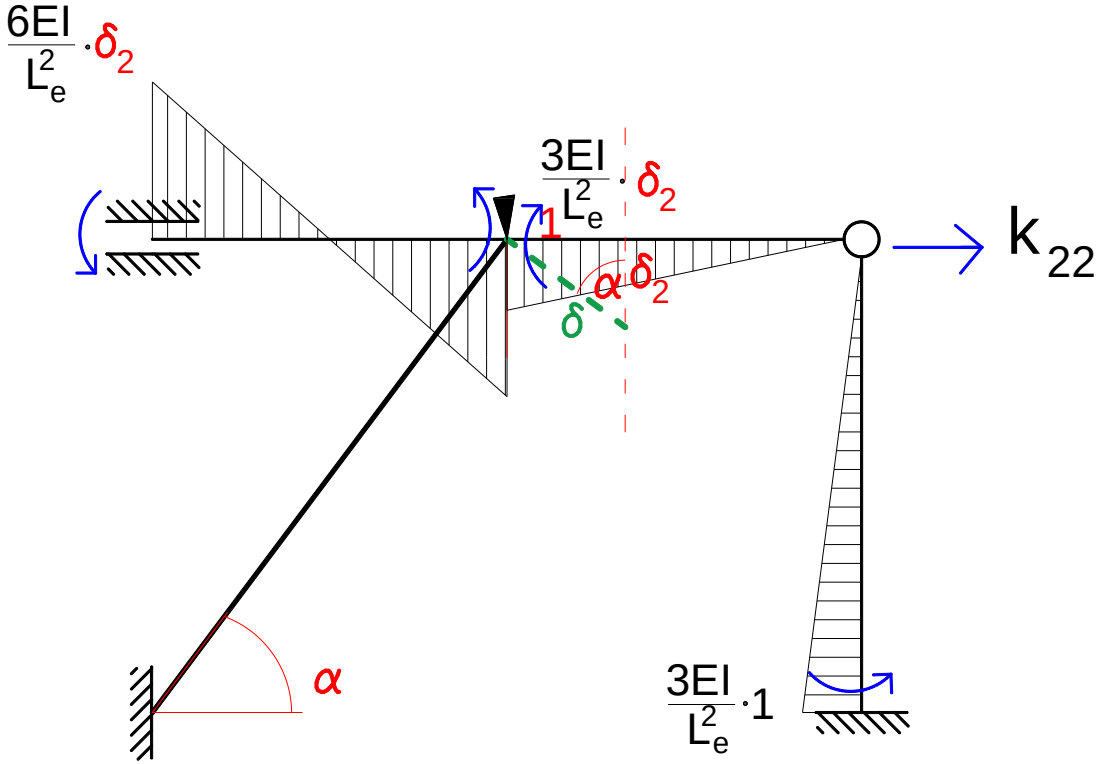

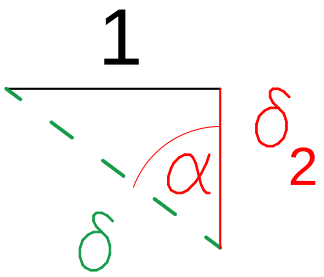

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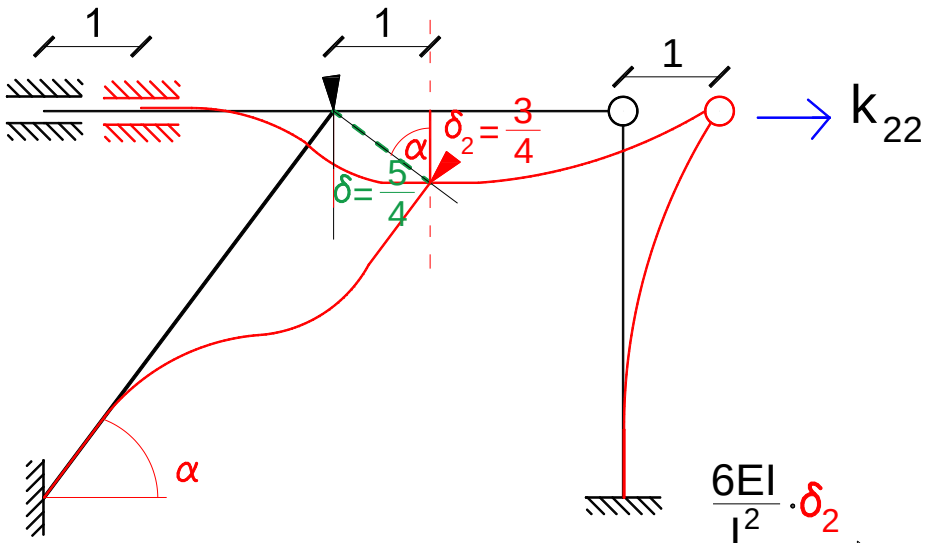

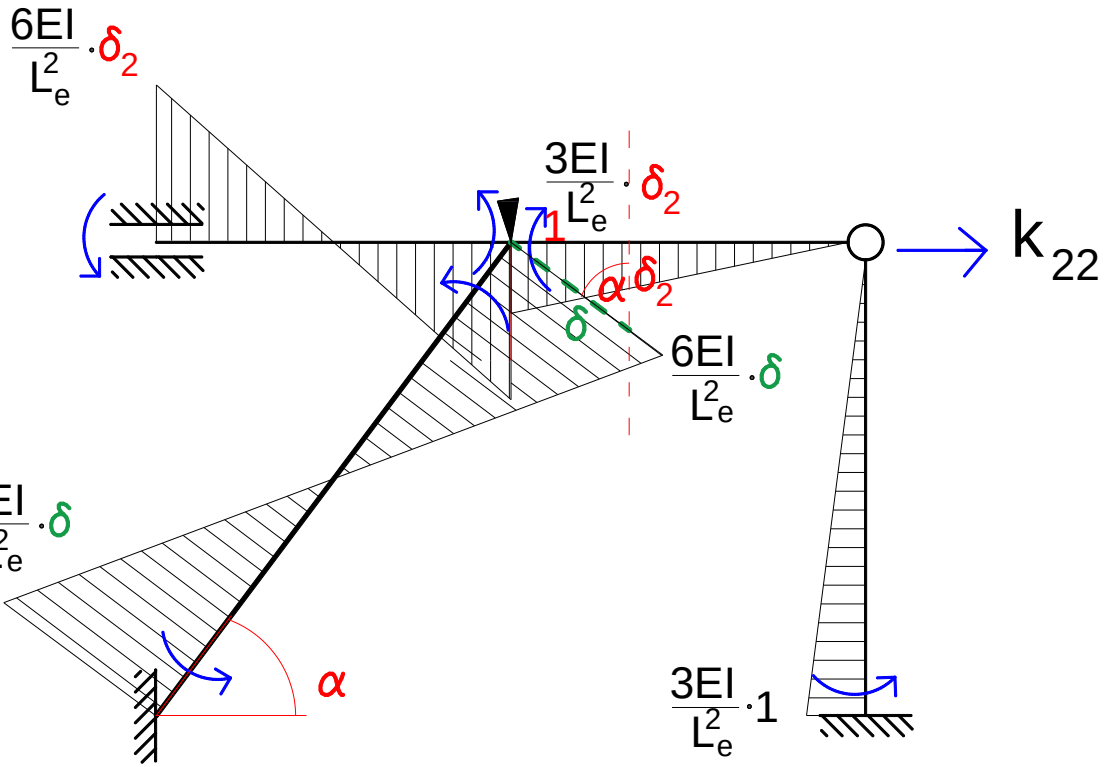

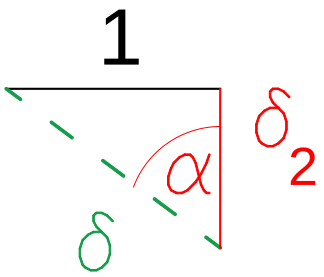

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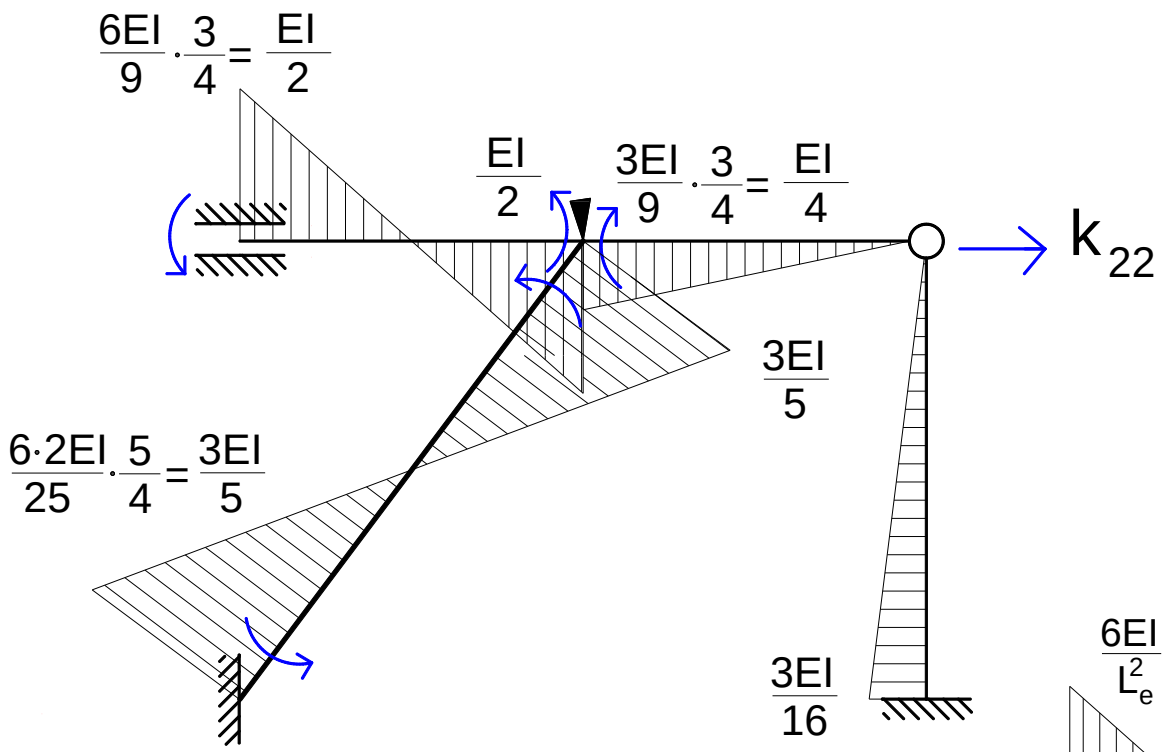
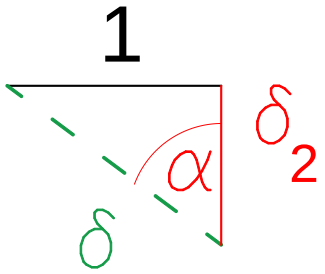
Stan $\triangle_2=1$





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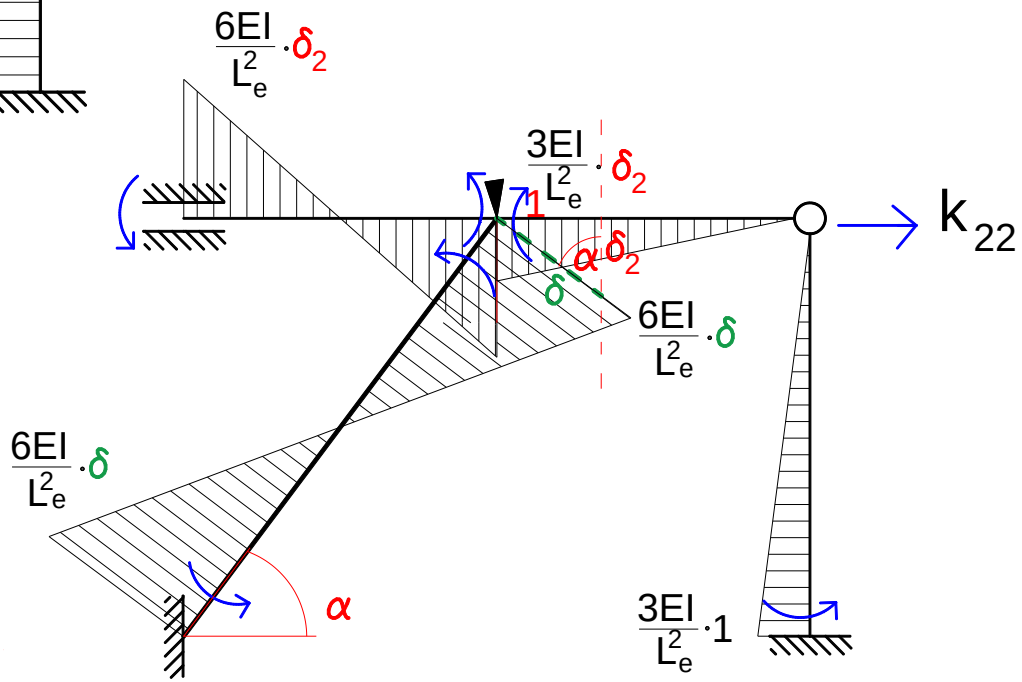

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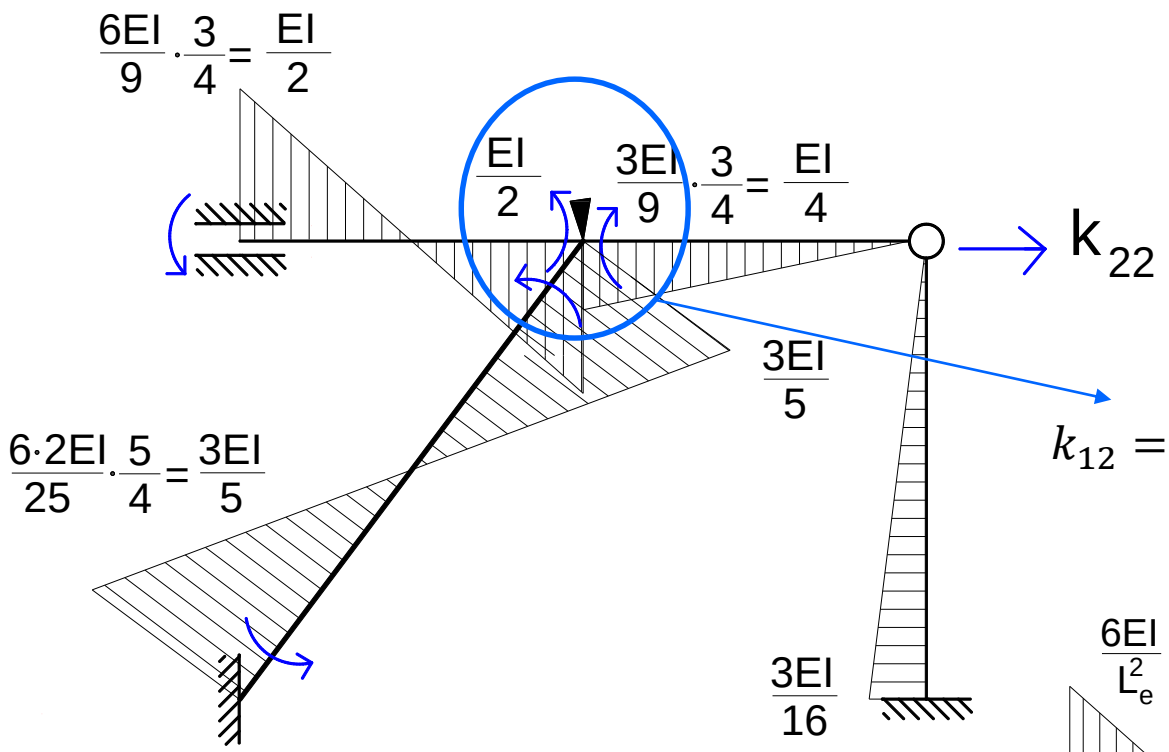
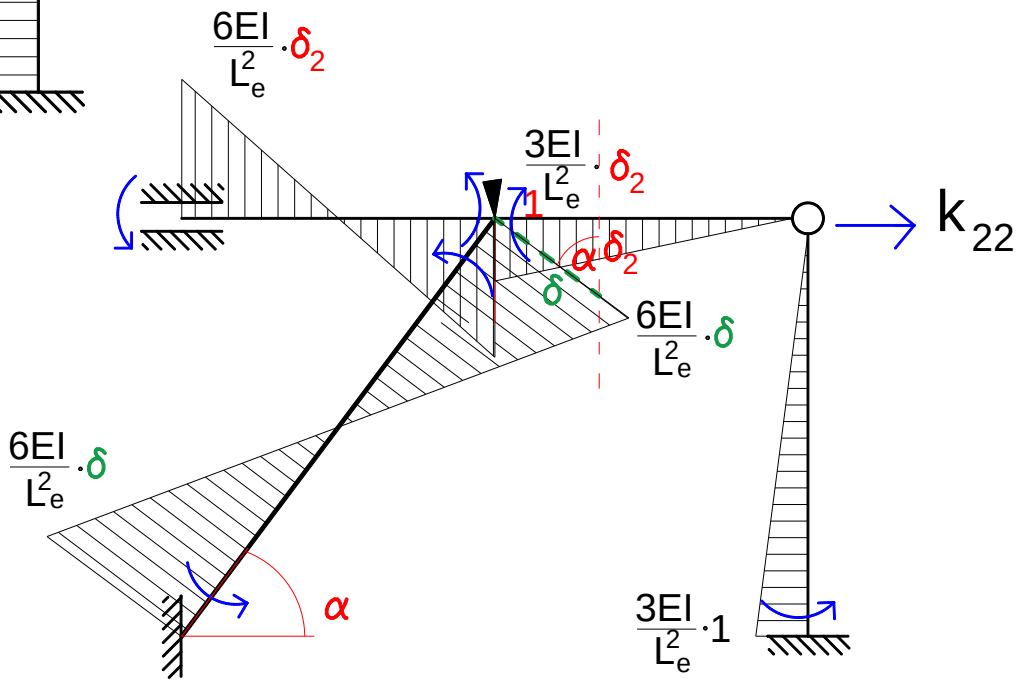
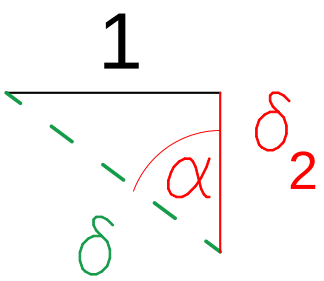
Stan $\triangle_2=1$



$$k_{12} = \frac{EI}{4} - \frac{EI}{2} - \frac{3EI}{5} = -\frac{17EI}{20} = k_{21}$$





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$$\frac{1}{\delta} = sin\alpha = \frac{4}{5} \rightarrow \delta = \frac{1}{sin\alpha} = \frac{5}{4}$$

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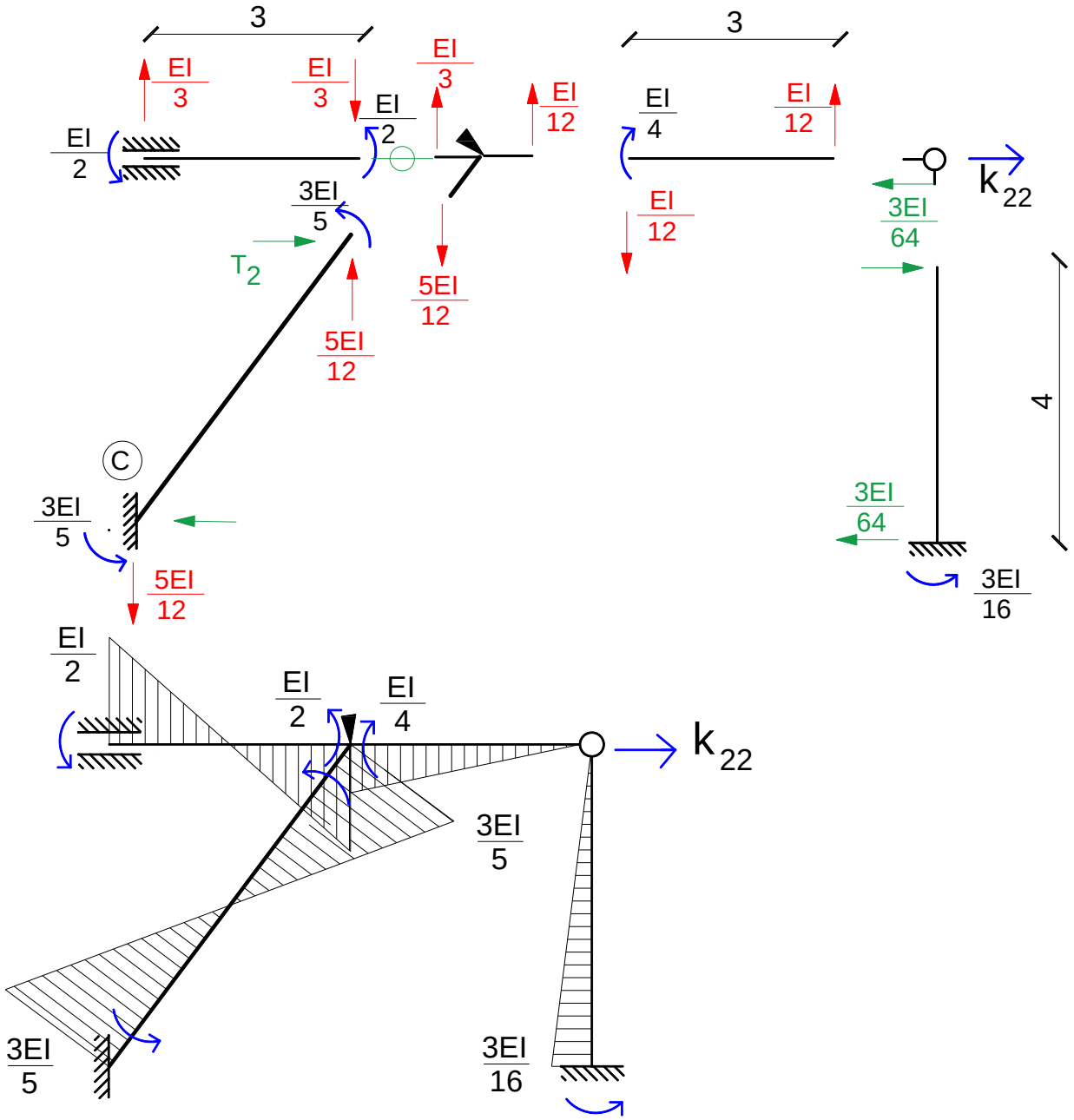
Wyznaczenie reakcji od przemieszczeń k_{22}



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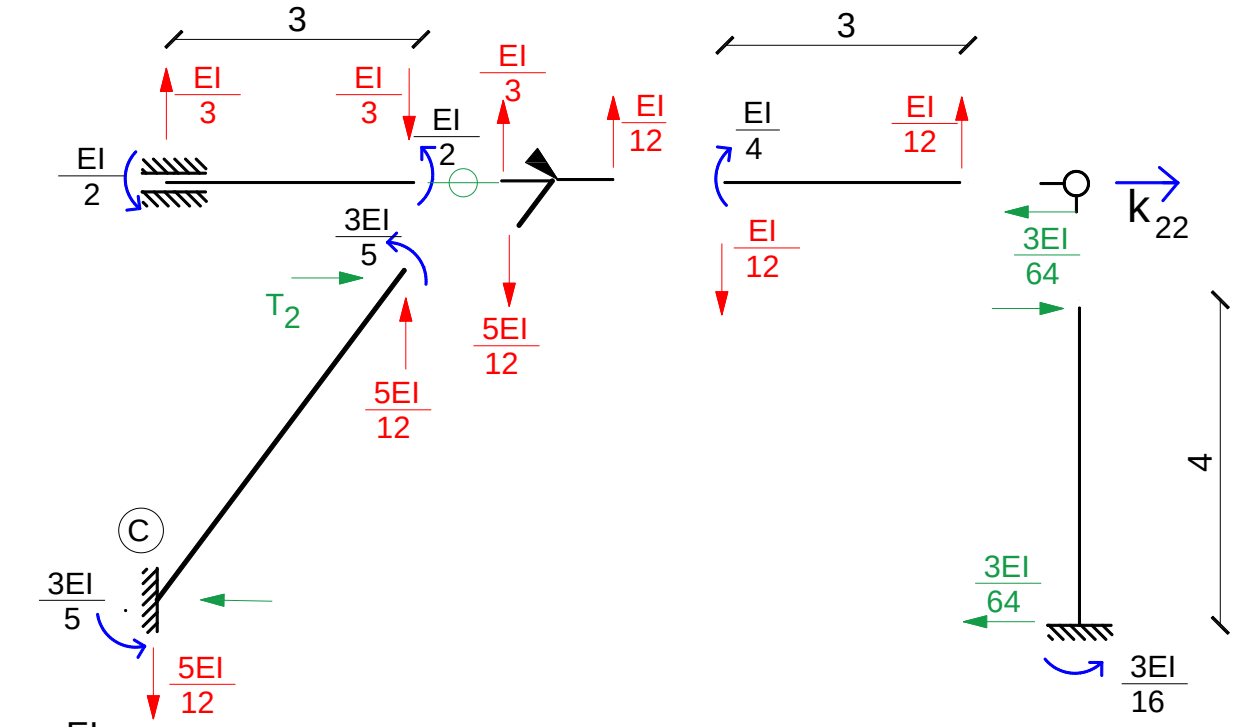
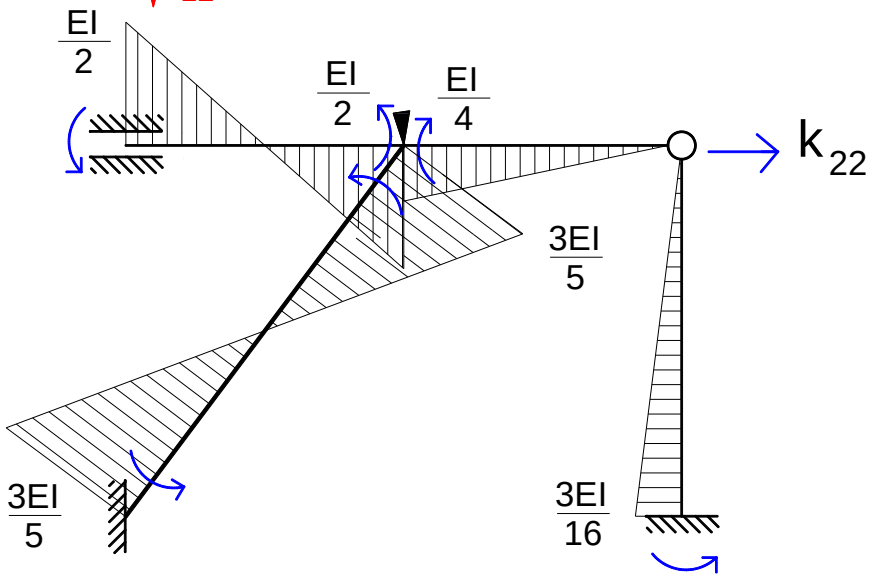
Wyznaczenie reakcji od przemieszczeń k_{22}



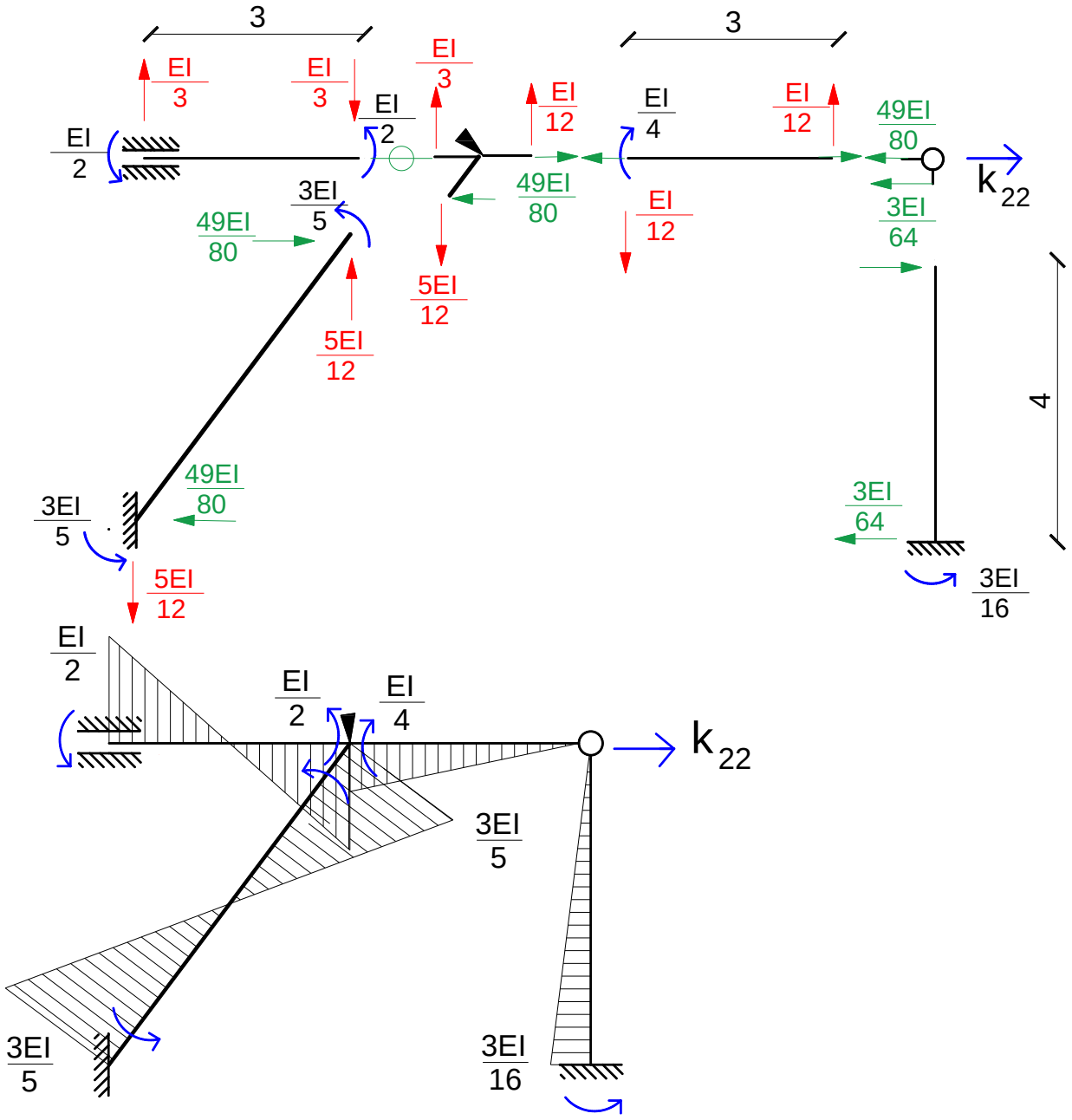


$$\sum M_C = -\frac{3EI}{5} - \frac{3EI}{5} - \frac{5EI}{12} \cdot 3 + T_2 \cdot 4 = 0$$

$$T_2 = \frac{49EI}{80}$$

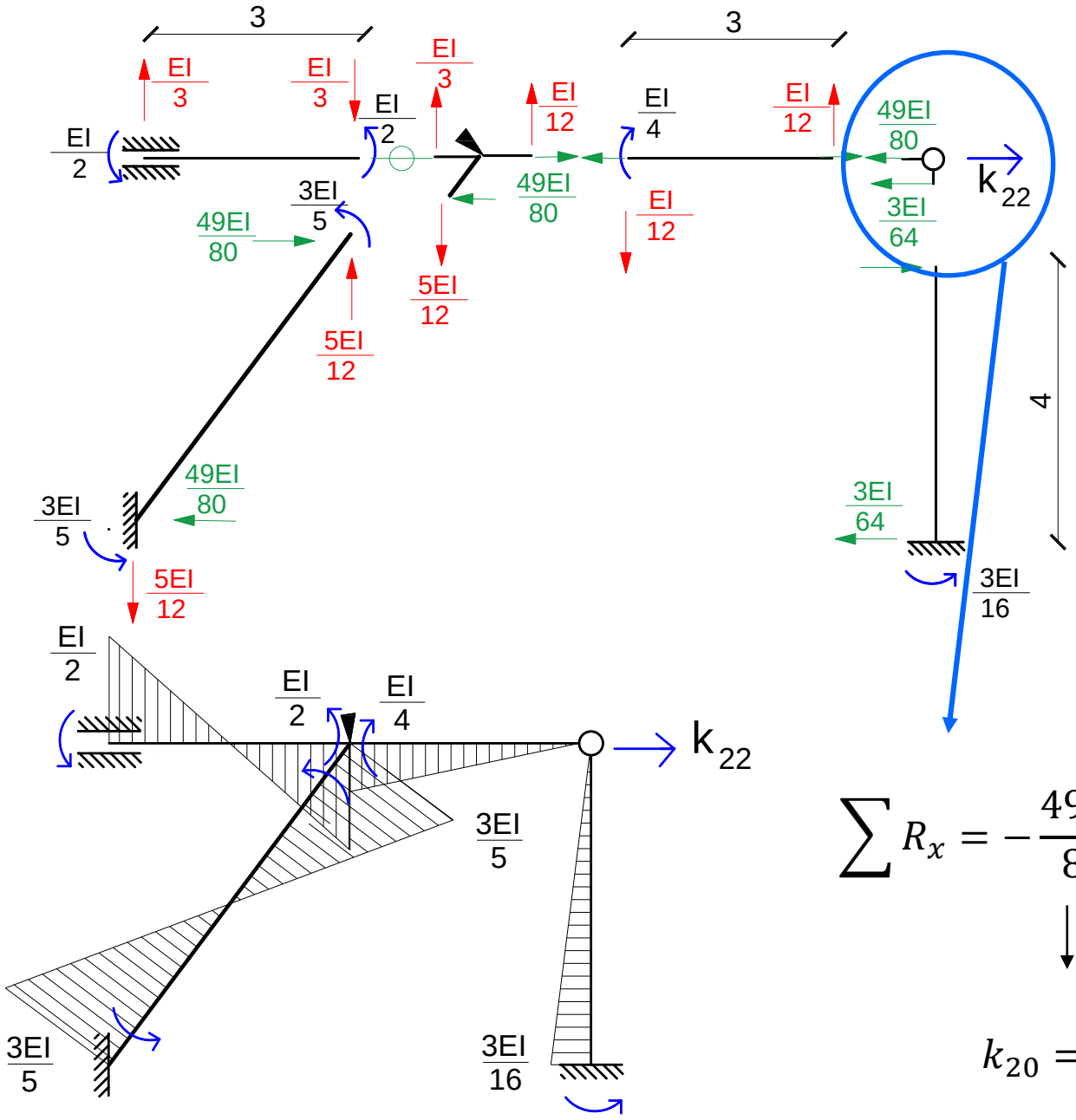
dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{22}



dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{22}



$$\sum R_x = -\frac{49EI}{80} - \frac{3EI}{64} + k_{22} = 0$$

$$\downarrow$$

$$k_{20} = \frac{211EI}{320}$$

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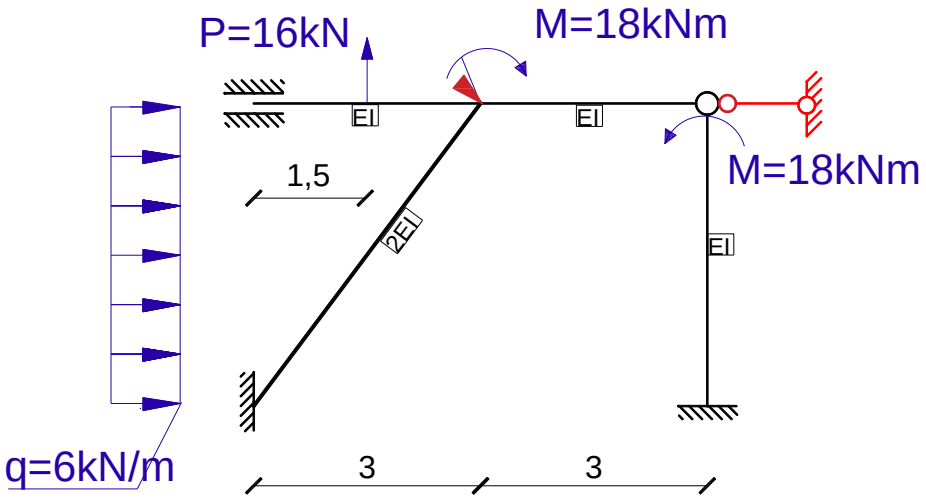
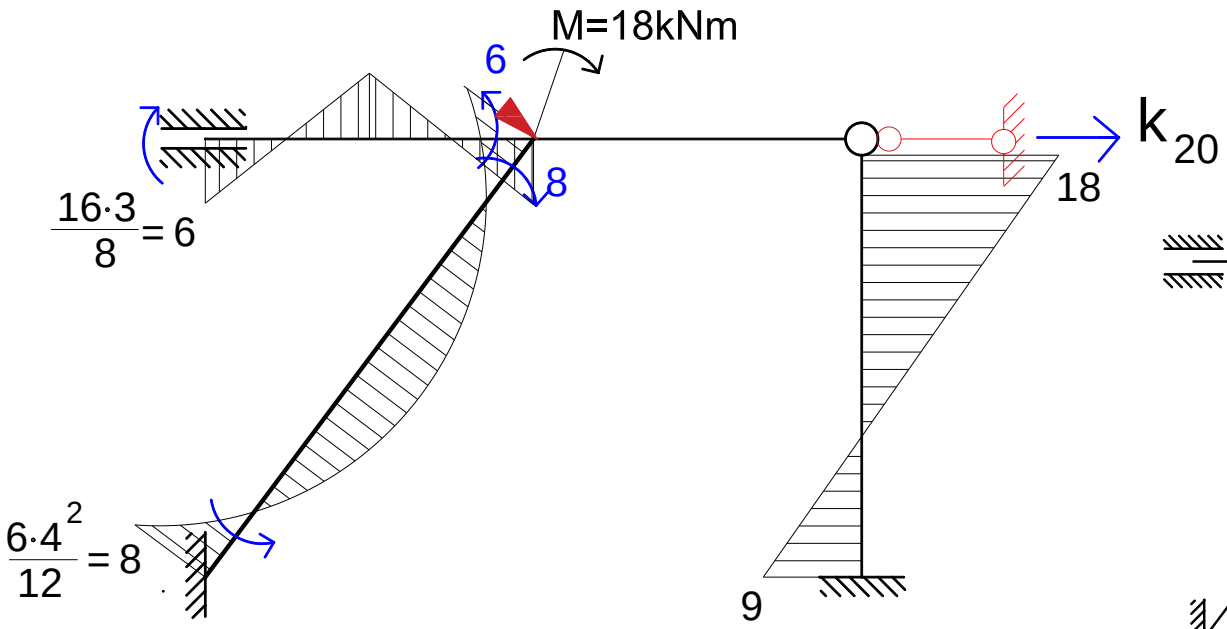
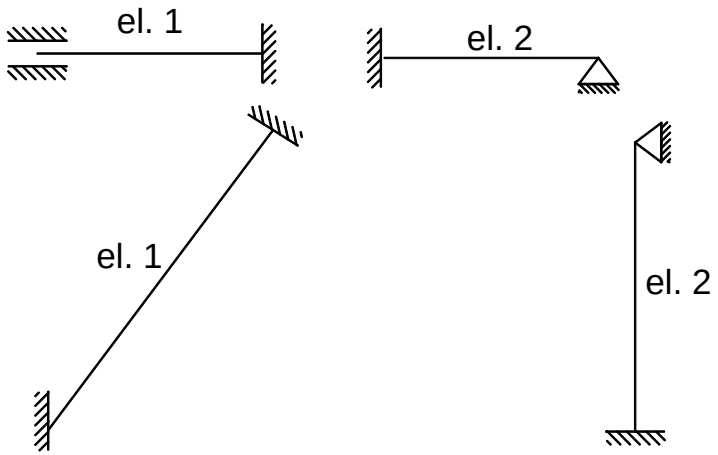
Obciążenie zewnętrzne

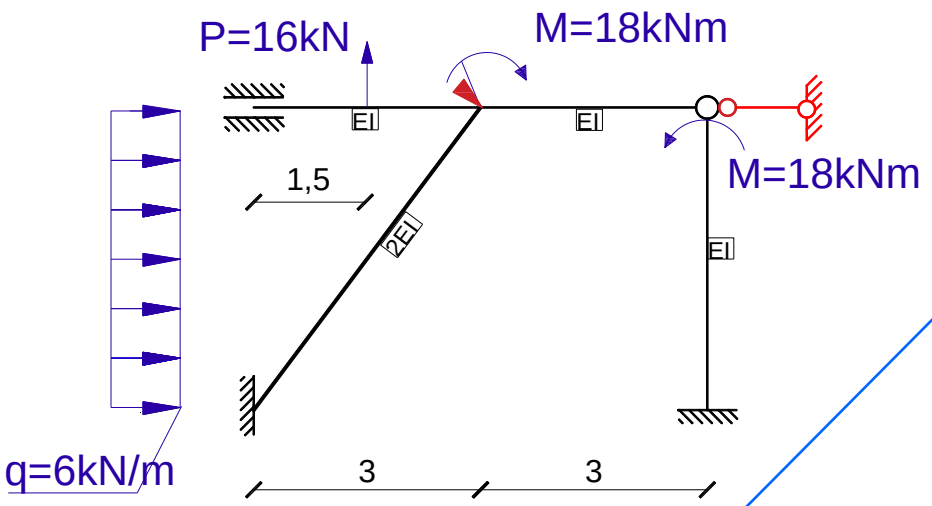




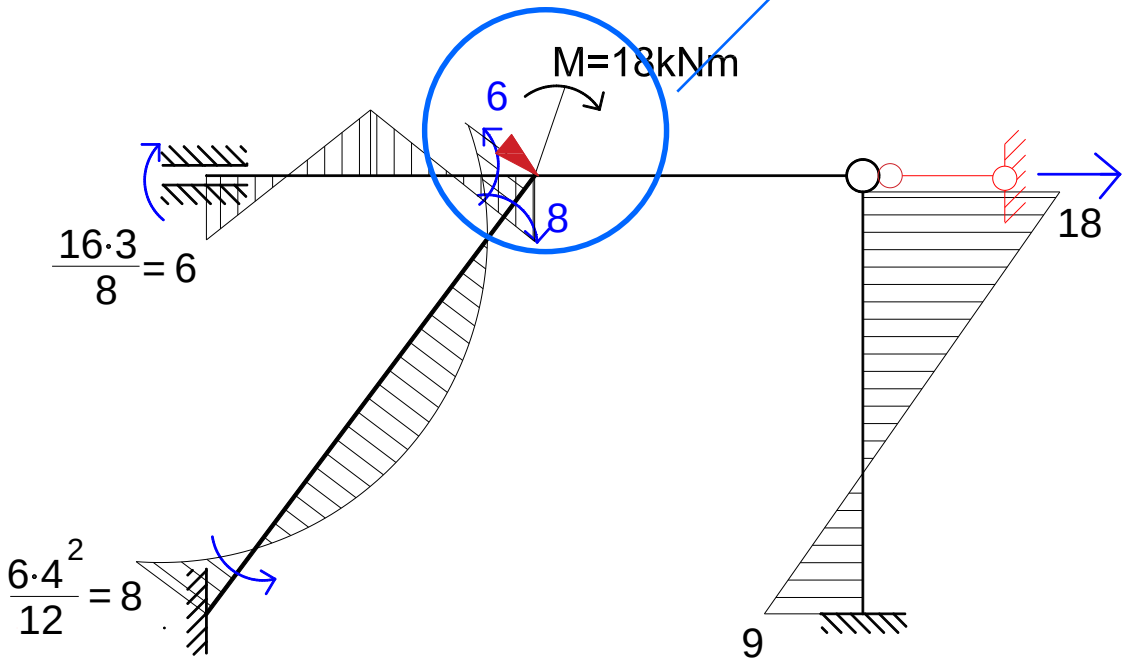


dr inż. Hanna Weber

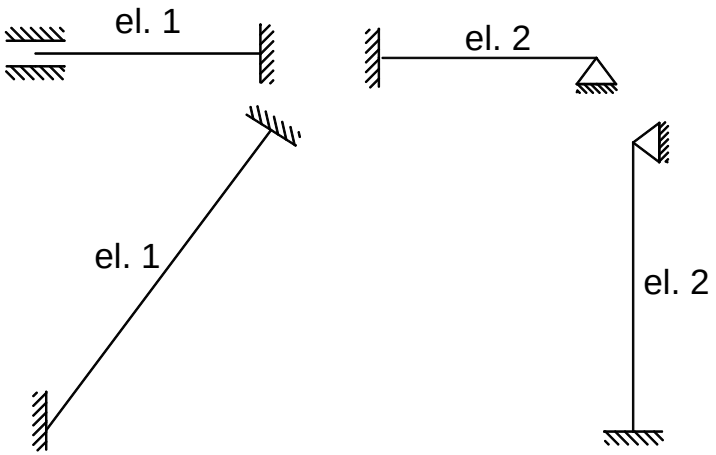
Obciążenie zewnętrzne



$$k_{10} = -6 + 8 - 18 = -16kNm$$

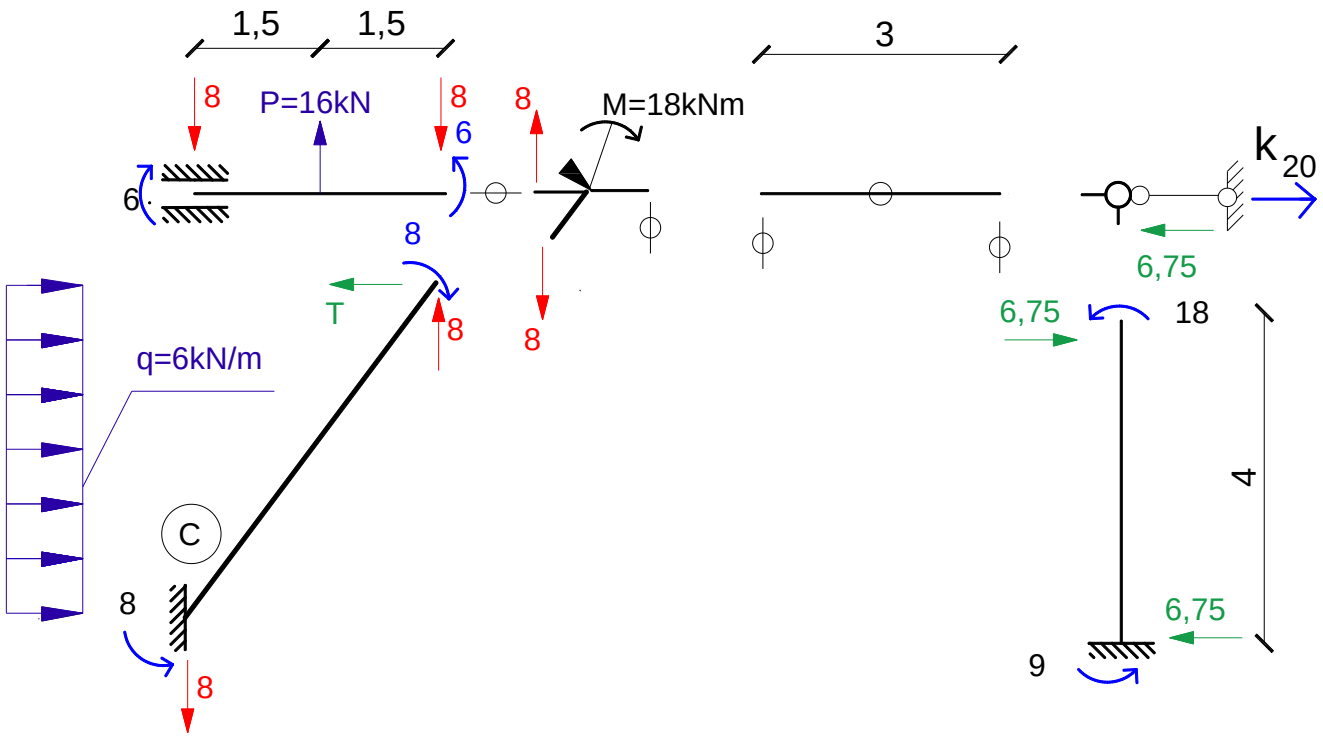


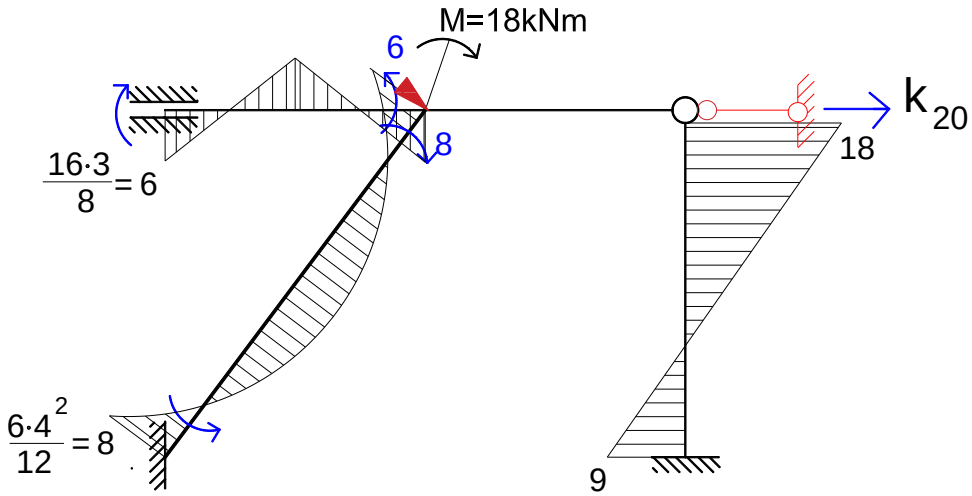
k_{20}



dr inż. Hanna Weber

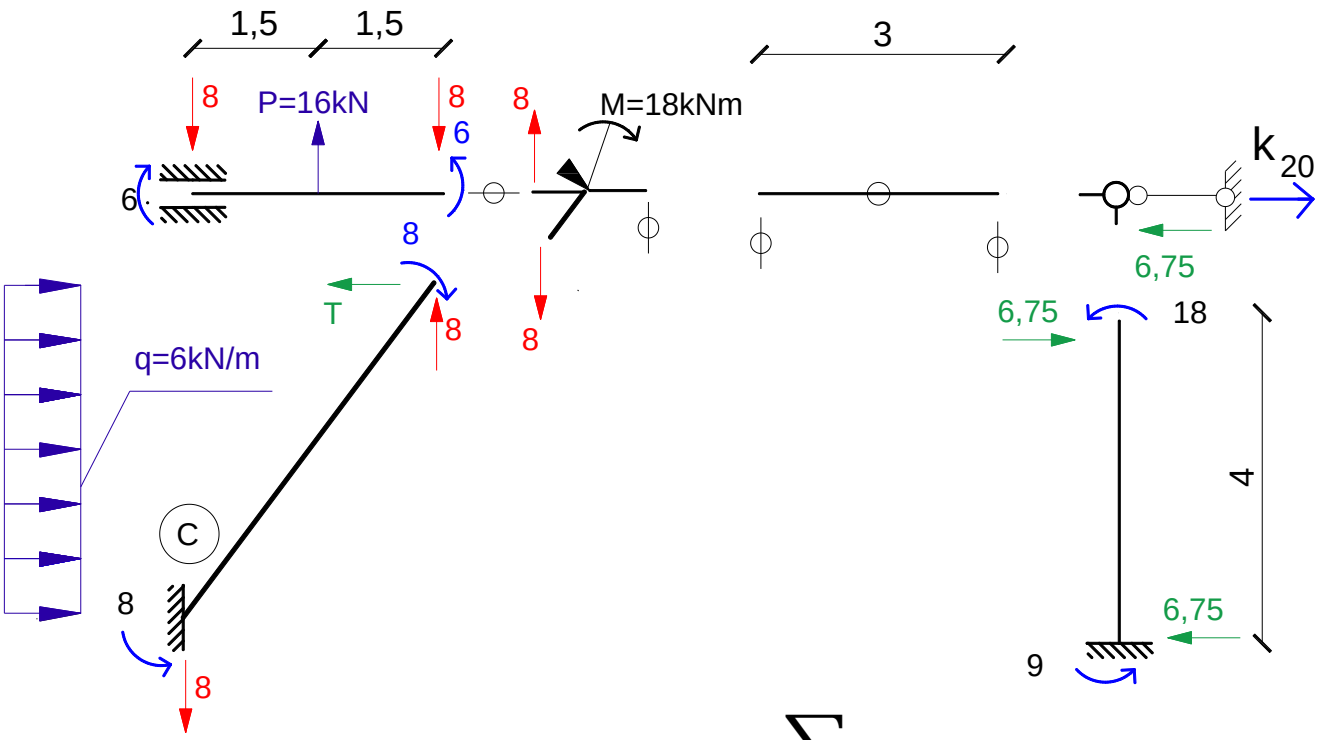
Wyznaczenie reakcji od przemieszczeń k_{20}

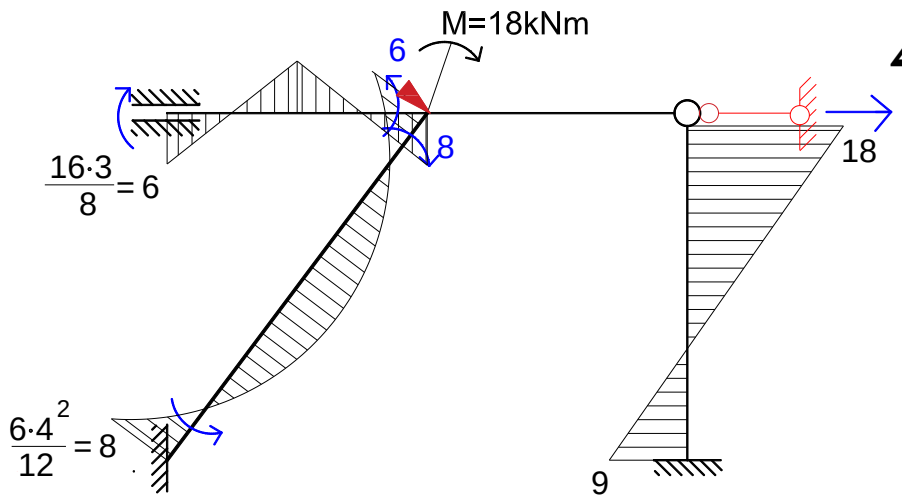




dr inż. Hanna Weber

Wyznaczenie reakcji od przemieszczeń k_{20}





$$\sum_{k_{20}} M_C = 6 \cdot 4 \cdot 2 - 8 \cdot 3 + 8 - 8 - T \cdot 4 = 0$$

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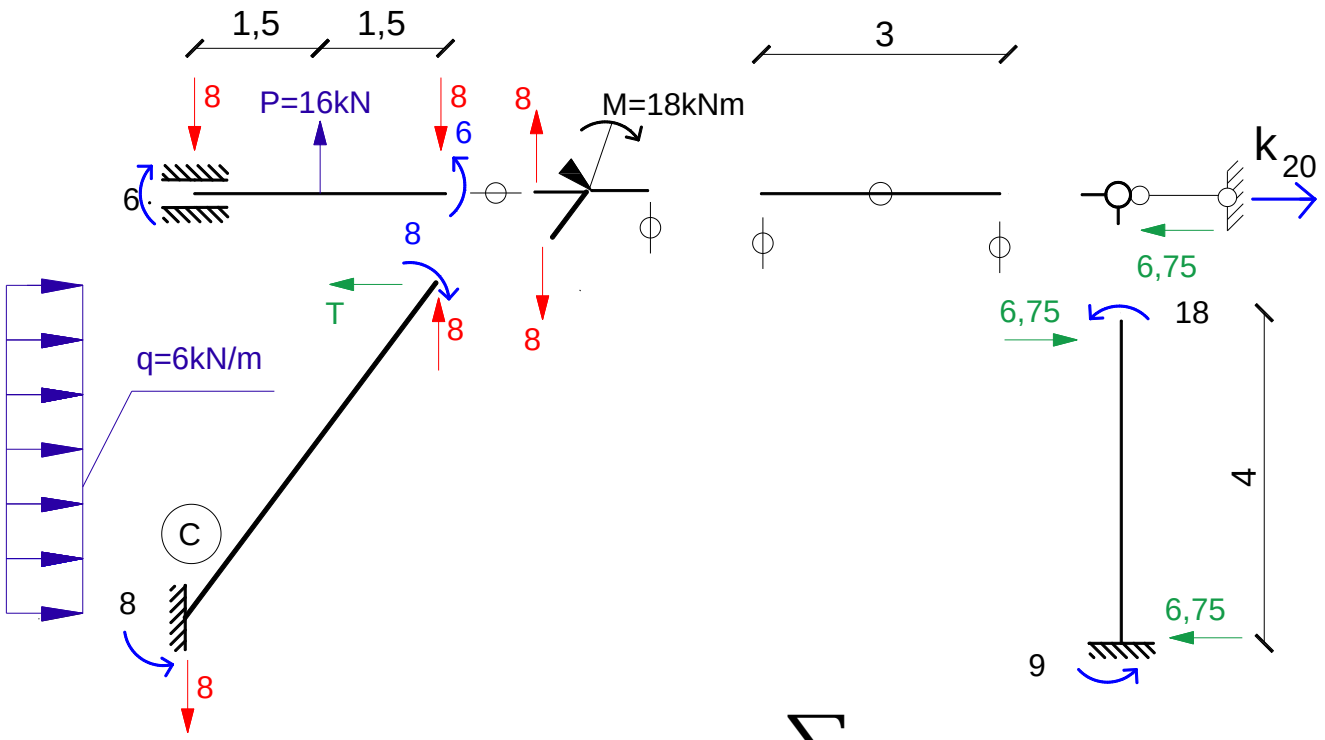
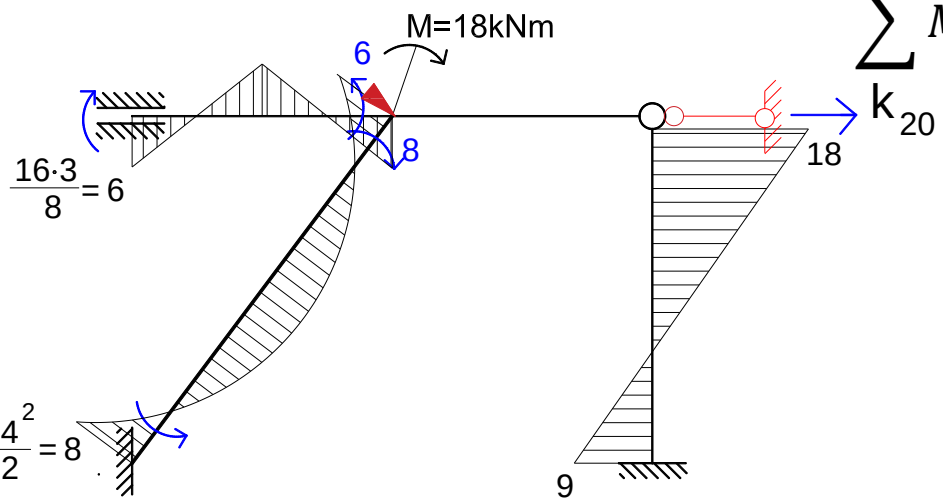
Wyznaczenie reakcji od przemieszczeń k_{20}





$$\sum_{k_{20}} M_C = 6 \cdot 4 \cdot 2 - 8 \cdot 3 + 8 - 8 - T \cdot 4 = 0$$

$$\downarrow$$

$$T = 6kN$$

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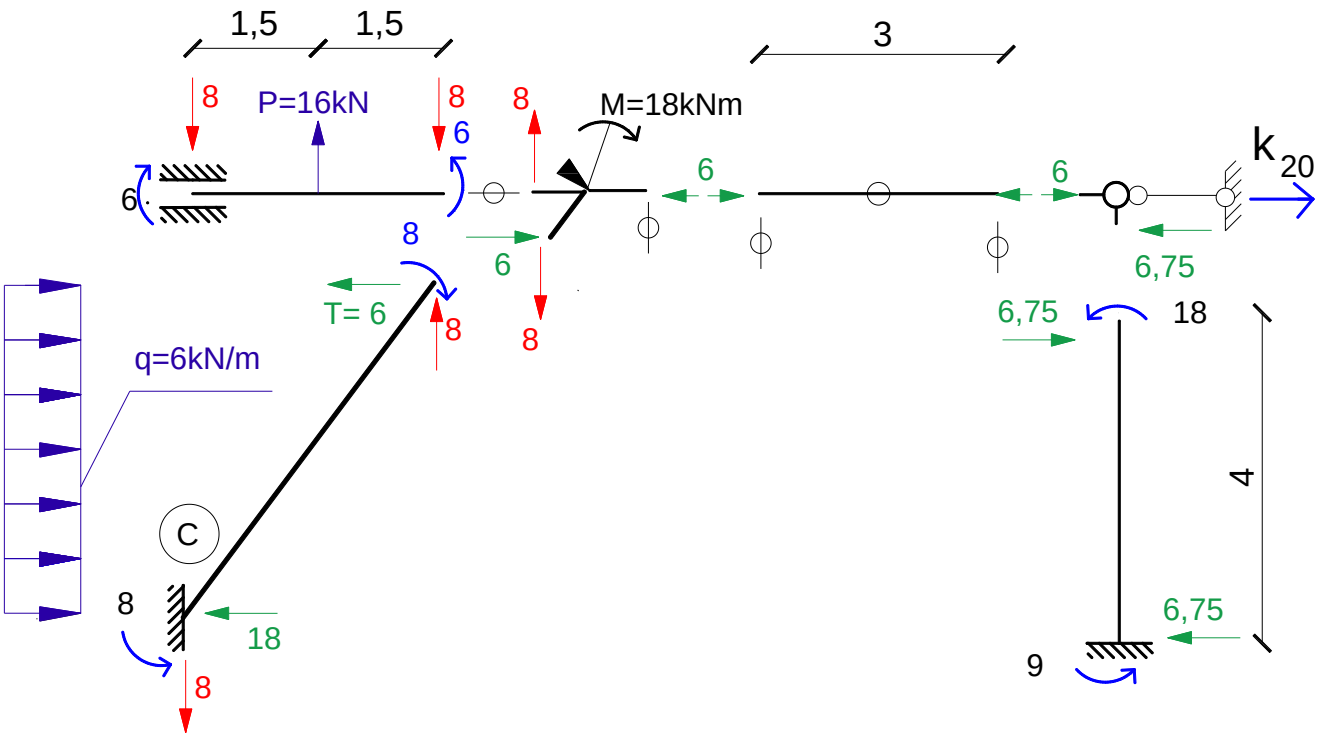
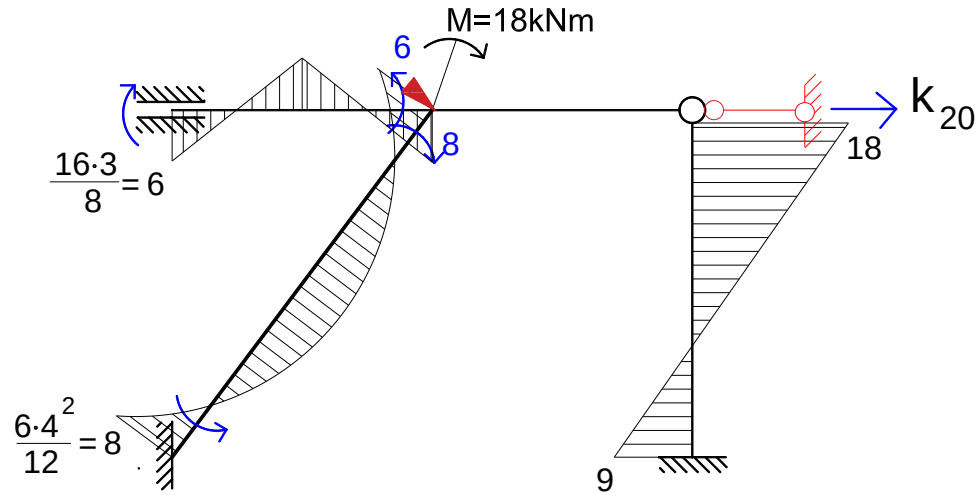
Wyznaczenie reakcji od przemieszczeń k_{20}





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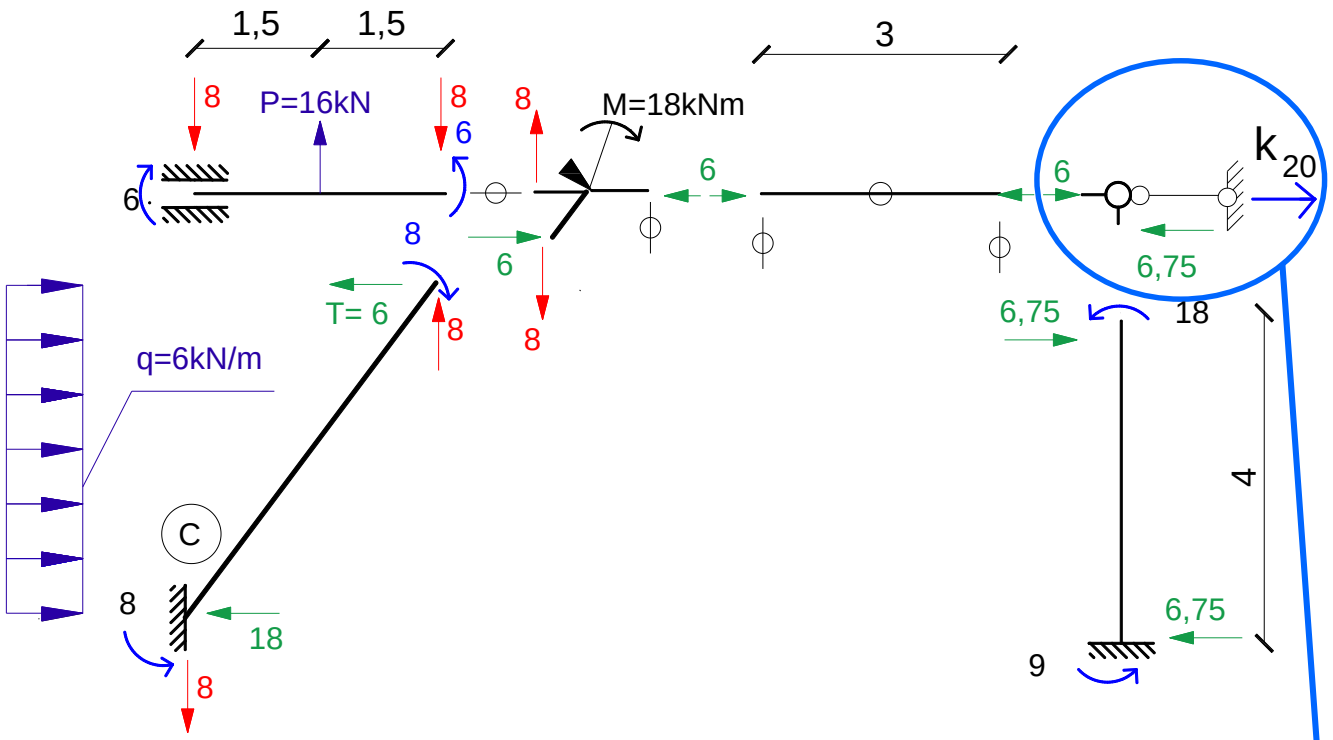
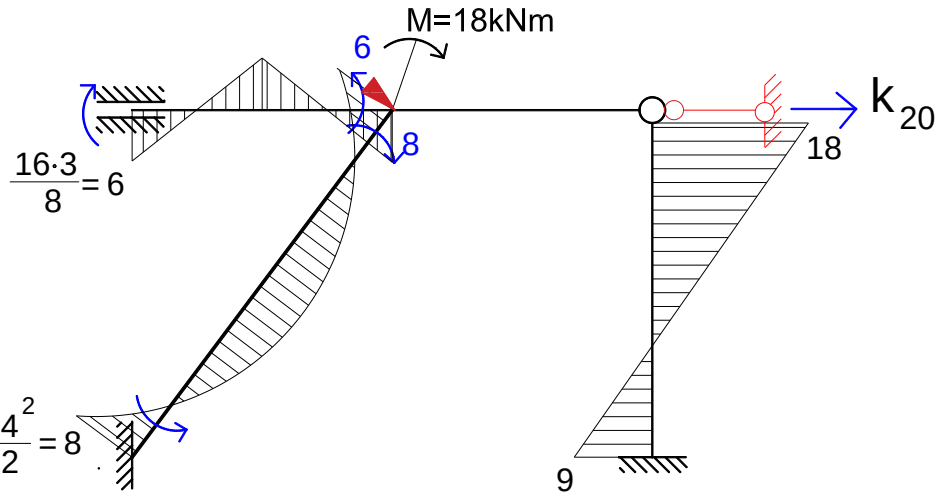
Wyznaczenie reakcji od przemieszczeń k_{20}





$$\sum R_x = 6 - 6{,}75 + k_{20} = 0$$

$$\downarrow$$

$$k_{20} = 0{,}75kN$$

dr inż. Hanna Weber

Wykład:

Liczenie przemieszczeń w belkach bezpośrednio z układu równań metody przemieszczeń

dr inż. Hanna Weber

Dobór schematu podstawowego metody przemieszczeń w belkach przy wyznaczaniu wykresu M w minimalnej ilości niewiadomych:

- Znajdujemy część statycznie wyznaczalną i pozostawiamy ją bez dodatkowych blokad,
- Wszystkie środkowe podpory przegubowe blokujemy na obrót,
- Środkowe przeguby blokujemy na przesunięcie na kierunku prostopadłym do osi belki,
- Sztywnie połączone punkty skokowej zmiany sztywności blokujemy na obrót i na przesunięcie (jeżeli nie ma podpory blokującej przesuw)

dr inż. Hanna Weber

Zadanie 1: Dobierz schemat podstawowy metody przemieszczeń dla poniższego układu i dokonaj podziału na elementy. Zadanie rozwiąż w minimalnej bazie niewiadomych.

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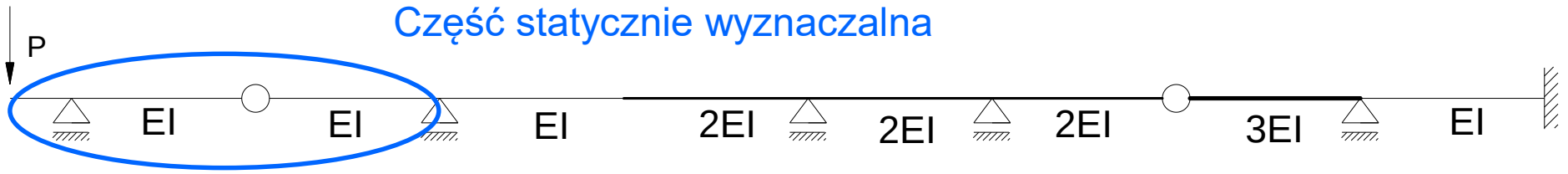

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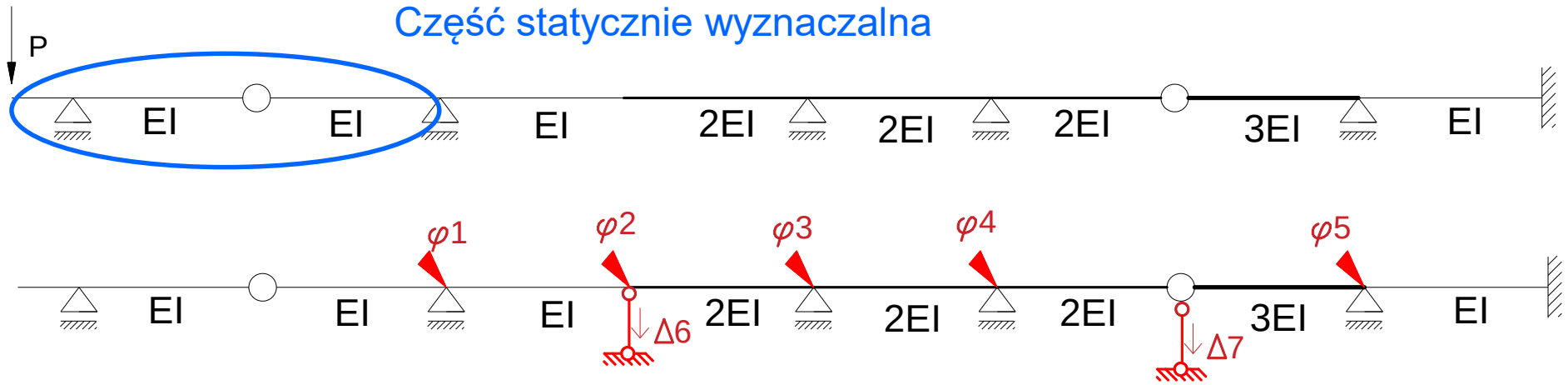

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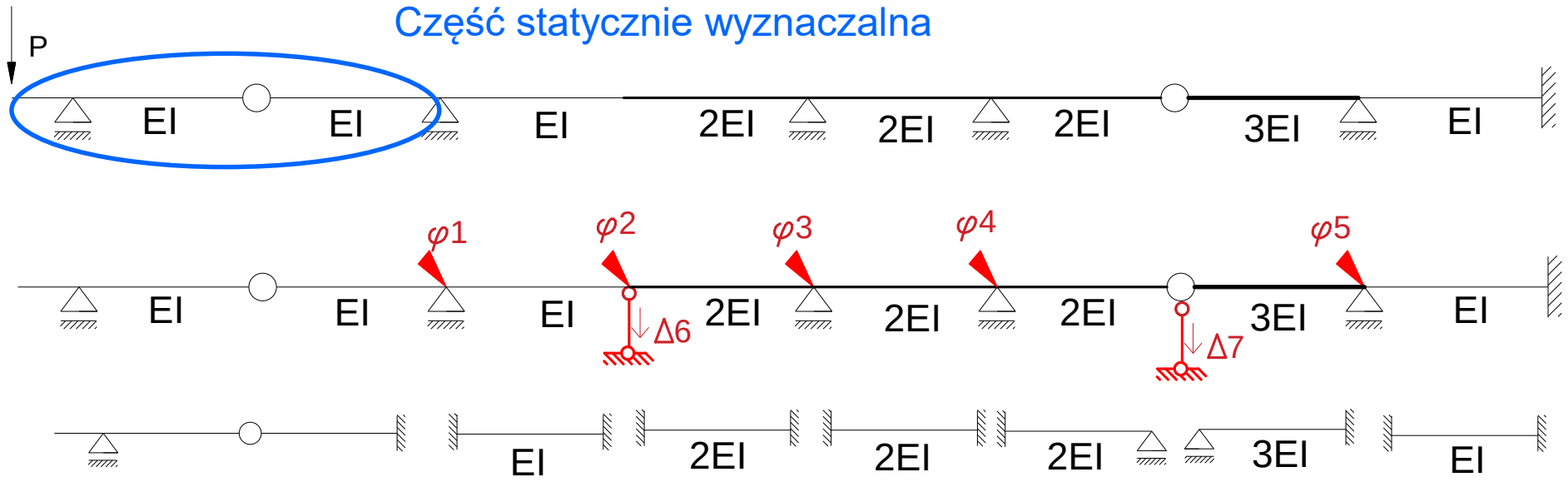

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Dobór schematu podstawowego metody przemieszczeń w belkach przy liczeniu przemieszczeń bezpośrednio z układu :

- Dobieramy niezbędne blokady w minimalnej ilości niewiadomych (tak jak dla wyznaczenia wykresu M)

- Przy liczeniu ugięcia na końcu wspornika wstawiamy samą blokadę na przesunięcie,

- Przy liczeniu kąta obrotu na końcu wspornika wstawiamy obie blokady (na obrót i na przesunięcie),

- Przy liczeniu dowolnego przemieszczenia na belce między podporami (w środku układu) – stawiamy obie blokady jednocześnie,

- Uzupełniamy układ brakującymi blokadami.

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