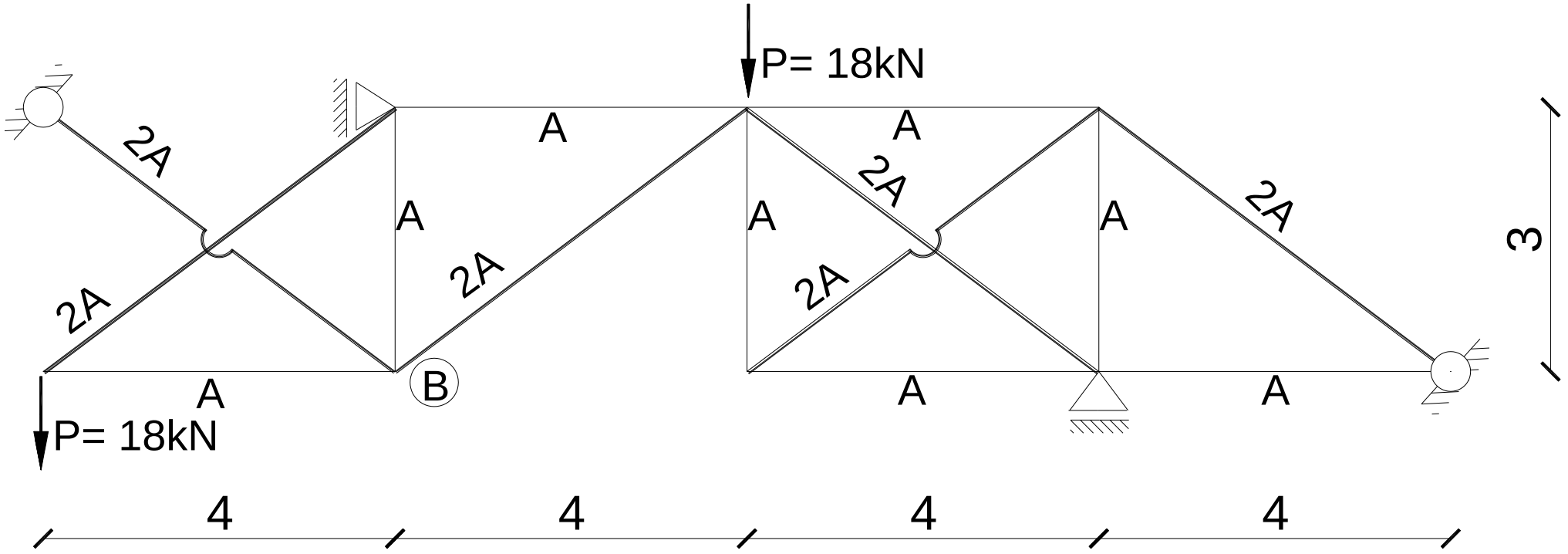
Zadanie: Narysuj wykres sił normalnych dla zadanej kratownicy. Zadanie rozwiąż metodą sił.



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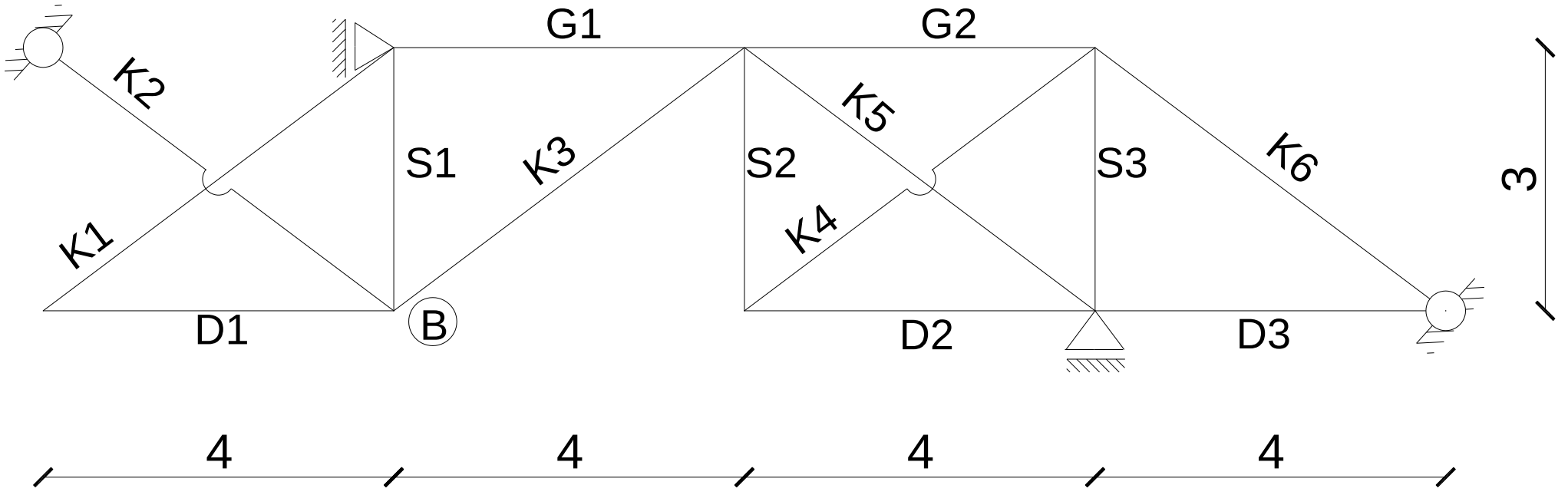
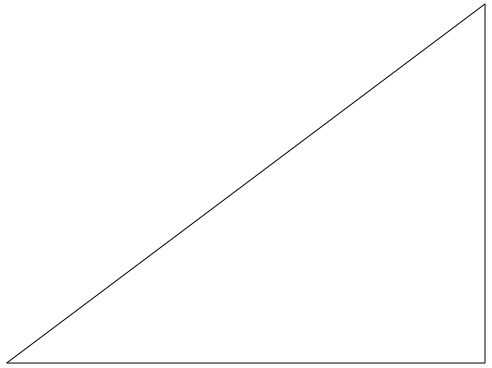
Oznaczenie prętów:

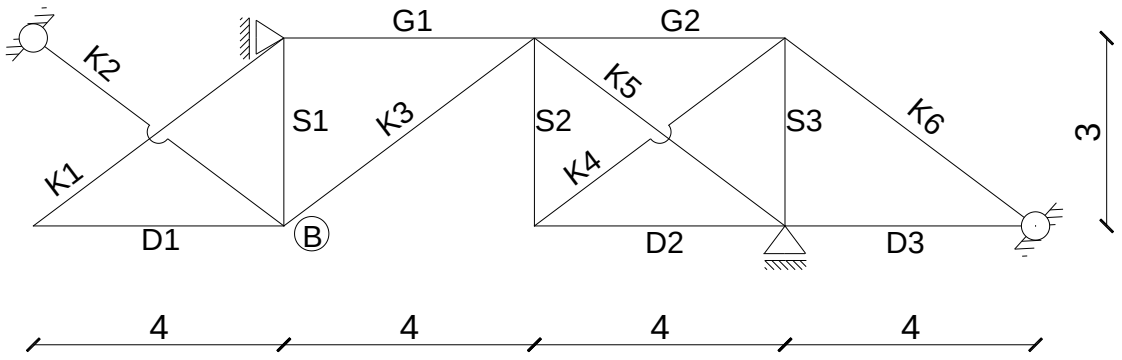


Stopień statycznej niewyznaczalności:

$$n_s = l_r + l_{pr} - 2 \cdot w = 6 + 14 - 2 \cdot 9 = 2$$

Dobór schematu podstawowego:





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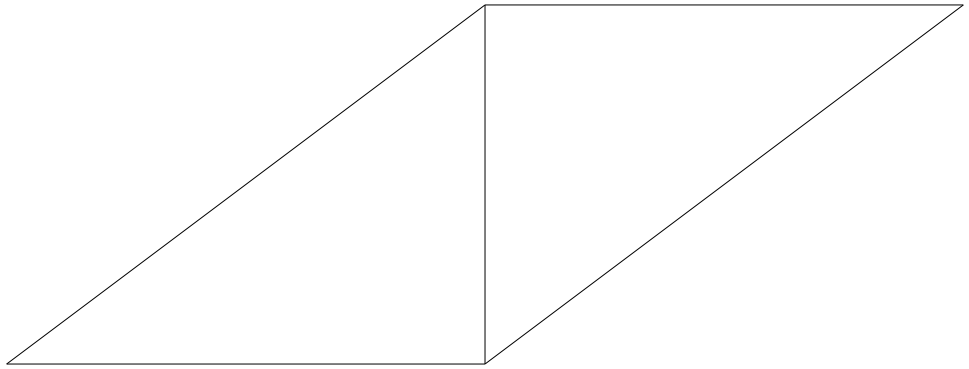
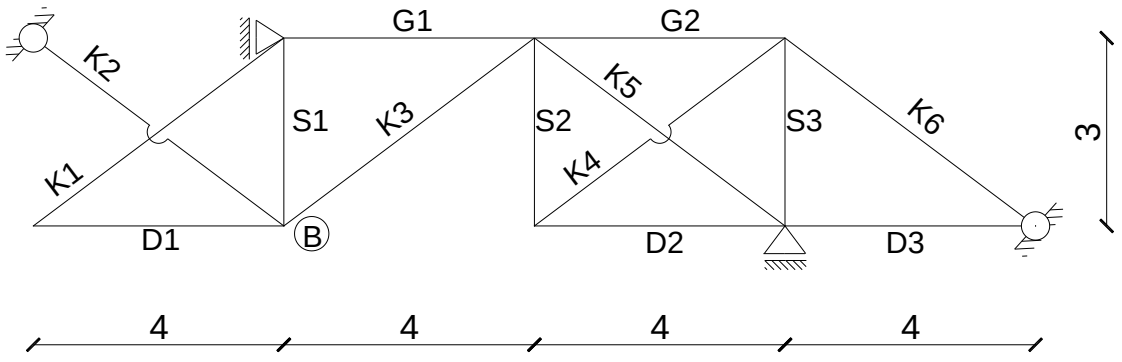
Dobór schematu podstawowego:





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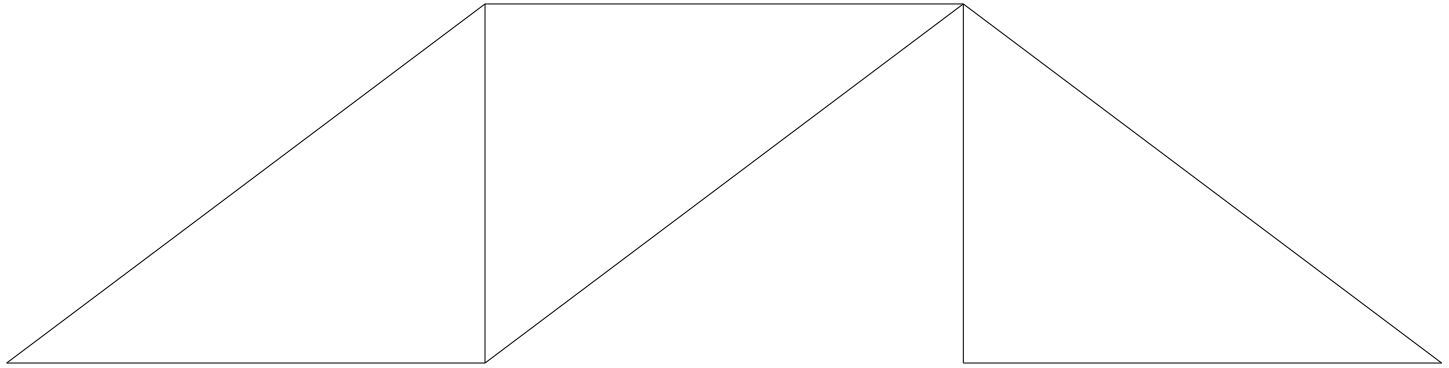
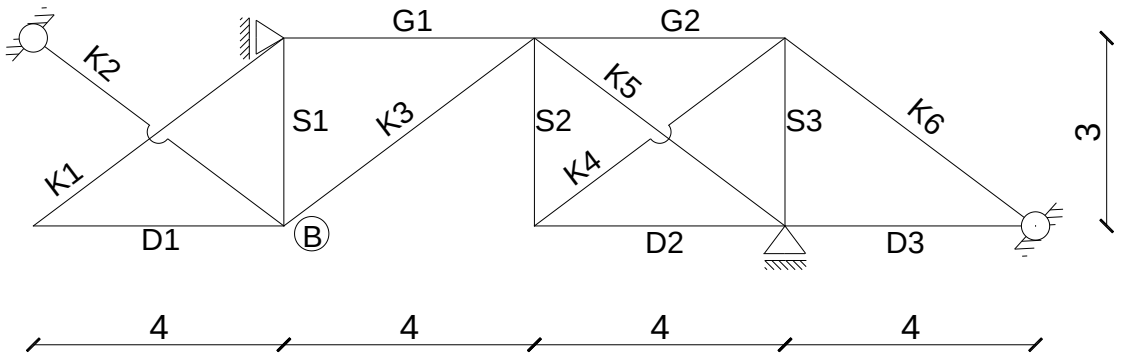
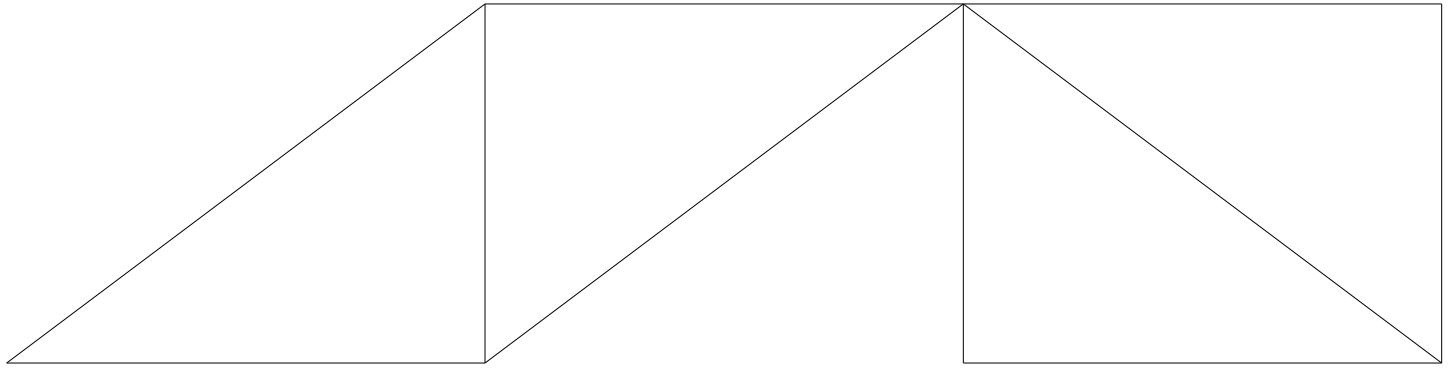
Dobór schematu podstawowego:

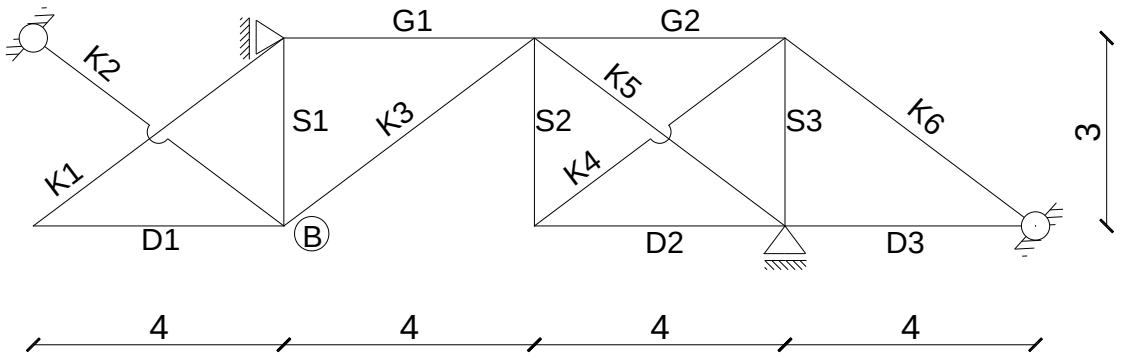




dr inż. Hanna Weber

Dobór schematu podstawowego:





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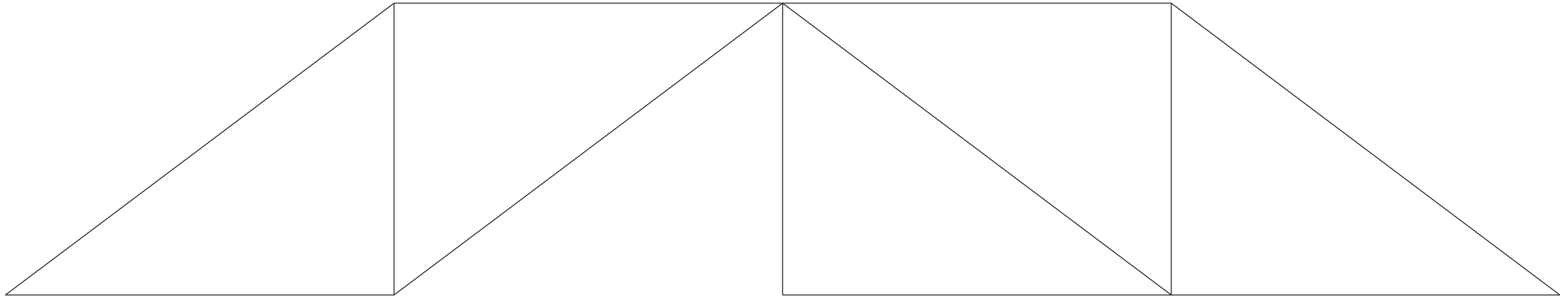

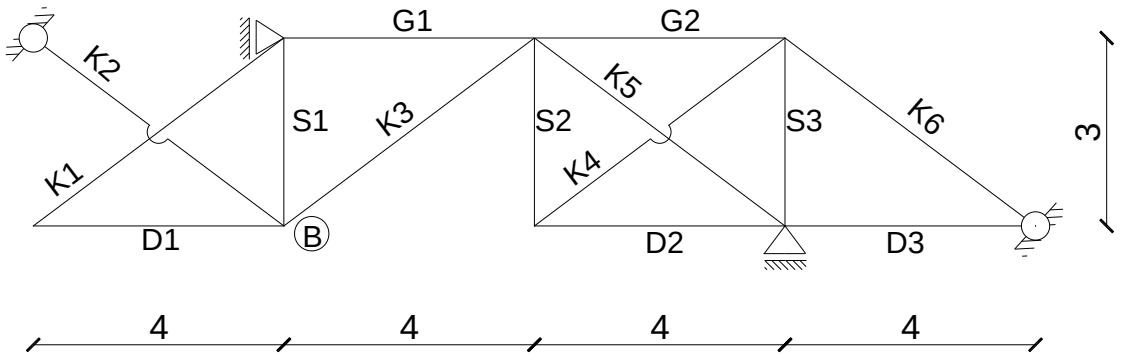

dr inż. Hanna Weber

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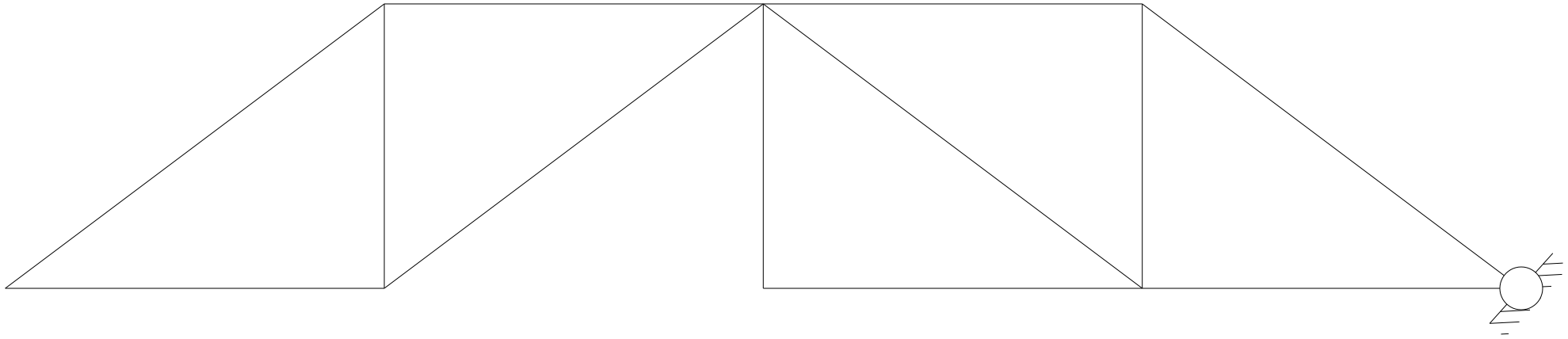

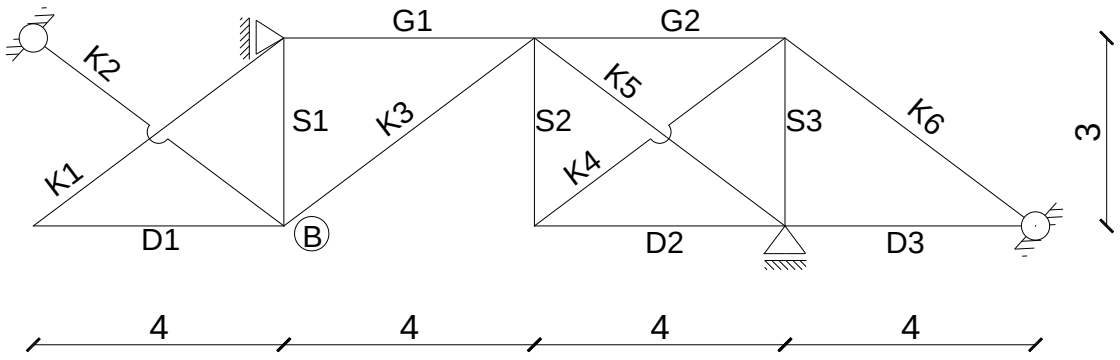

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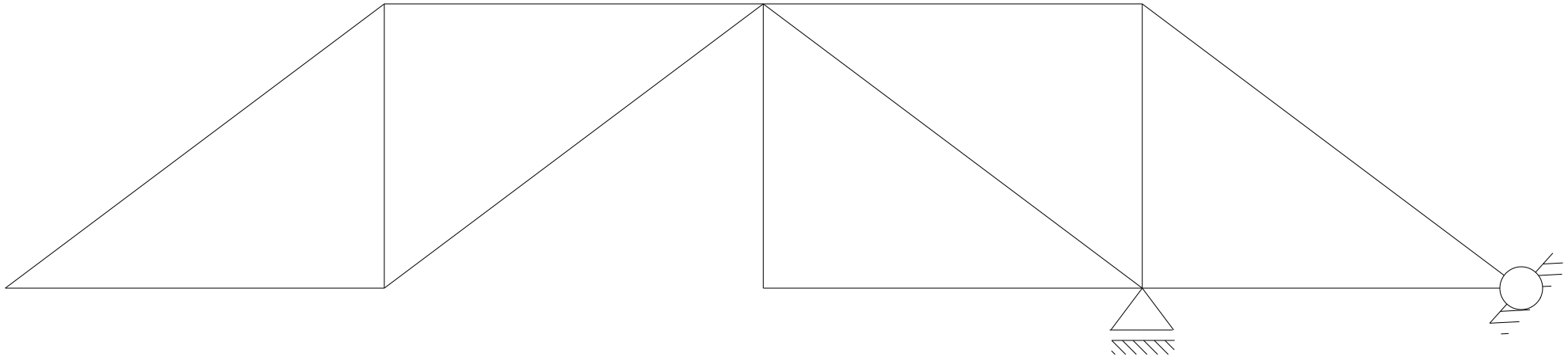
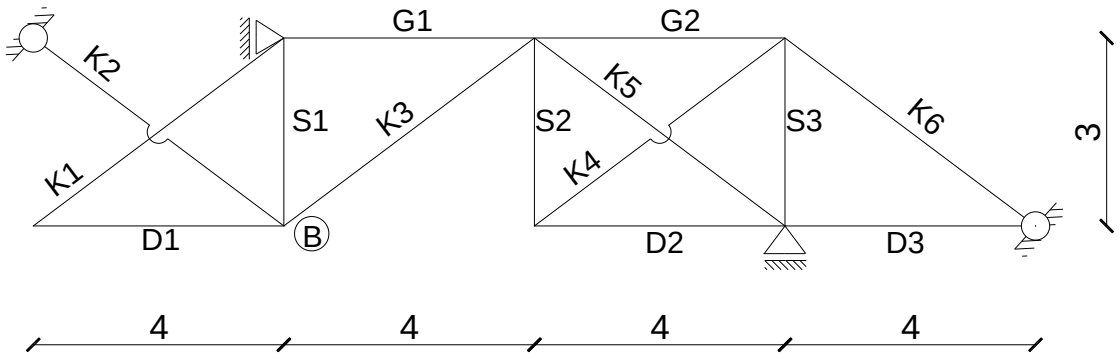
Dobór schematu podstawowego:





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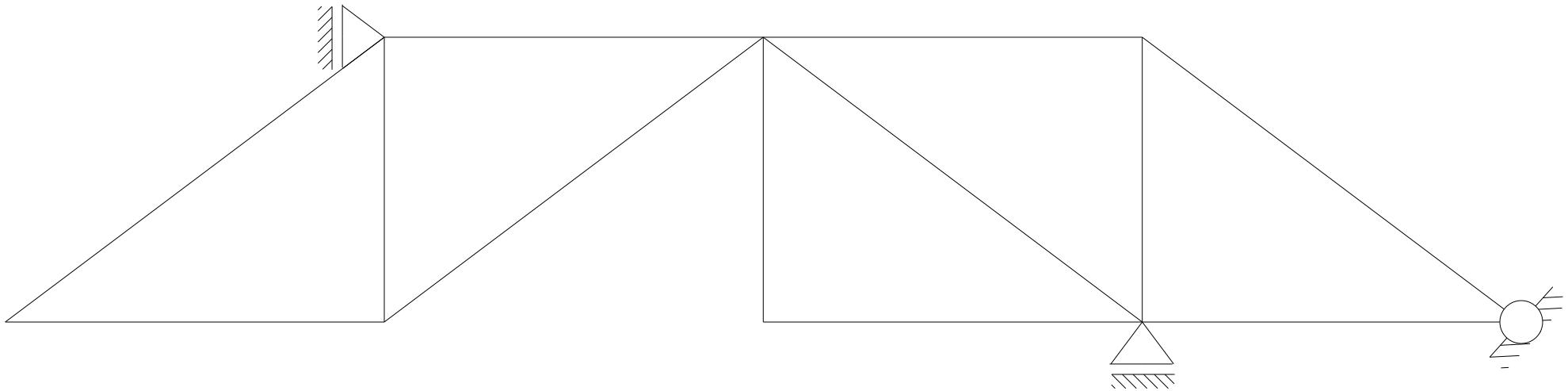
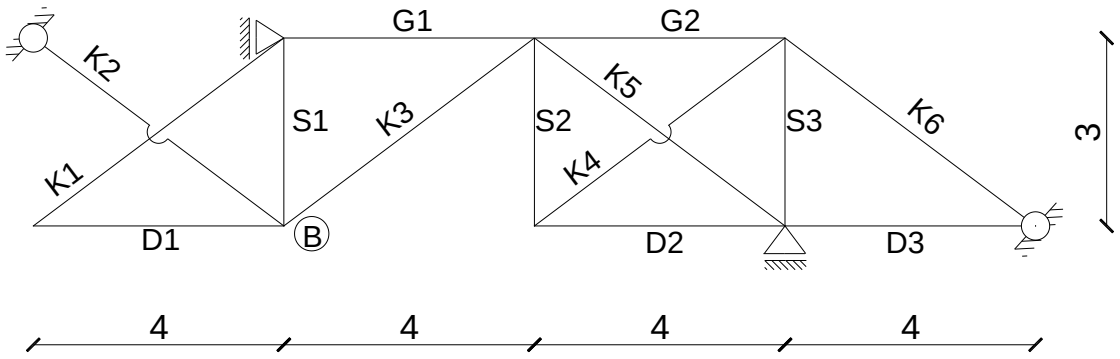
Dobór schematu podstawowego:





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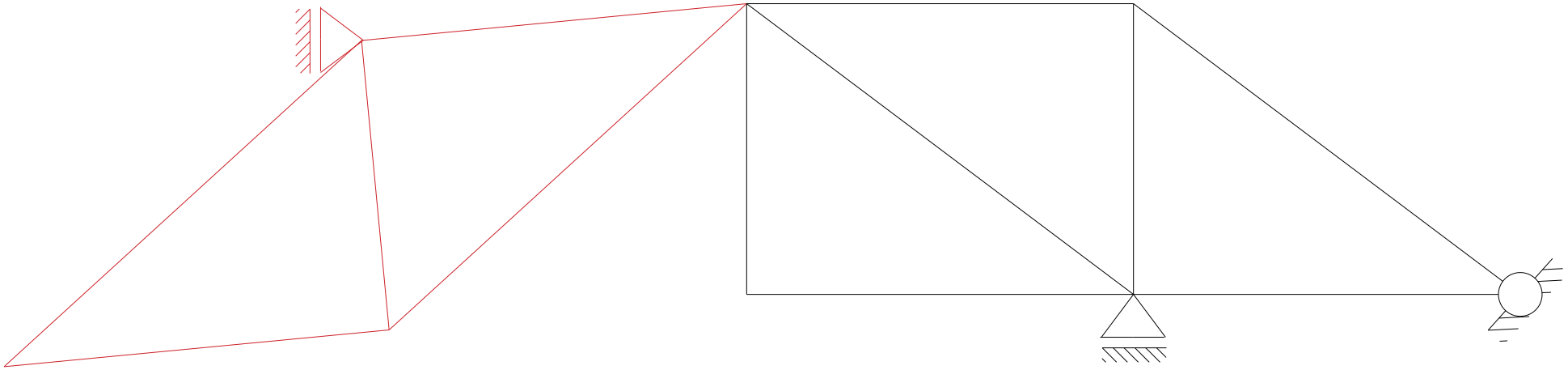

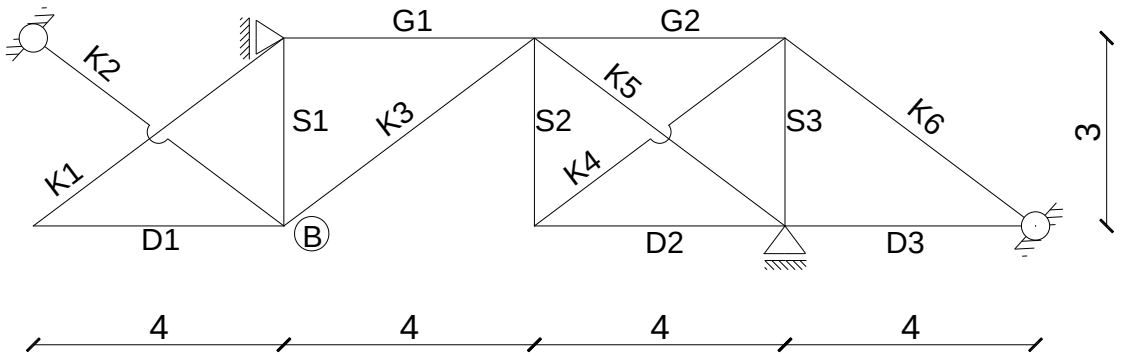

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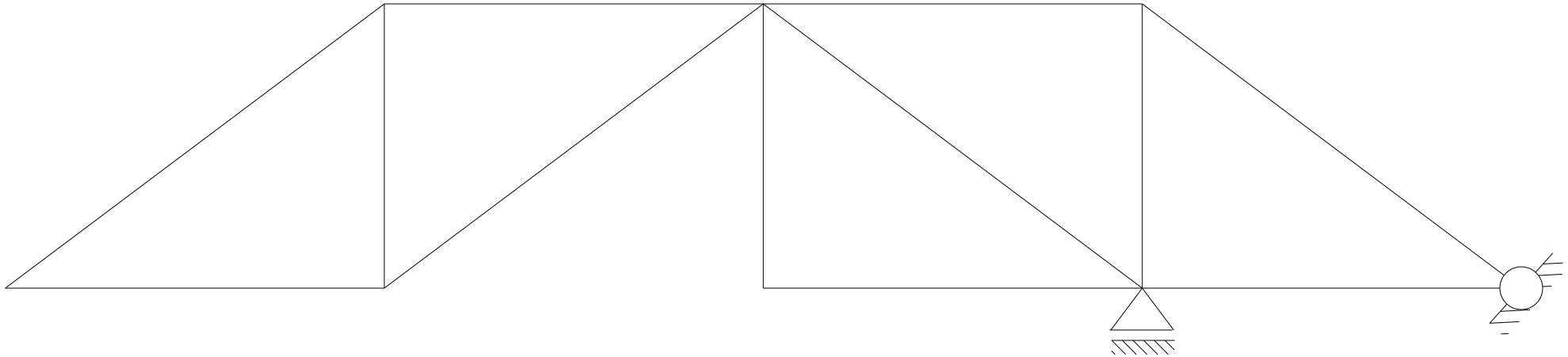

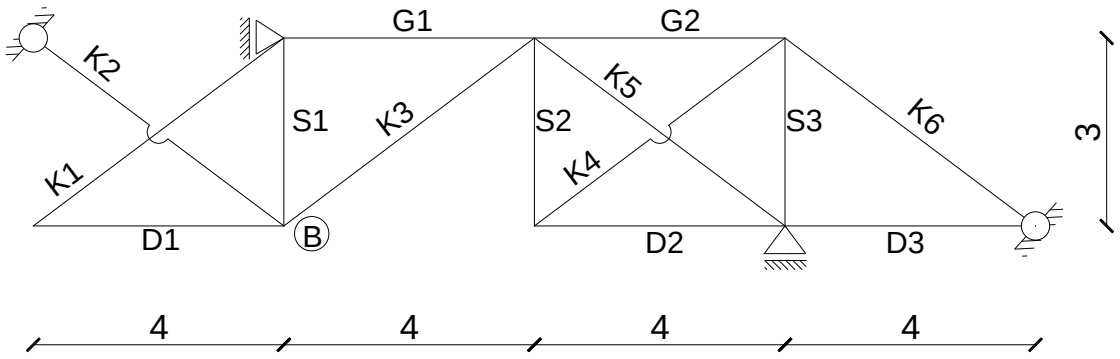

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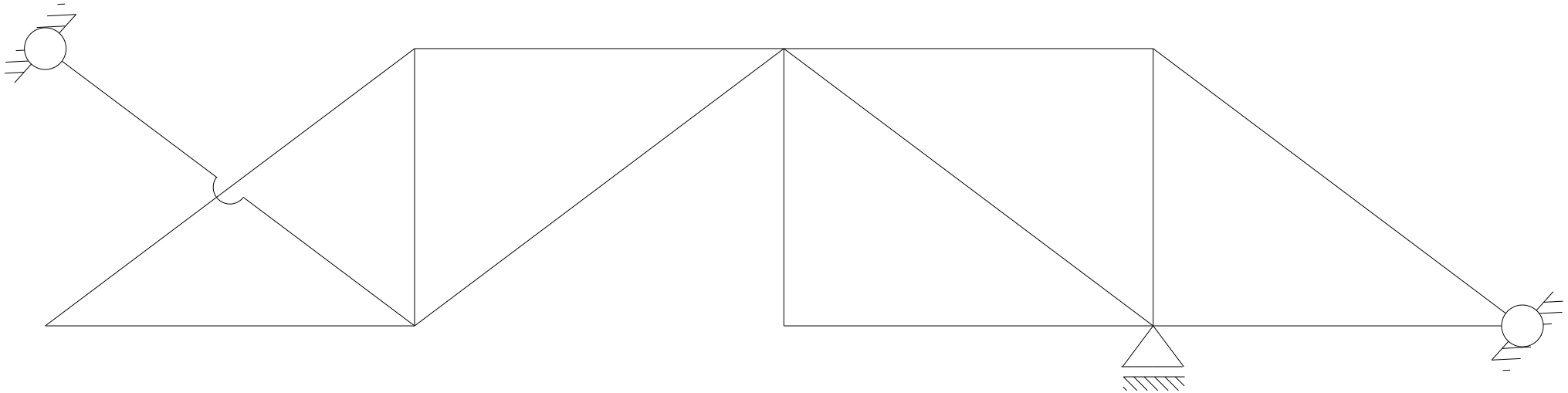

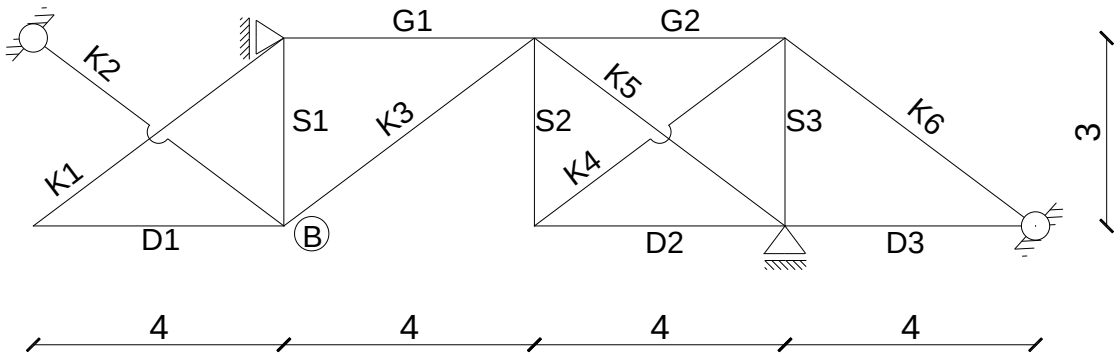

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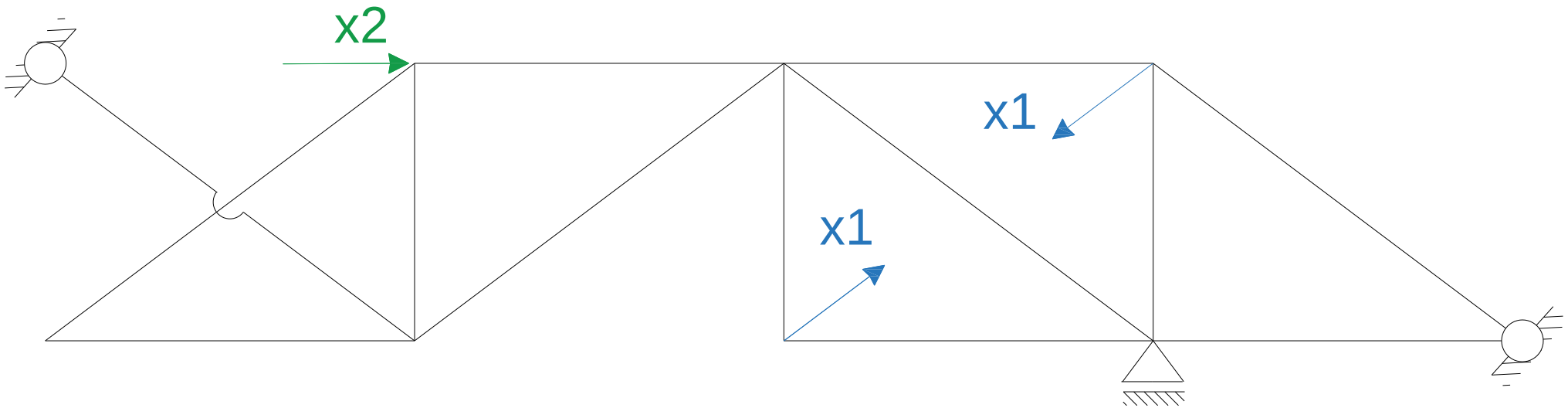

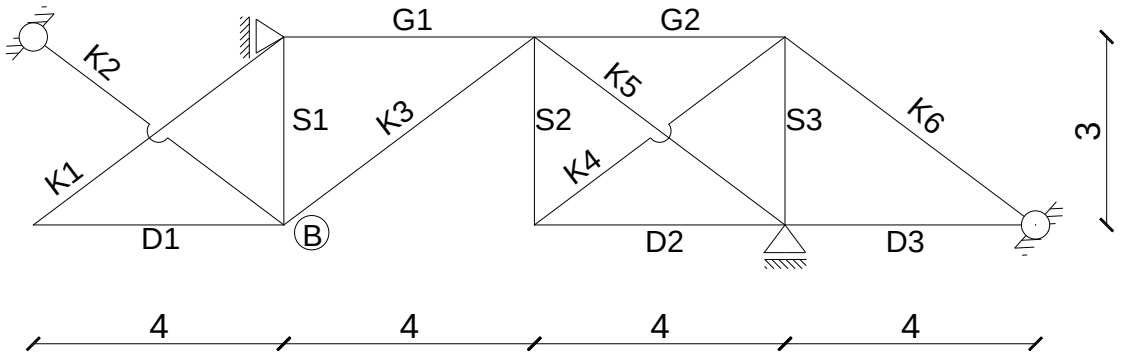

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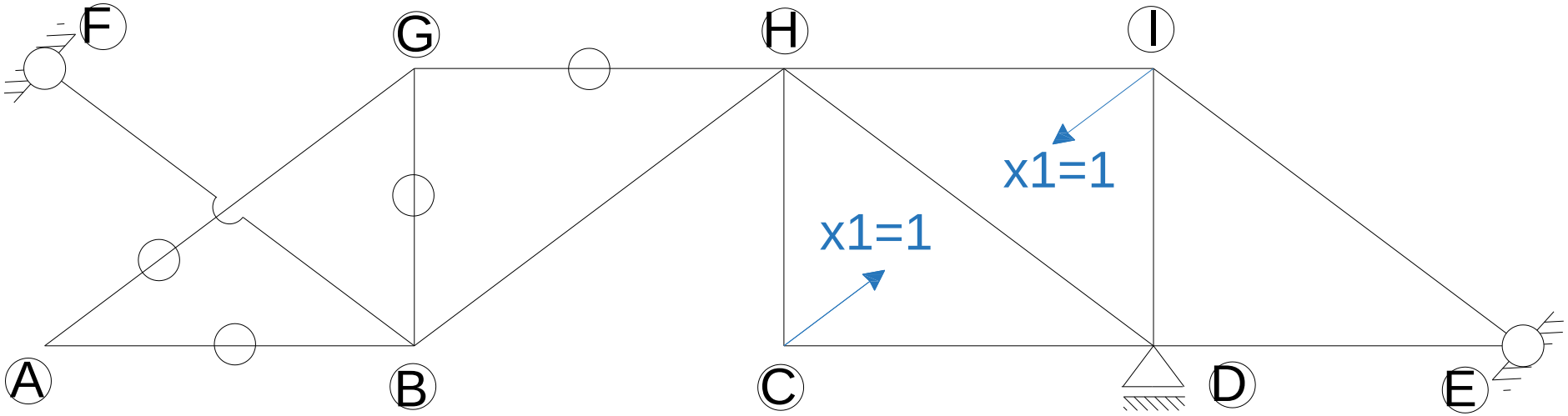
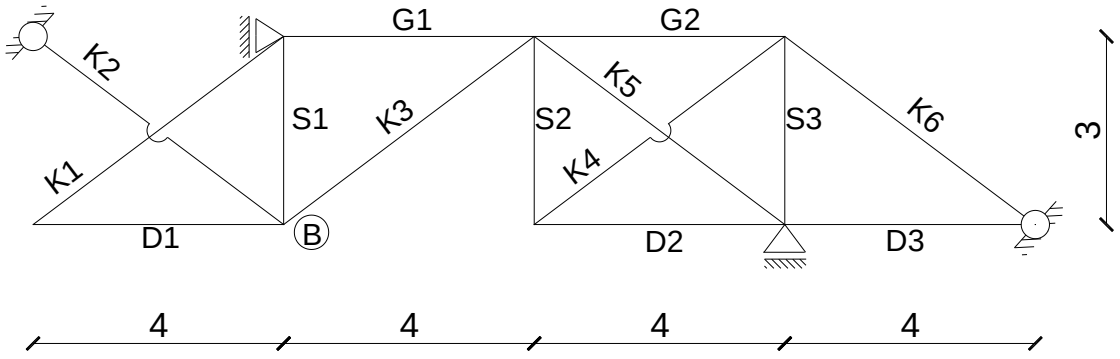
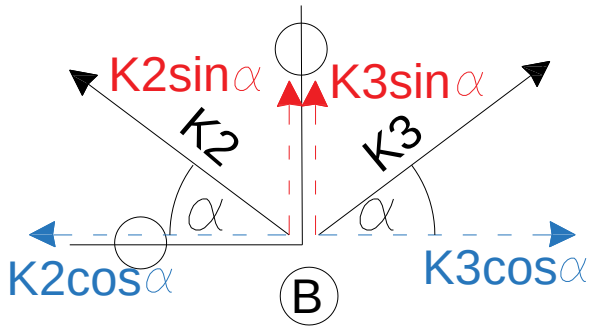
Wykresy: stan x1=1, siły N1







$$\sum R_y = K2\sin\alpha + K3\sin\alpha = 0$$

$$\downarrow$$

$$K2 = -K3$$

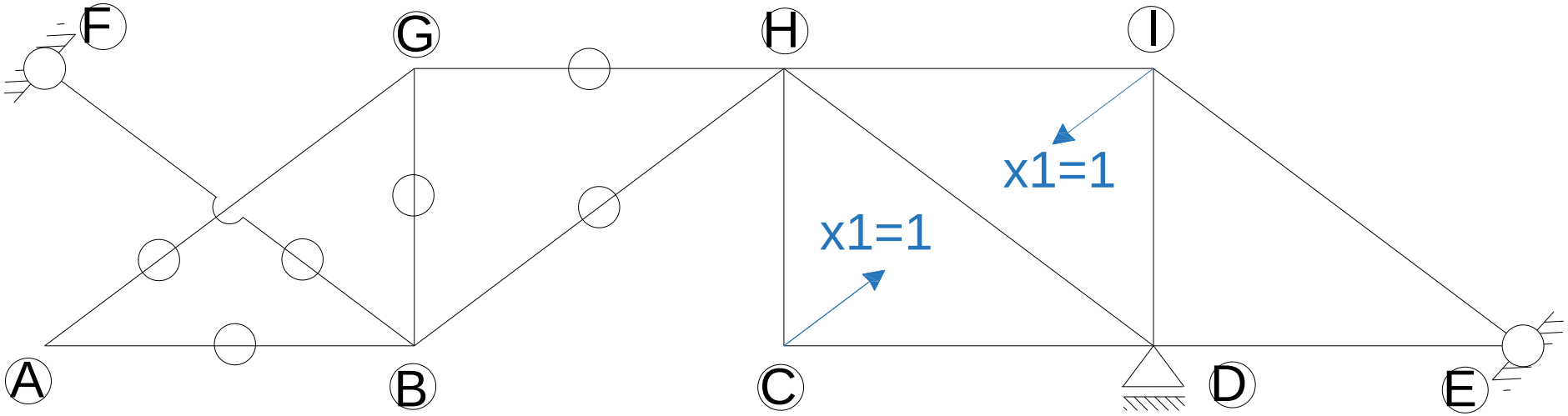
$$\sum R_x = -K2\cos\alpha + K3\cos\alpha = 0$$

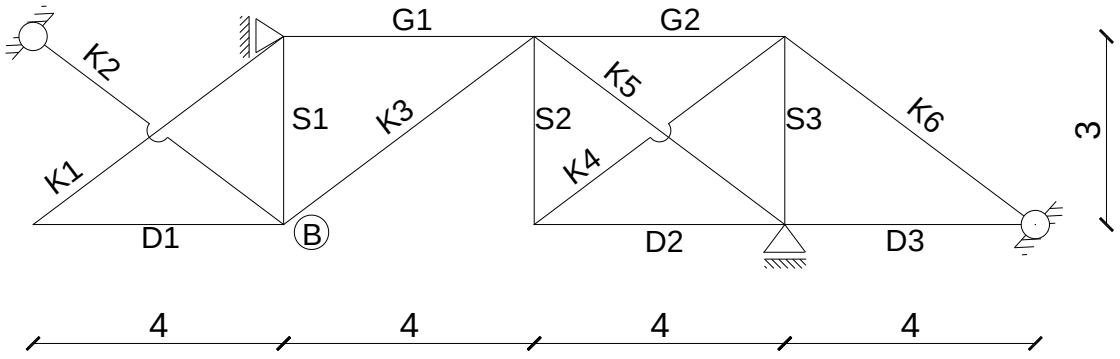
$$-(-K3)\cos\alpha + K3\cos\alpha = 0$$

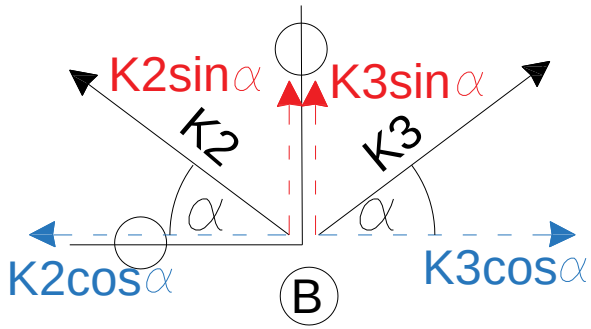
$$2 \cdot K3\cos\alpha = 0$$

$$K3 = 0$$

Wykresy: stan x1=1, siły N1







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$$\downarrow$$

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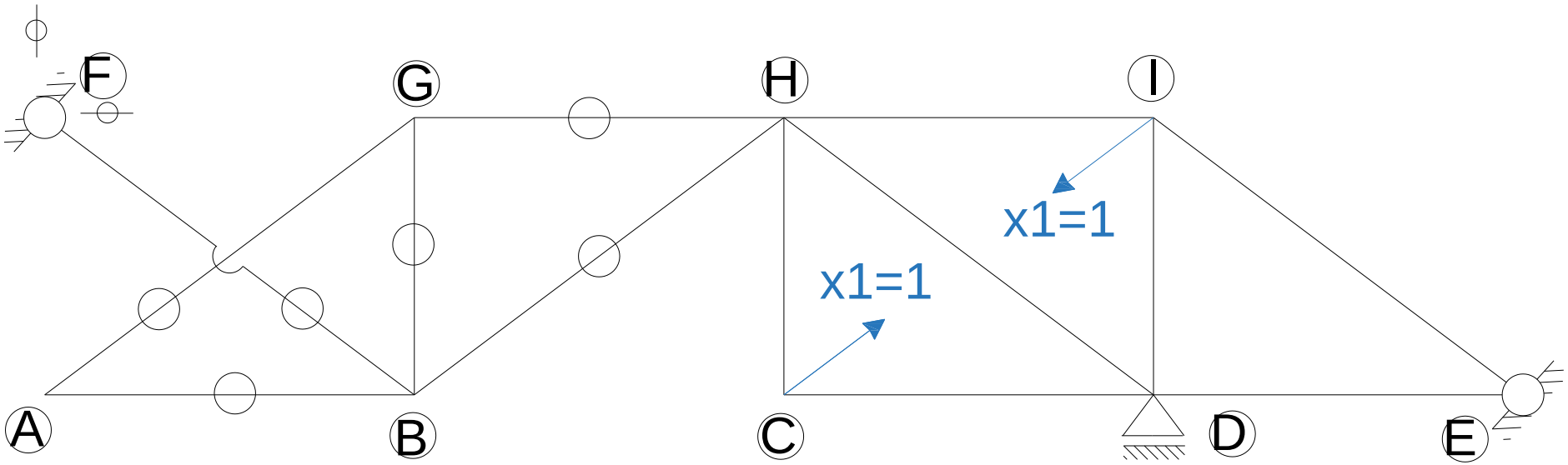
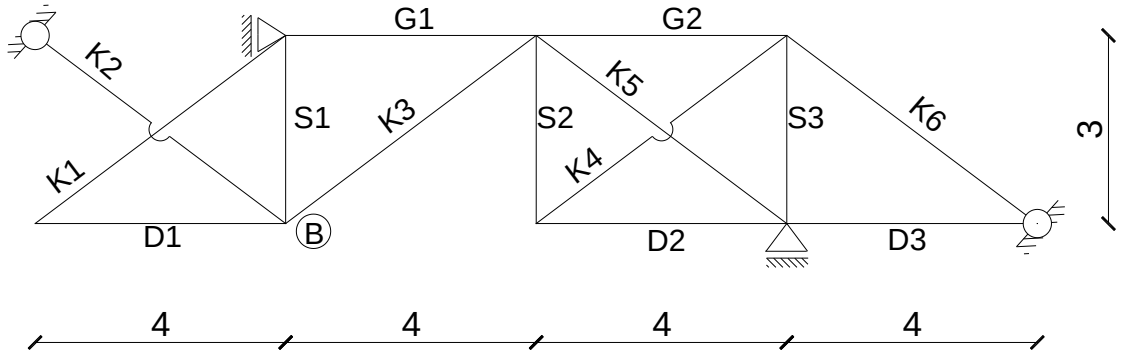
$$-(-K3)\cos\alpha + K3\cos\alpha = 0$$

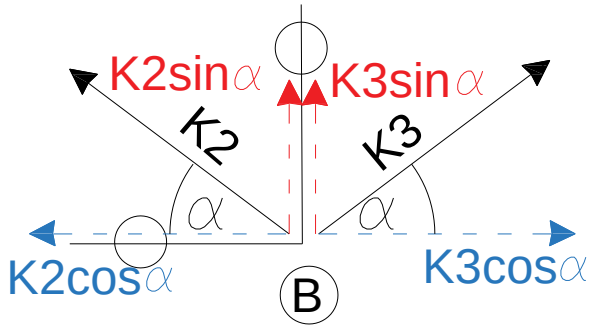
$$2 \cdot K3\cos\alpha = 0$$

$$K3 = 0$$

Wykresy: stan x1=1, siły N1







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$$\downarrow$$

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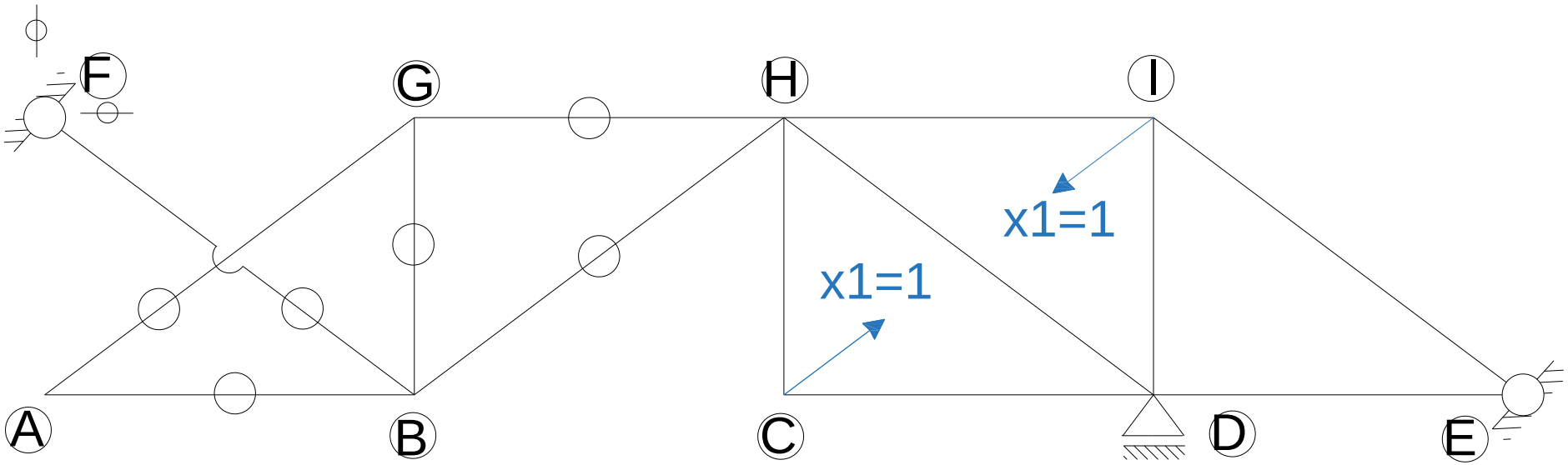
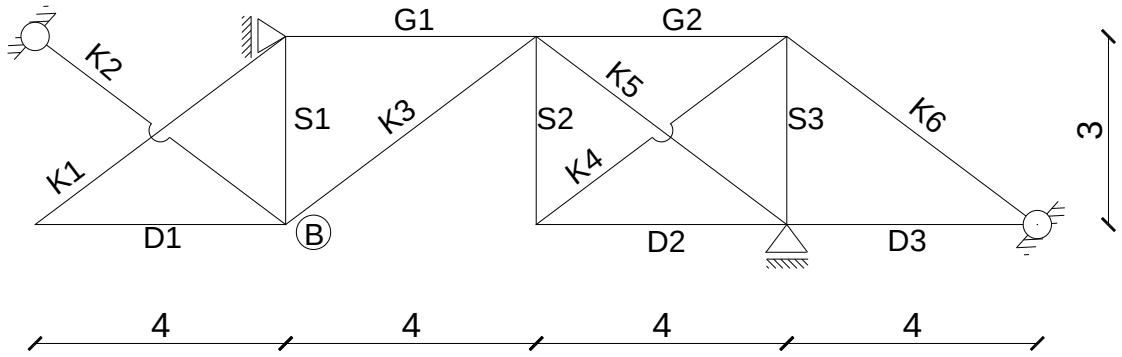
$$-(-K3)\cos\alpha + K3\cos\alpha = 0$$

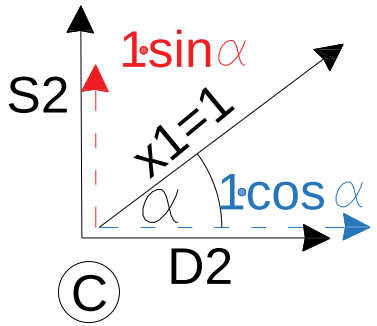
$$2 \cdot K3\cos\alpha = 0$$

$$K3 = 0$$

Wykresy: stan x1=1, siły N1







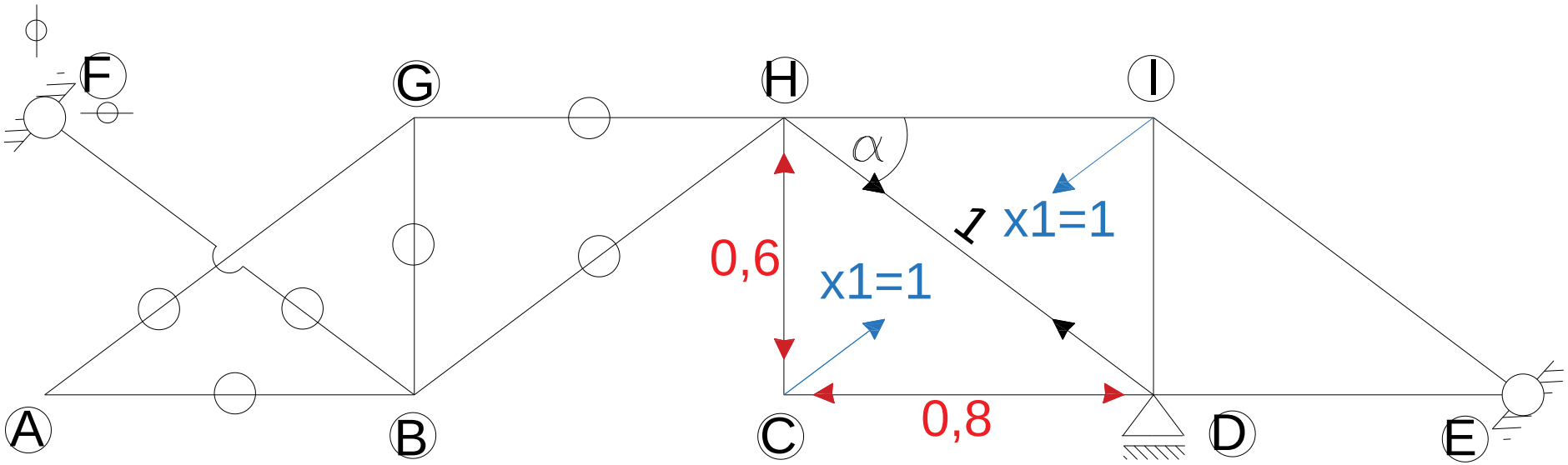
$$\sum R_y = S2 + x1 \cdot \sin\alpha = 0$$

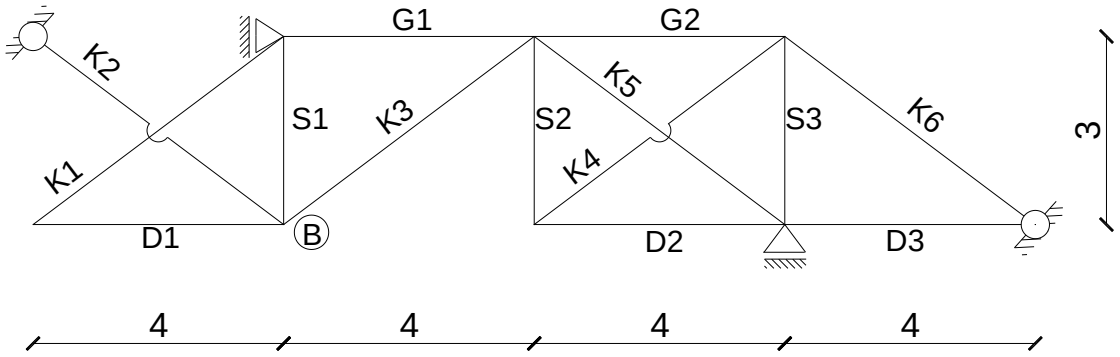
$$S2 = -x1 \cdot \sin\alpha = -1 \cdot \frac{3}{5} = -0{,}6$$

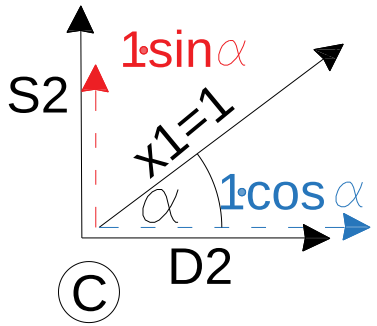
$$\sum R_x = D2 + x1 \cdot \cos\alpha = 0$$

$$D2 = -x1 \cdot \cos\alpha = -1 \cdot \frac{4}{5} = -0{,}8$$

Wykresy: stan x1=1, siły N1







$$\sum R_y = S2 + x1 \cdot \sin\alpha = 0$$

$$\downarrow$$

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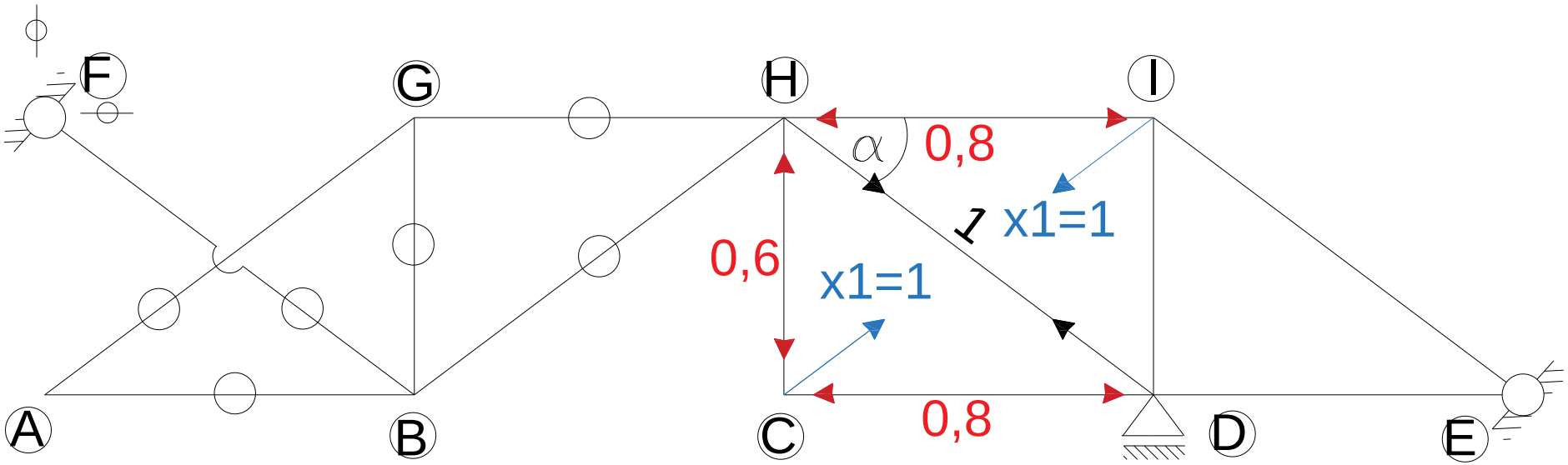
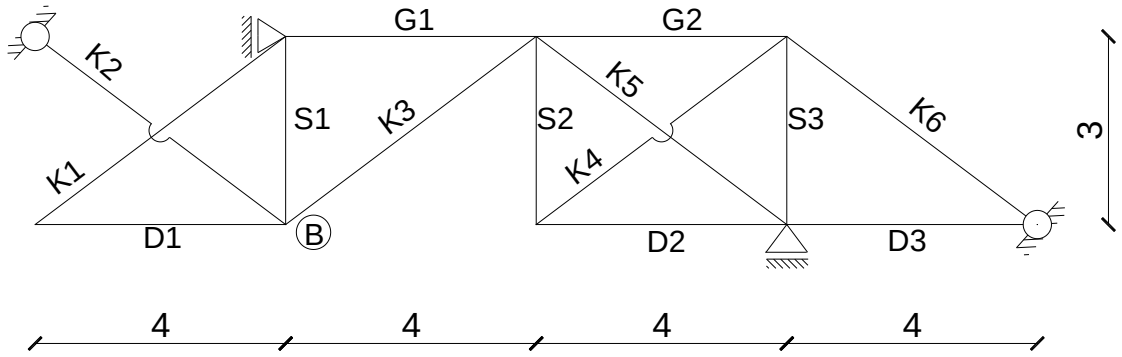
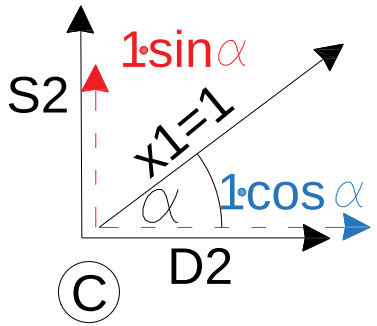
Wykresy: stan x1=1, siły N1







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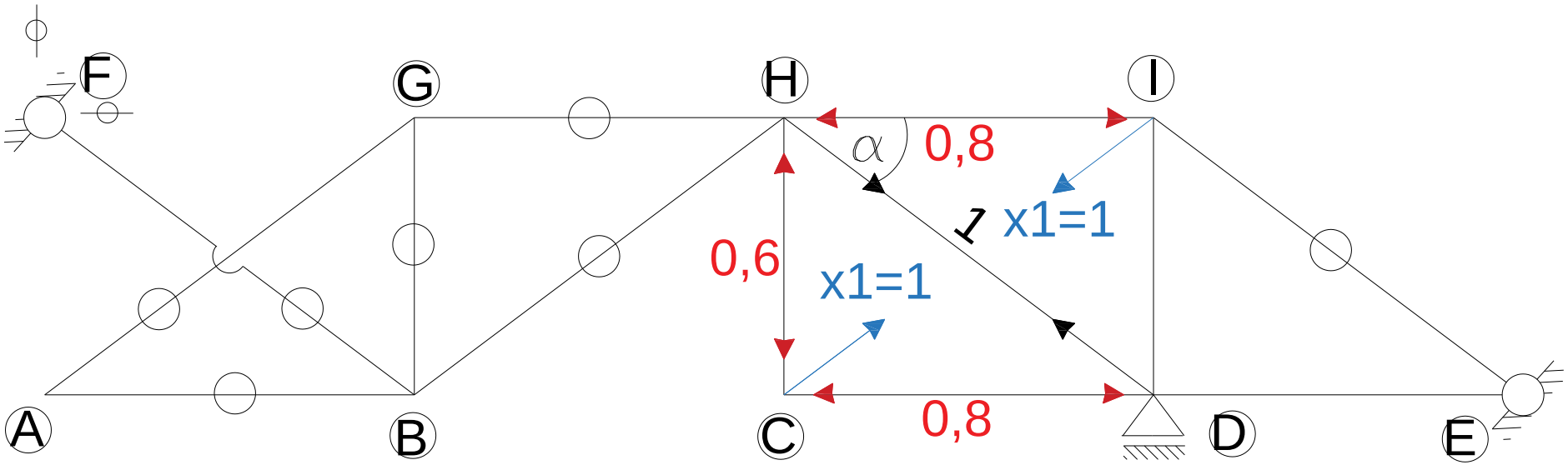
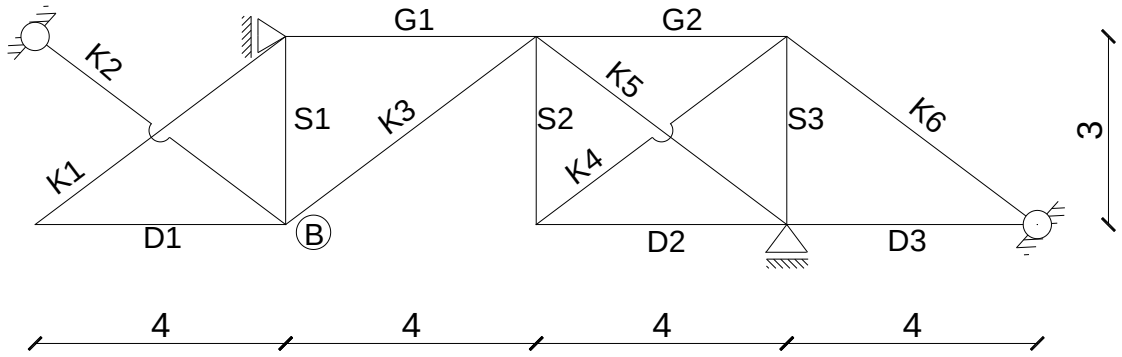
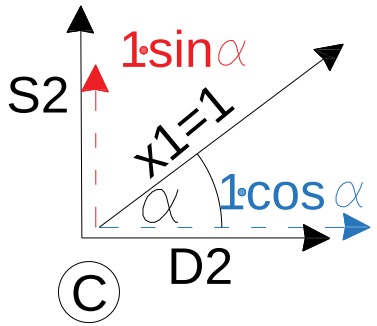
Wykresy: stan x1=1, siły N1







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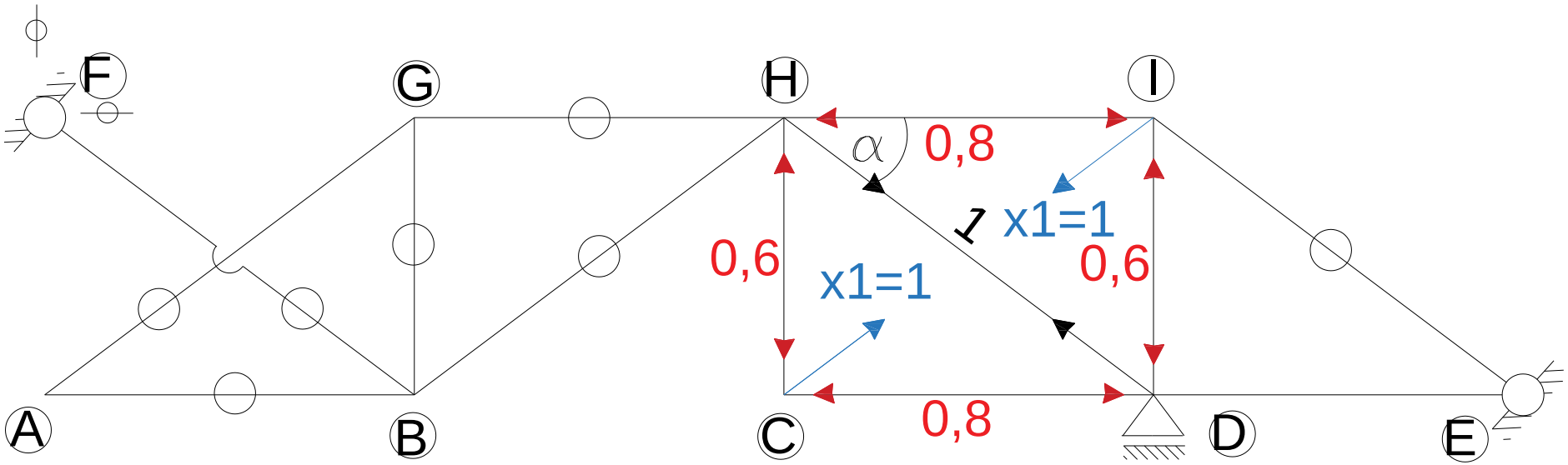
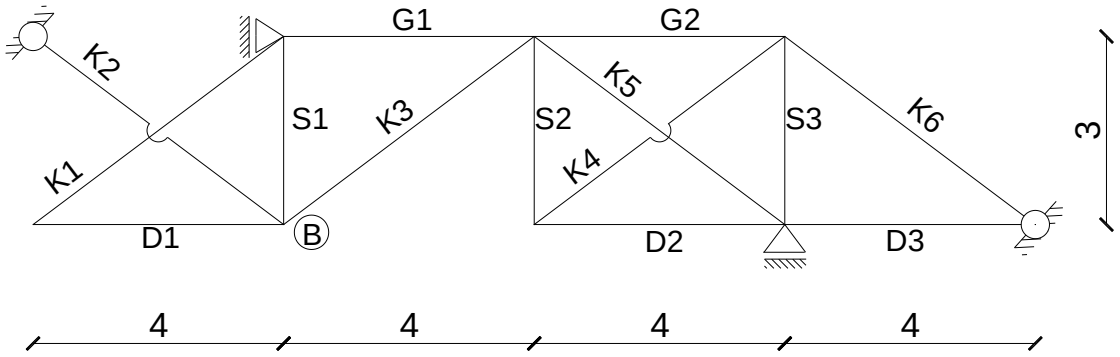
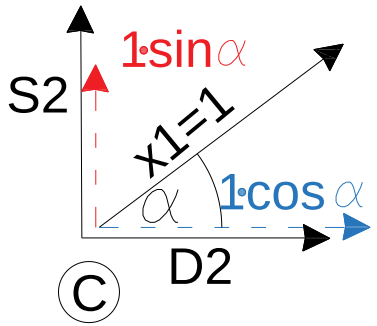
Wykresy: stan x1=1, siły N1







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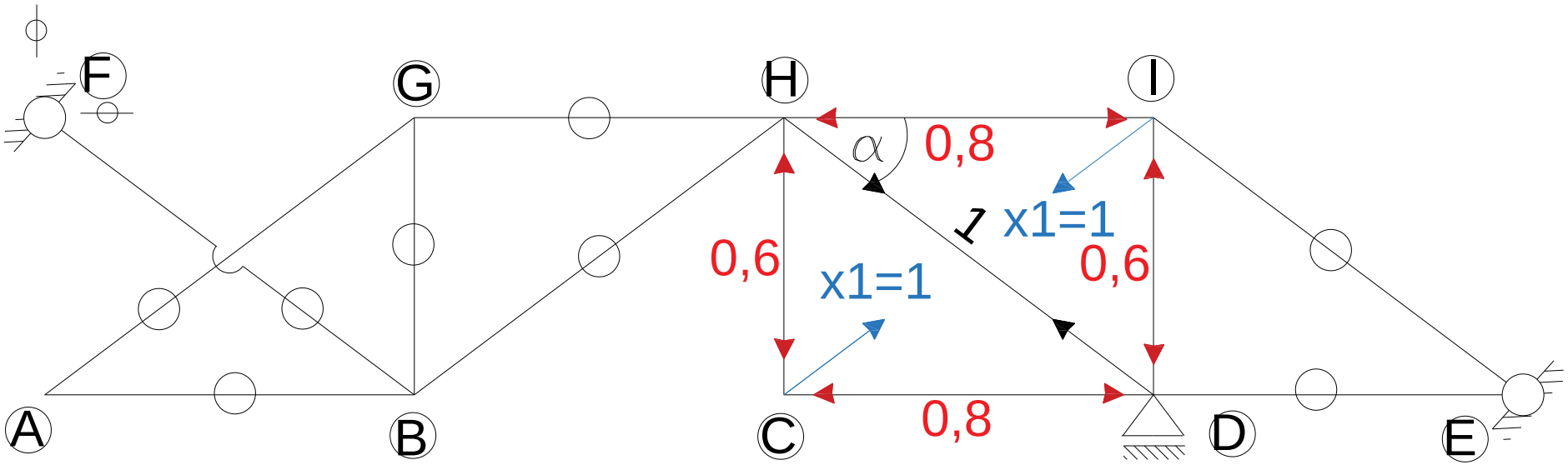
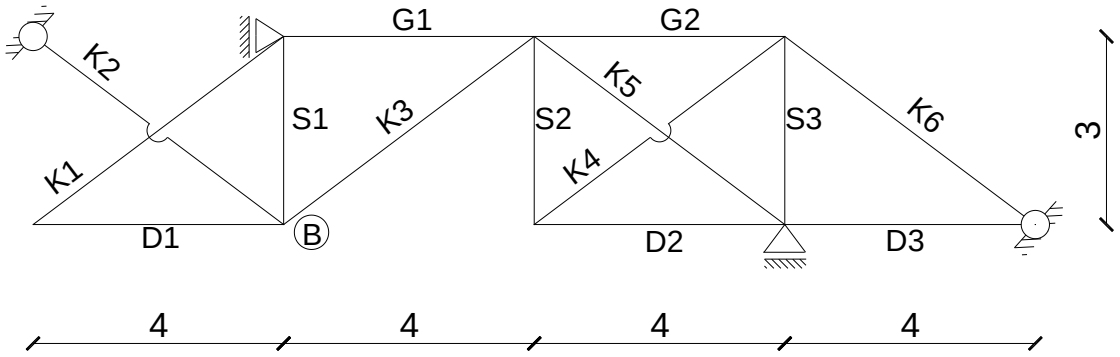
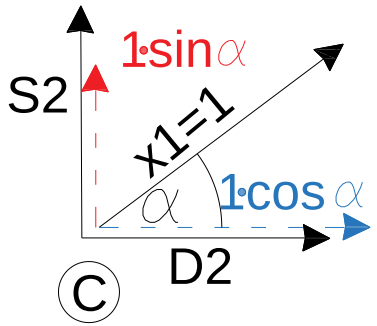






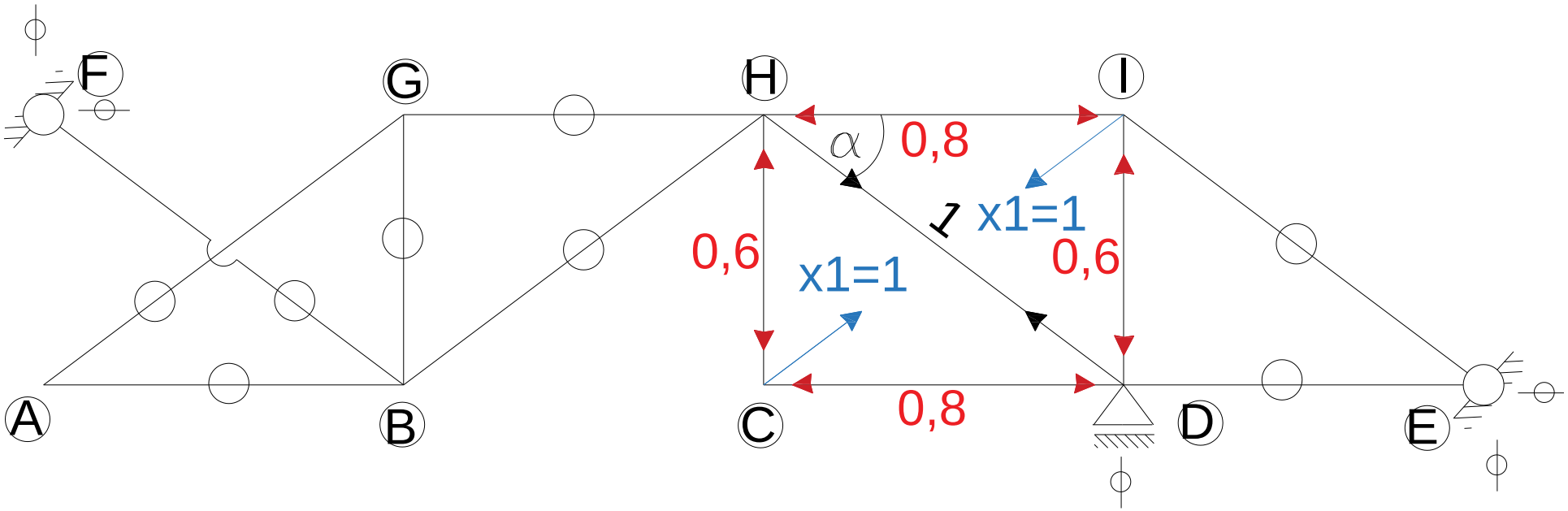
$$\sum R_y = S2 + x1 \cdot \sin\alpha = 0$$

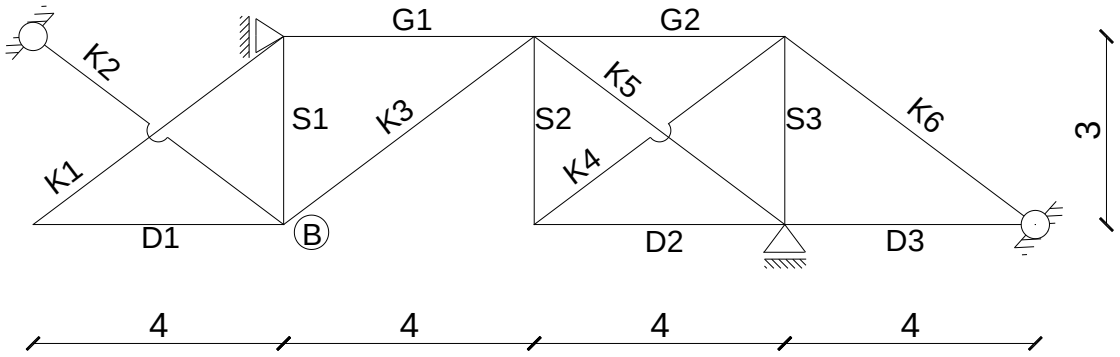
$$S2 = -x1 \cdot \sin\alpha = -1 \cdot \frac{3}{5} = -0{,}6$$

$$\sum R_x = D2 + x1 \cdot \cos\alpha = 0$$

$$D2 = -x1 \cdot \cos\alpha = -1 \cdot \frac{4}{5} = -0{,}8$$

Wykresy: stan x1=1, siły N1





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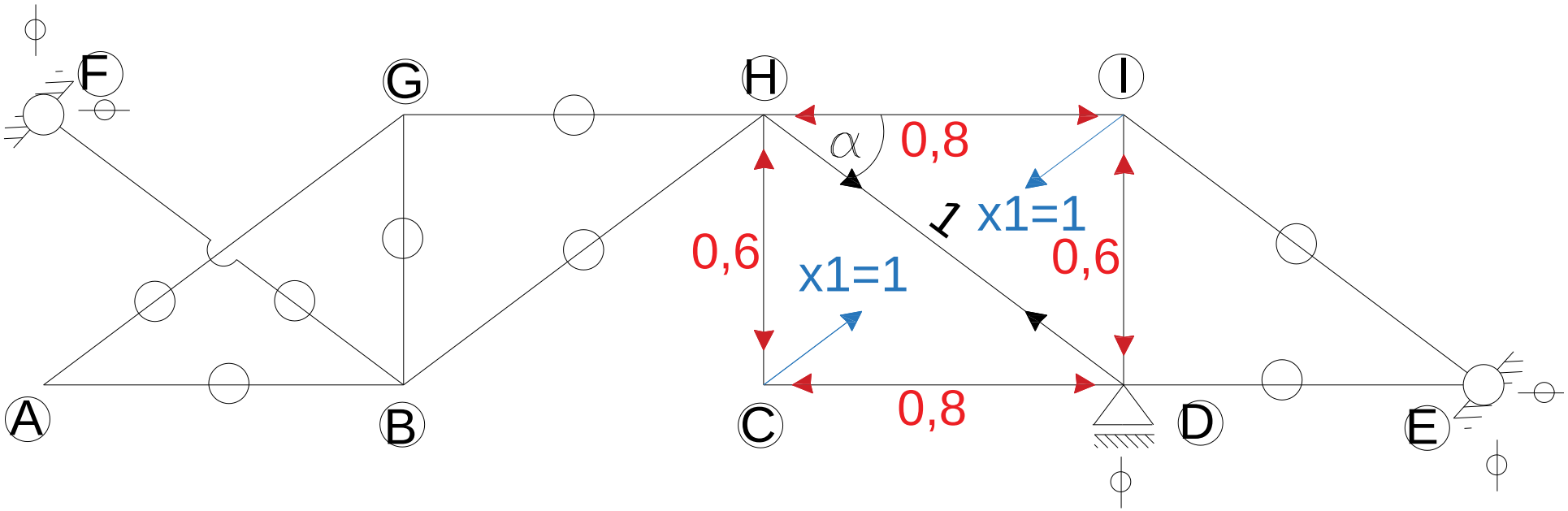
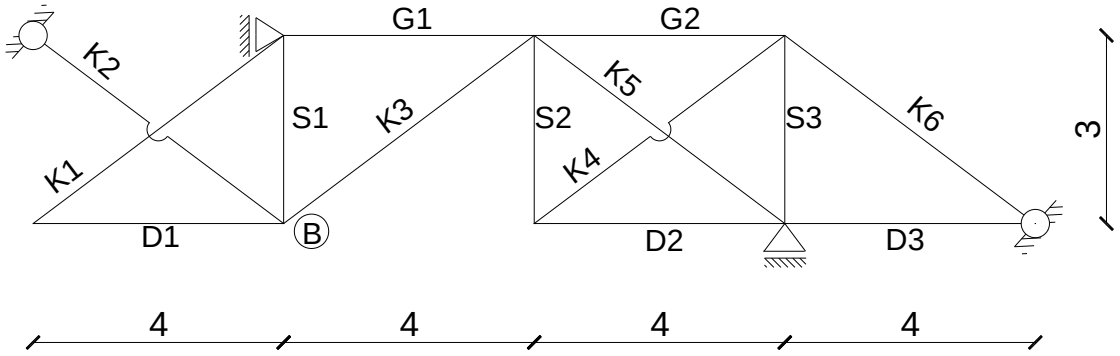
Wykresy: stan x1=1, siły N1





Pręt	L/EA	N1
D1	4	0
D2	4	-0,8
D3	4	0
G1	4	0
G2	4	-0,8
S1	3	0
S2	3	-0,6
S3	3	-0,6
K1	2,5	0
K2	2,5	0
K3	2,5	0
K4	2,5	1
K5	2,5	1
K6	2,5	0

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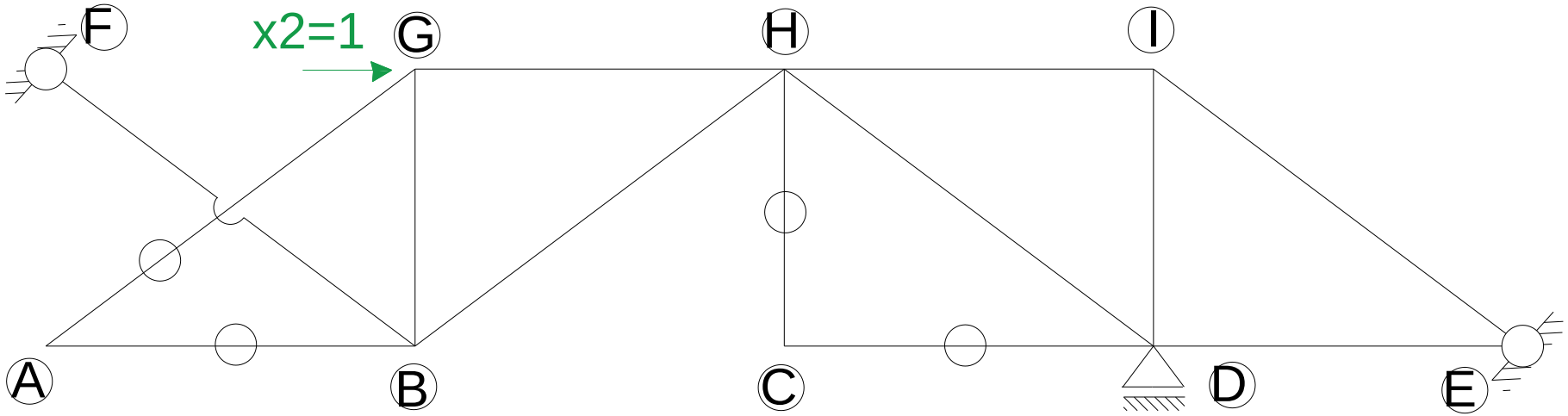
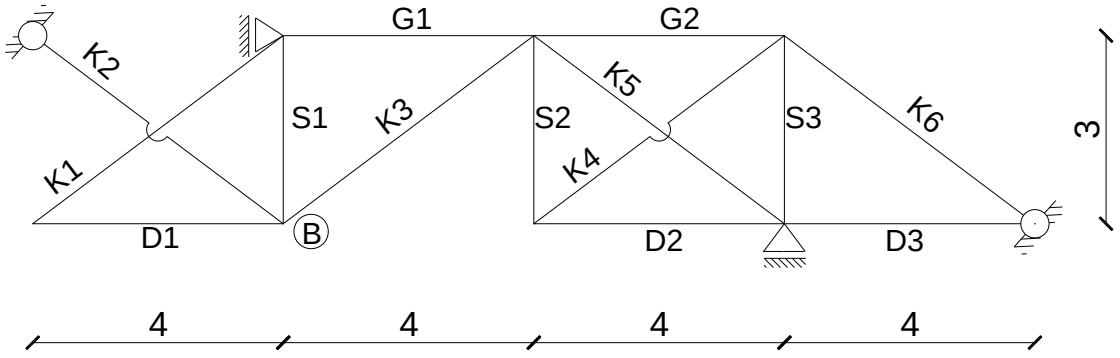
Wykresy: stan x2=1 – siły N2





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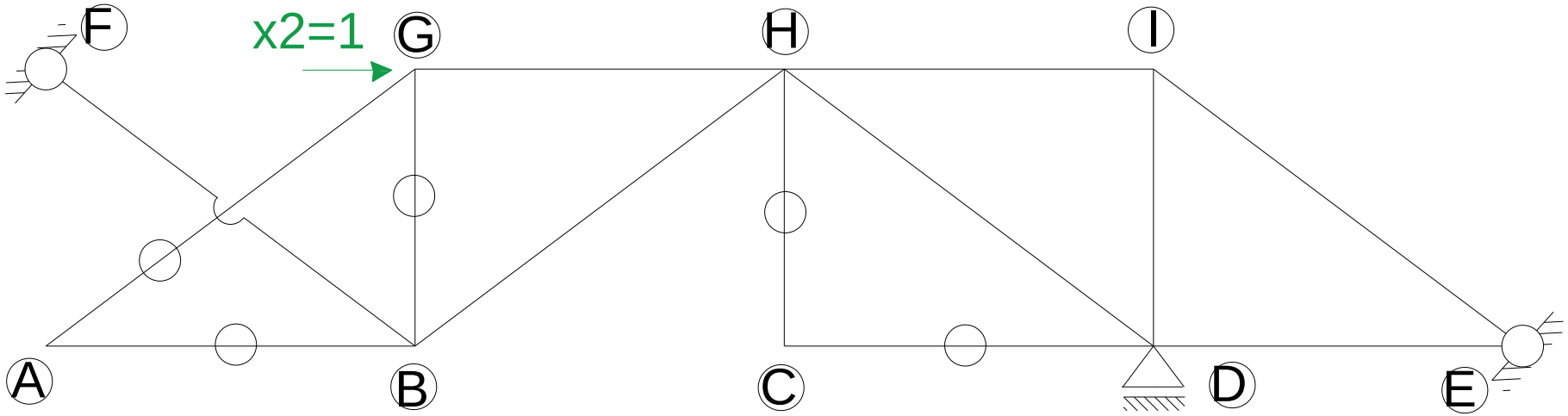
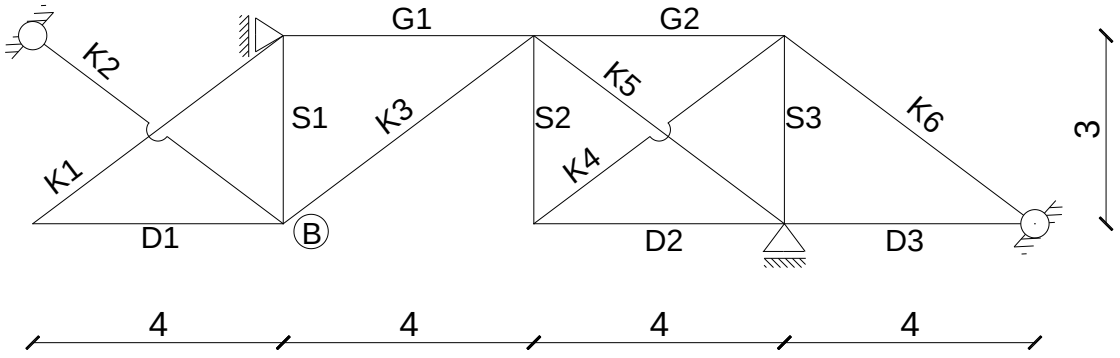
Wykresy: stan x2=1 – siły N2





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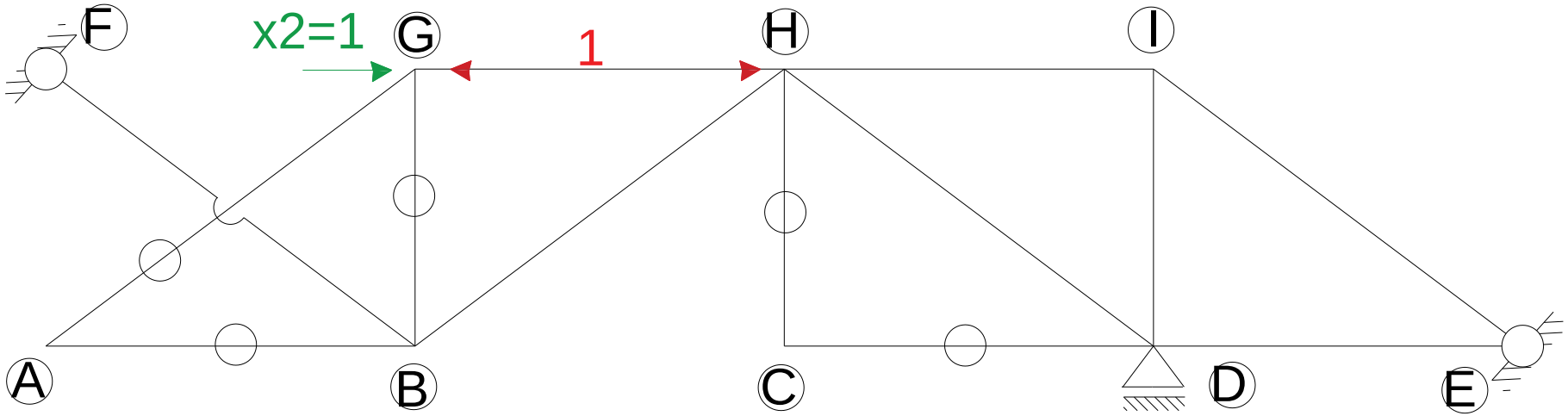
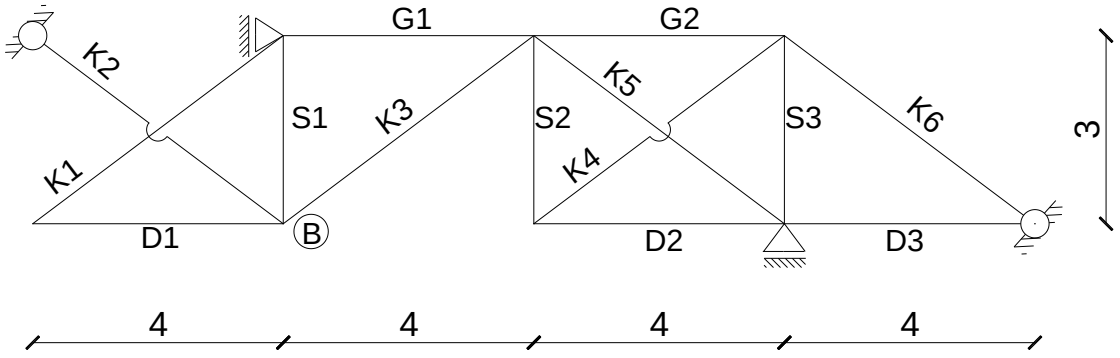
Wykresy: stan x2=1 – siły N2





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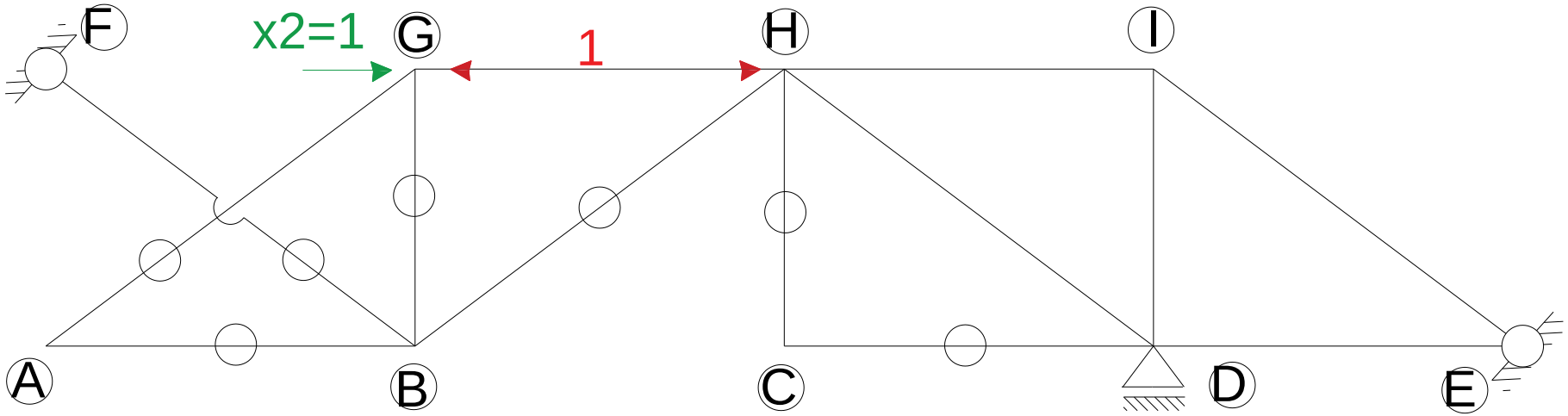
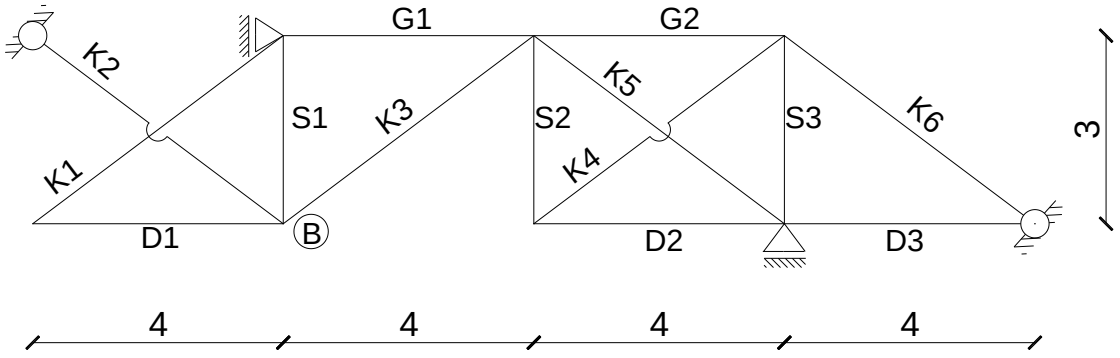
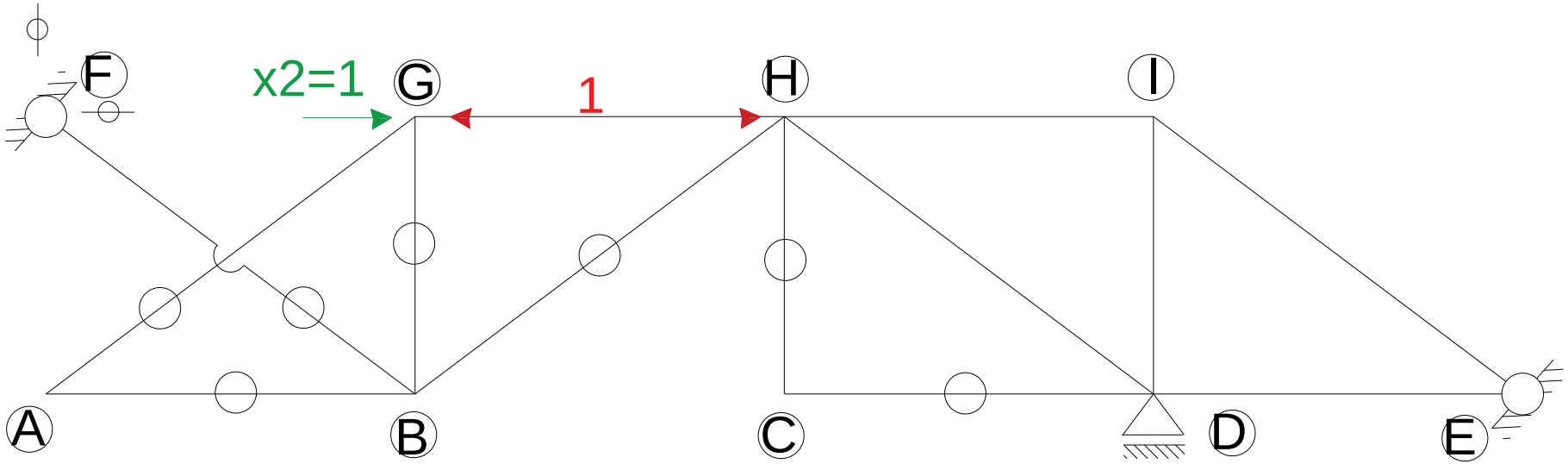
Wykresy: stan x2=1 – siły N2

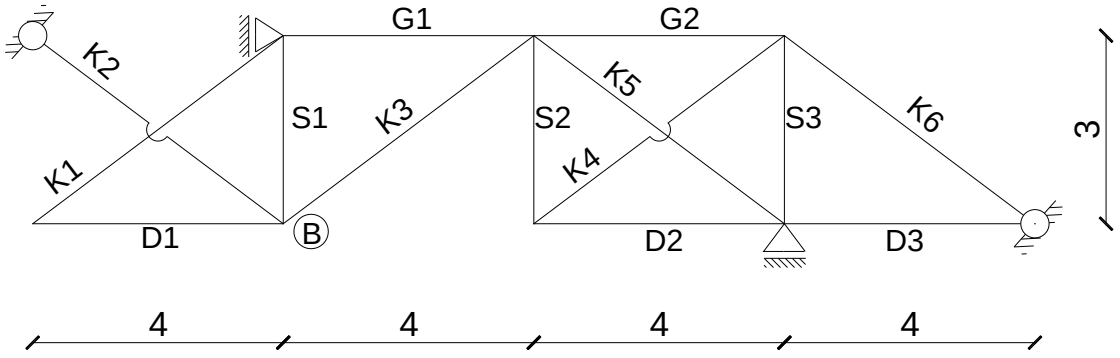




dr inż. Hanna Weber

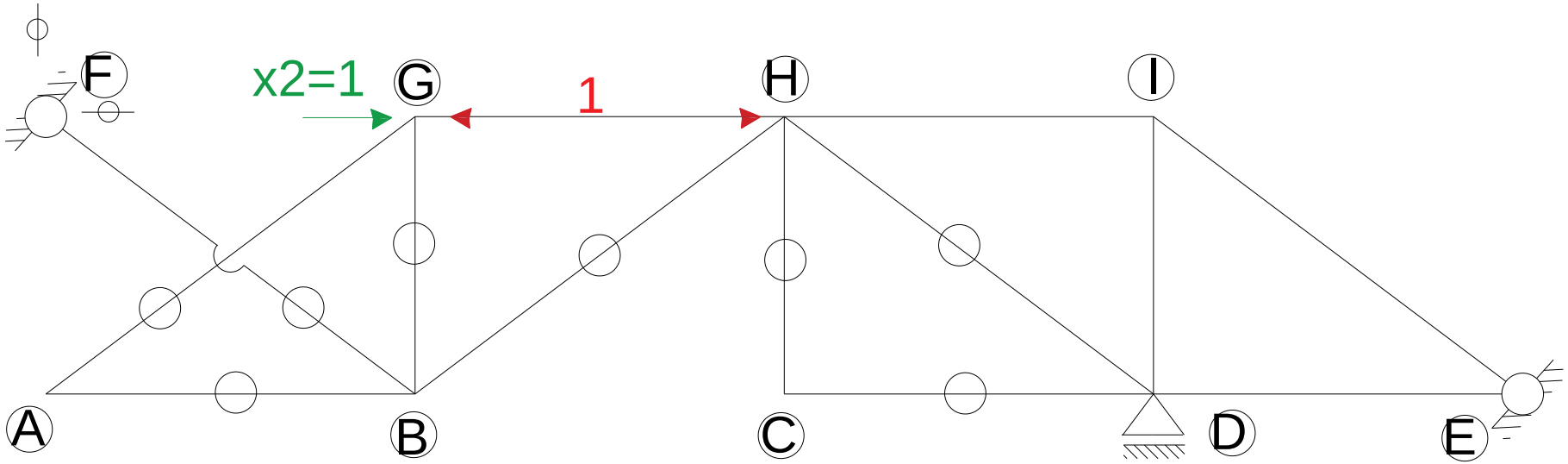
Wykresy: stan x2=1 – siły N2

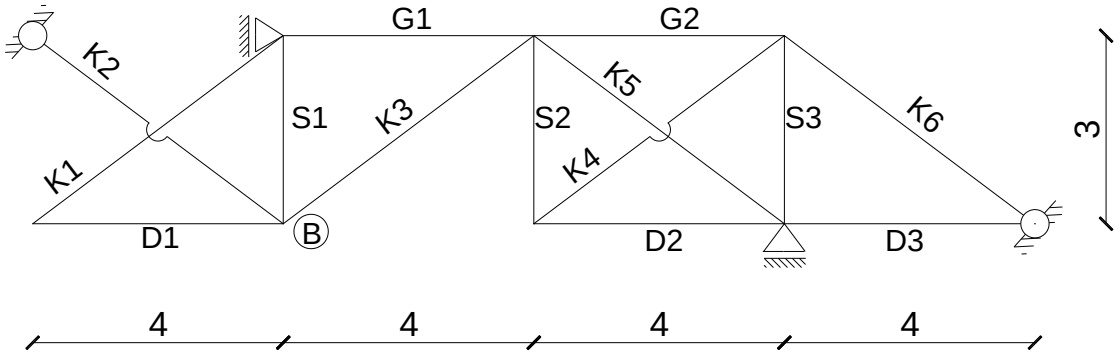




dr inż. Hanna Weber

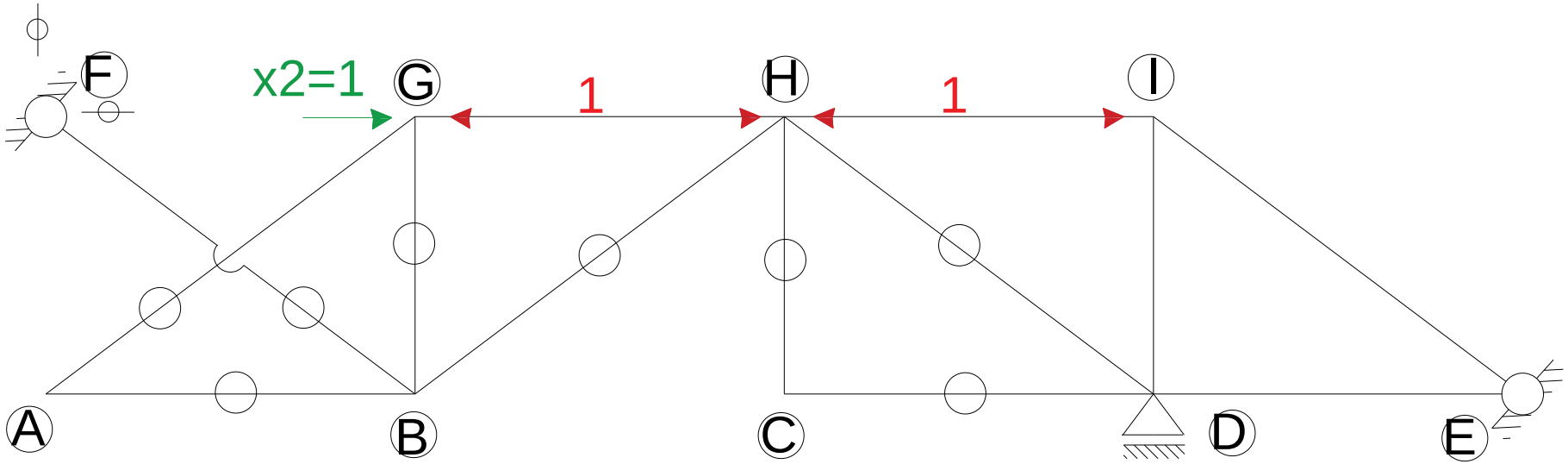
Wykresy: stan x2=1 – siły N2

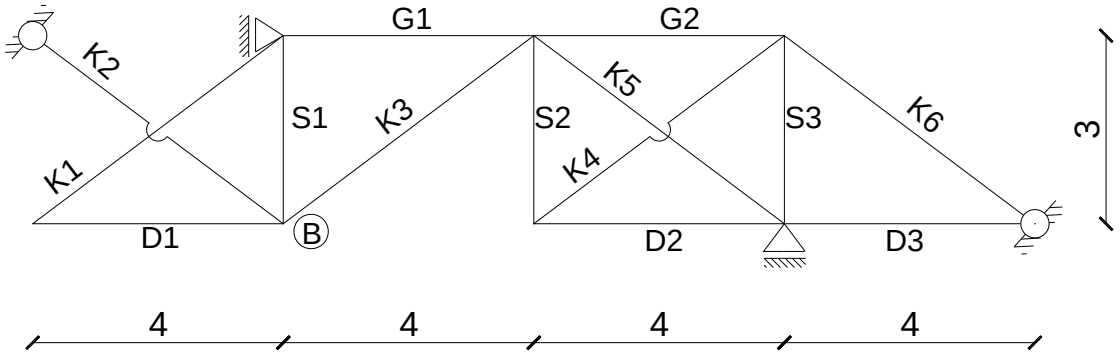




dr inż. Hanna Weber

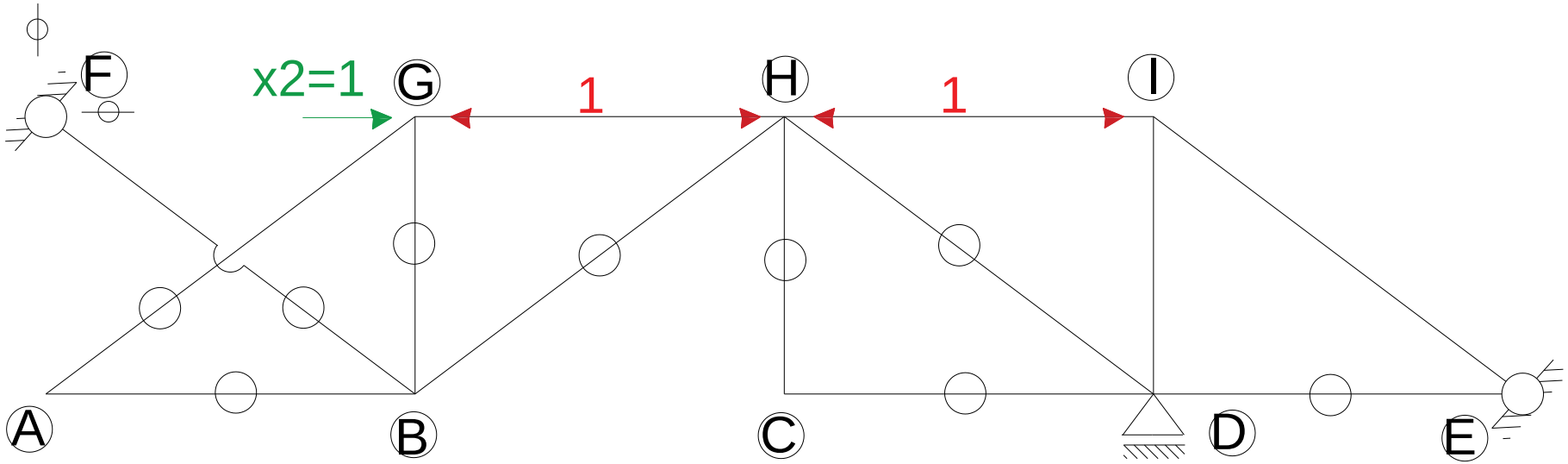
Wykresy: stan x2=1 – siły N2

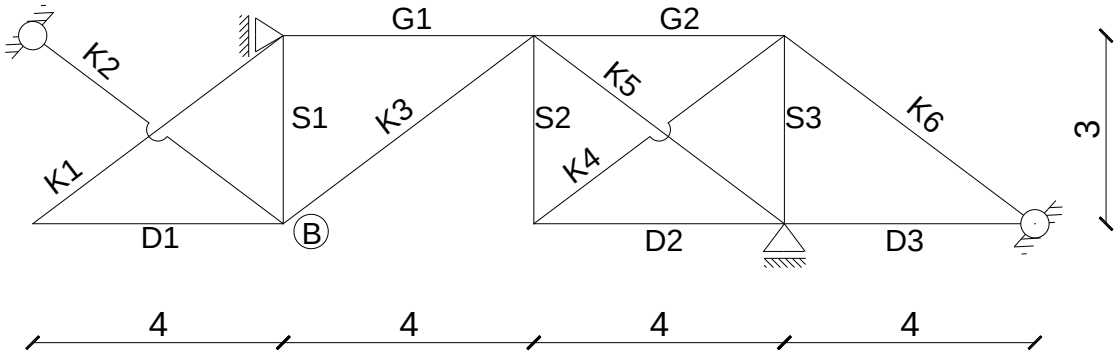




dr inż. Hanna Weber

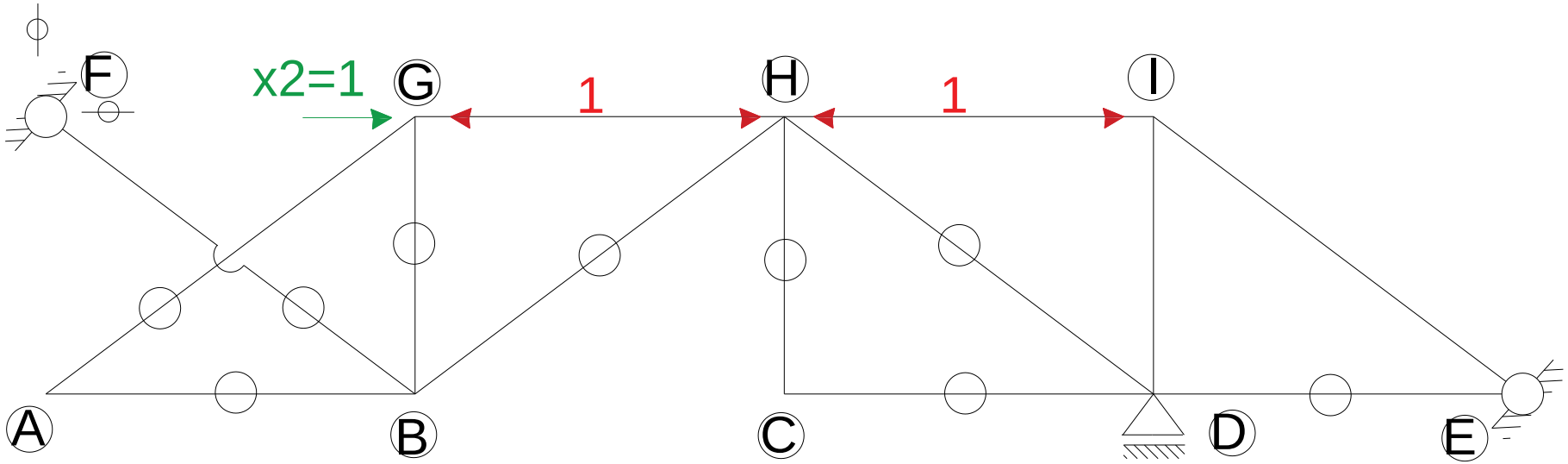
Wykresy: stan x2=1 – siły N2

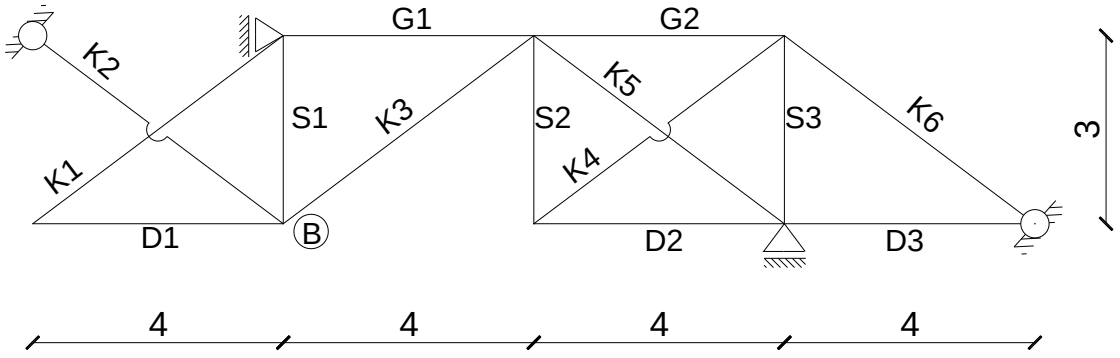


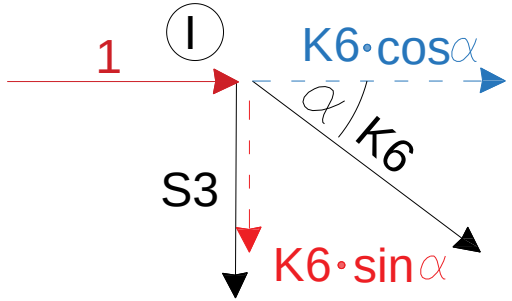


dr inż. Hanna Weber

Wykresy: stan x2=1 – siły N2







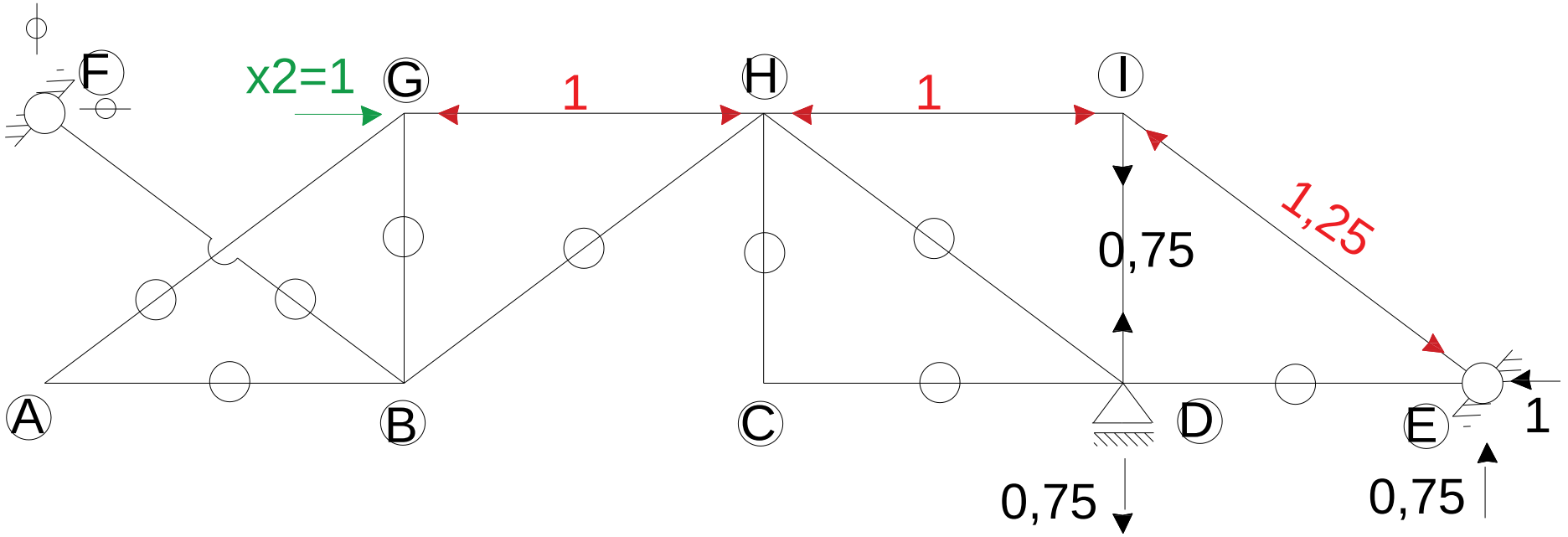
$$\sum R_x = 1 + K6 \cdot \cos\alpha = 0$$

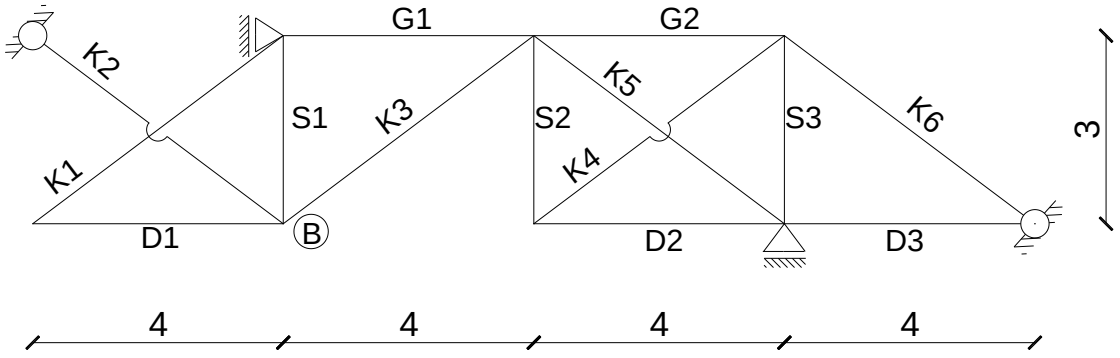
$$K6 = -\frac{1}{\cos\alpha} = -\frac{5}{4} = -1{,}25$$

$$\sum R_Y = -S3 - K6 \cdot \sin\alpha = 0$$

$$S3 = -K6 \cdot \sin\alpha = -\left(-\frac{5}{4}\right) \cdot \frac{3}{5} = \frac{3}{4} = 0{,}75$$

Wykresy: stan x2=1 – siły N2





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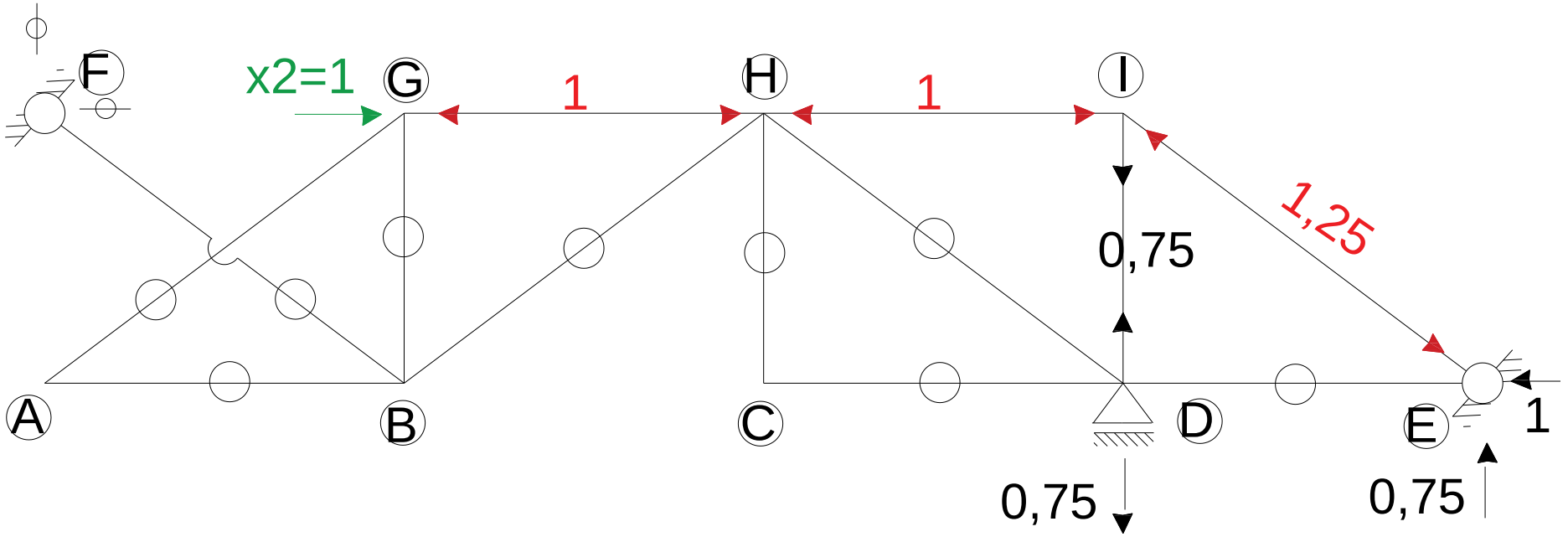
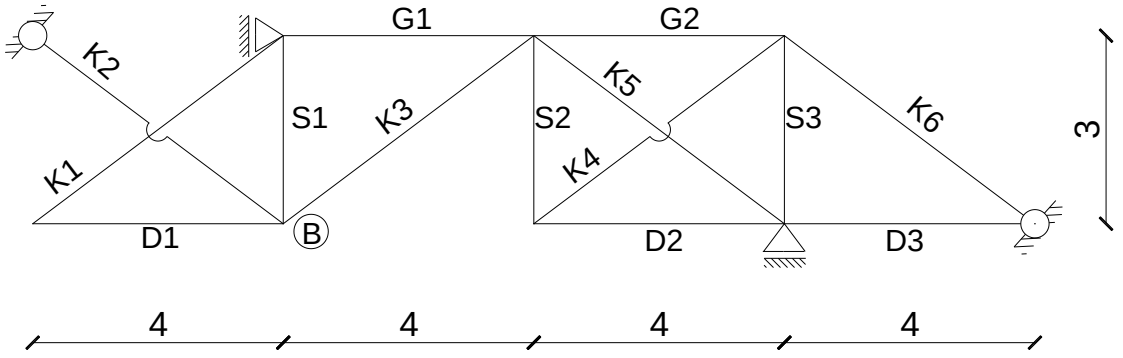
Wykresy: stan x2=1 – siły N2

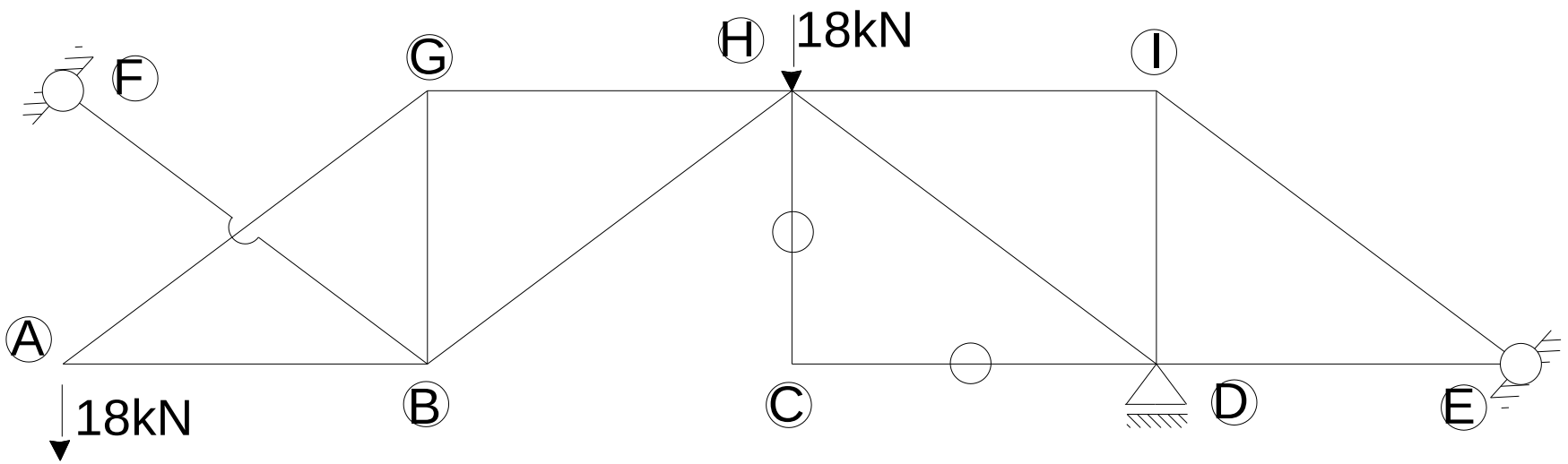


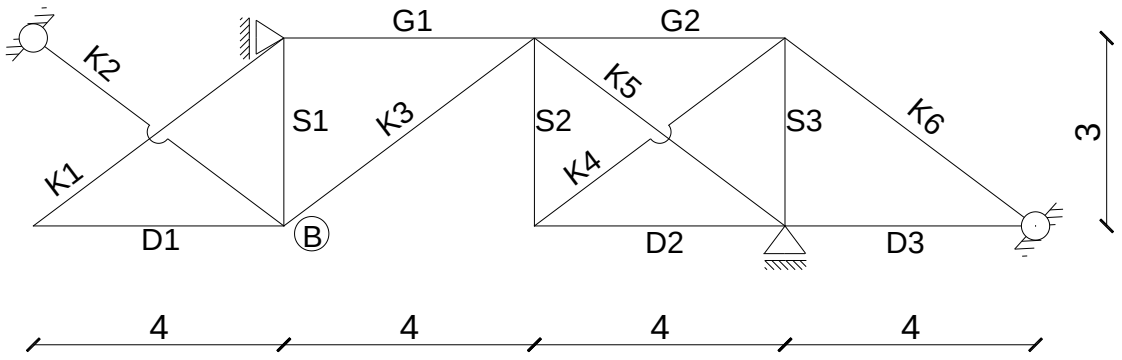


Pręt	L/EA	N2
D1	4	0
D2	4	0
D3	4	0
G1	4	-1
G2	4	-1
S1	3	0
S2	3	0
S3	3	0,75
K1	2,5	0
K2	2,5	0
K3	2,5	0
K4	2,5	0
K5	2,5	0
K6	2,5	-1,25

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]





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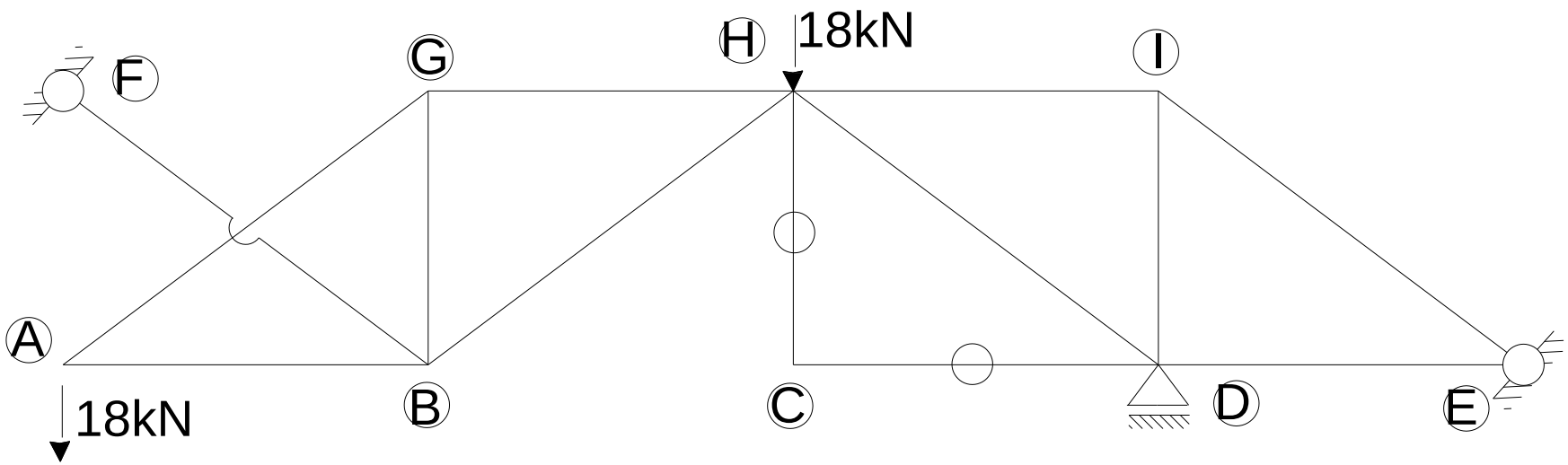
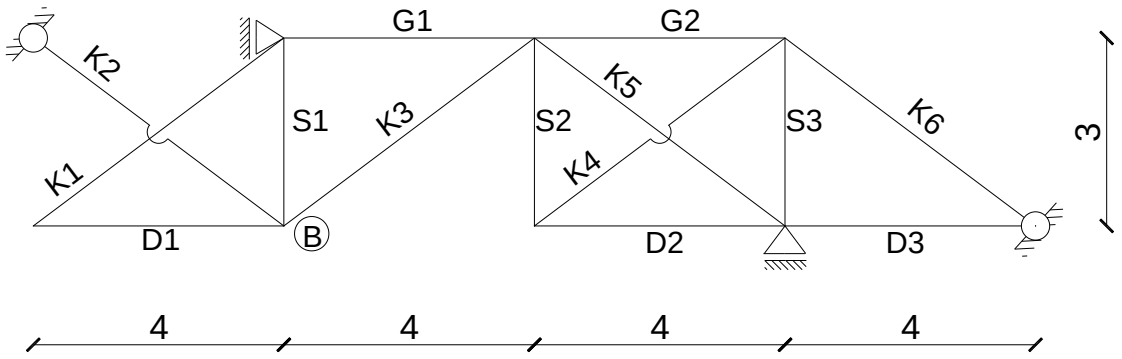
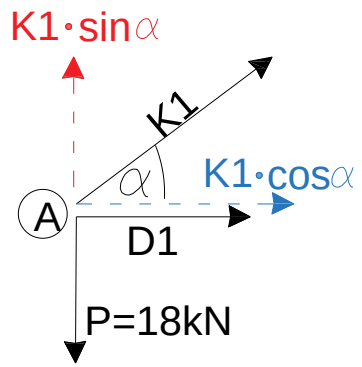
Wykresy: obciążenie zewnętrzne – siły N0 [kN]







$$\sum R_Y = -18 + K1 \cdot \sin\alpha = 0$$

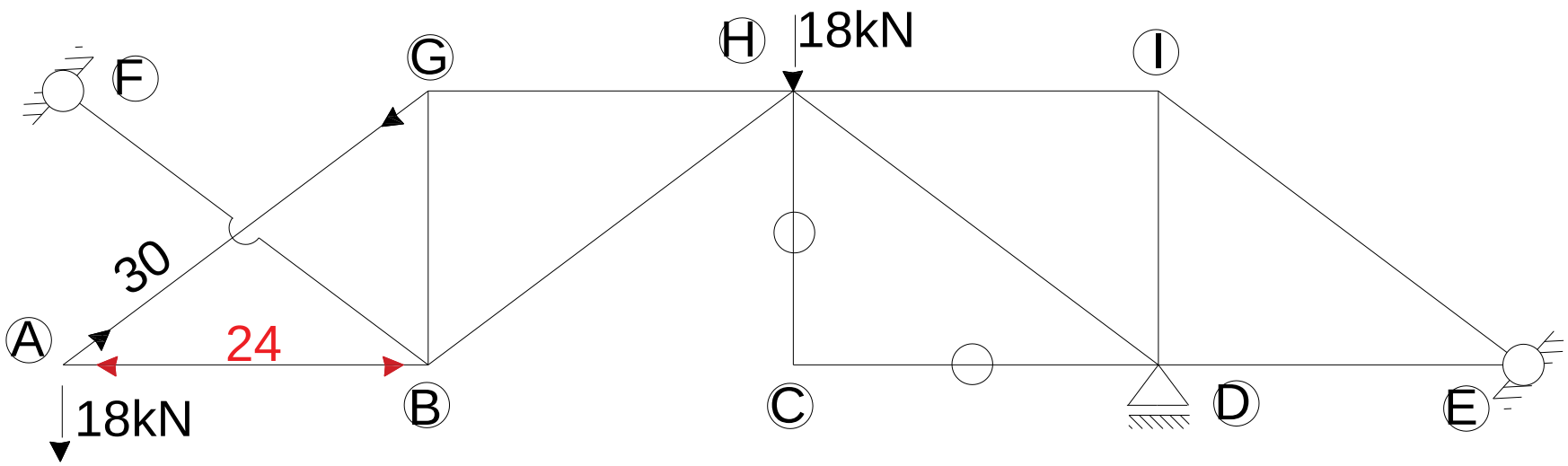
$$\downarrow$$

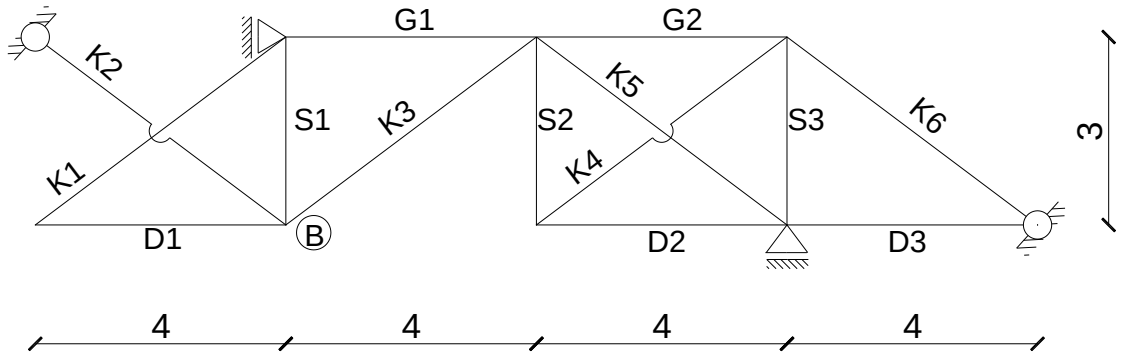
$$K1 = \frac{18}{\sin\alpha} = 18 \cdot \frac{5}{3} = 30\text{kN}$$

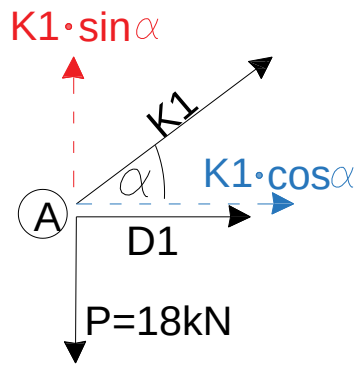
$$\sum R_X = D1 + K1 \cdot \cos\alpha = 0$$

$$D1 = -K1 \cdot \cos\alpha = -30 \cdot \frac{4}{5} = -24\text{kN}$$

Wykresy: obciążenie zewnętrzne – siły N0 [kN]







$$\sum R_Y = -18 + K1 \cdot \sin\alpha = 0$$

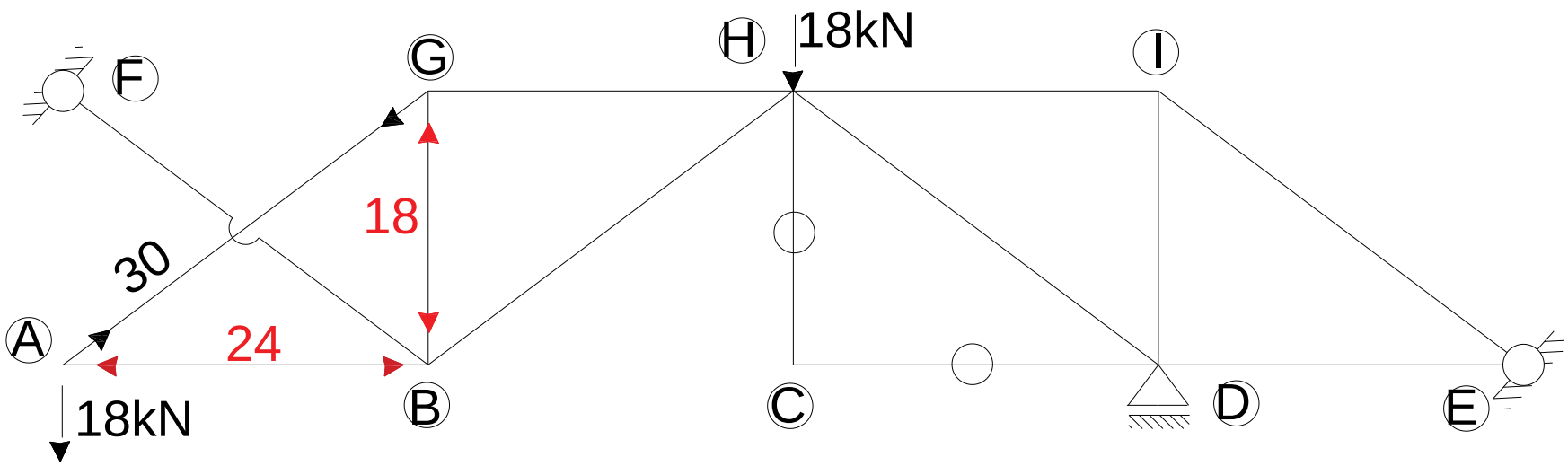
$$\downarrow$$

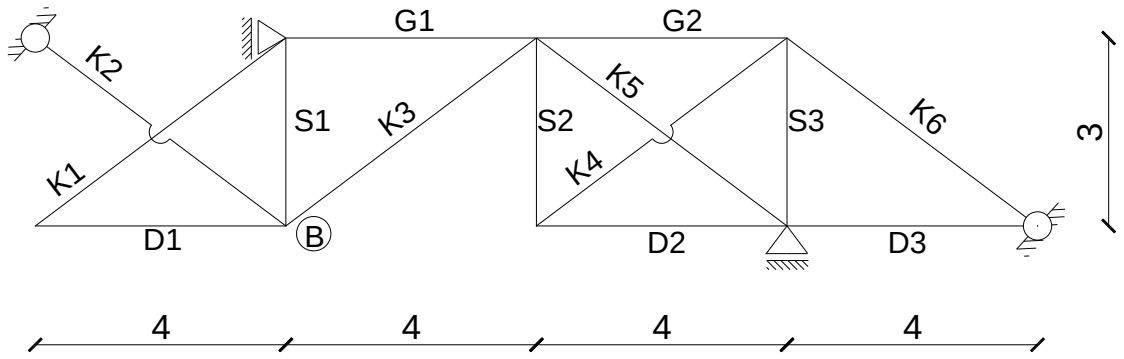
$$K1 = \frac{18}{\sin\alpha} = 18 \cdot \frac{5}{3} = 30\text{kN}$$

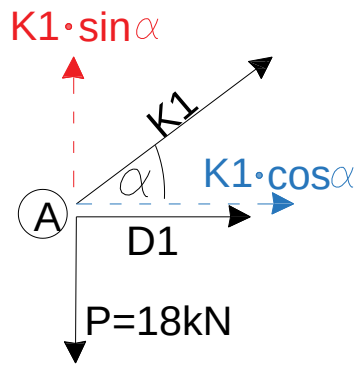
$$\sum R_X = D1 + K1 \cdot \cos\alpha = 0$$

$$D1 = -K1 \cdot \cos\alpha = -30 \cdot \frac{4}{5} = -24\text{kN}$$

Wykresy: obciążenie zewnętrzne – siły N0 [kN]







$$\sum R_Y = -18 + K1 \cdot \sin\alpha = 0$$

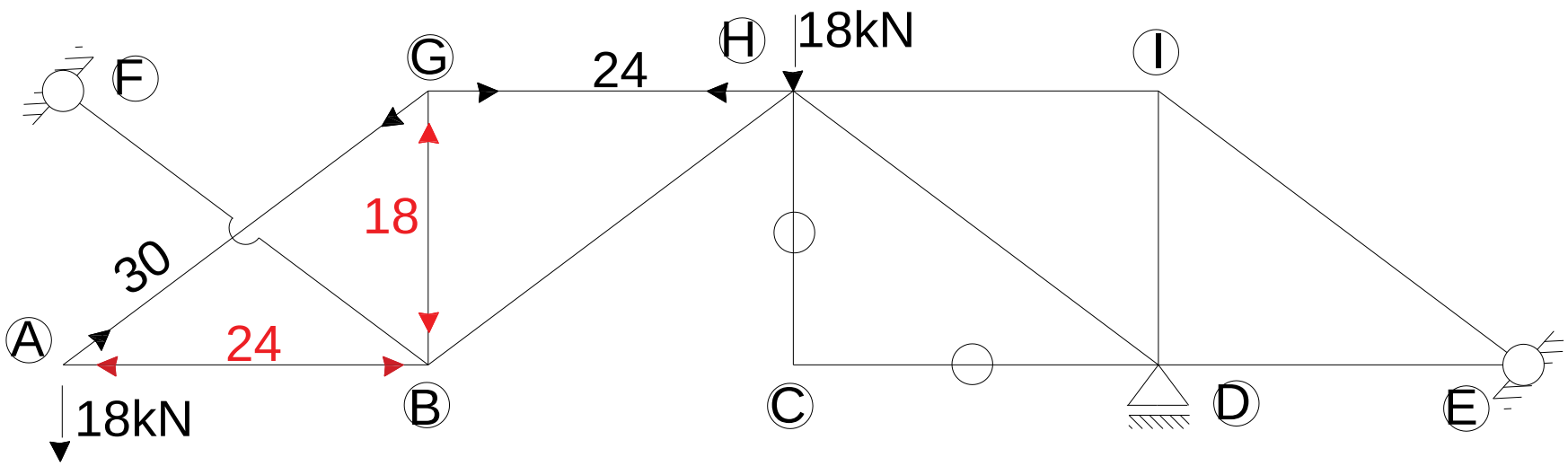
$$\downarrow$$

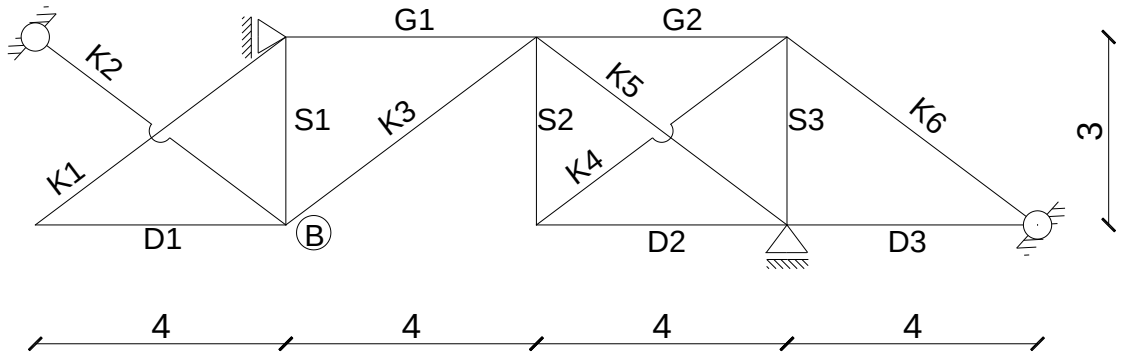
$$K1 = \frac{18}{\sin\alpha} = 18 \cdot \frac{5}{3} = 30\text{kN}$$

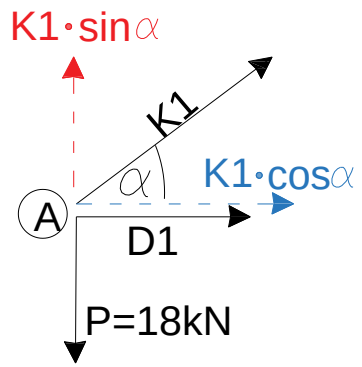
$$\sum R_X = D1 + K1 \cdot \cos\alpha = 0$$

$$D1 = -K1 \cdot \cos\alpha = -30 \cdot \frac{4}{5} = -24\text{kN}$$

Wykresy: obciążenie zewnętrzne – siły N0 [kN]







$$\sum R_Y = -18 + K1 \cdot \sin\alpha = 0$$

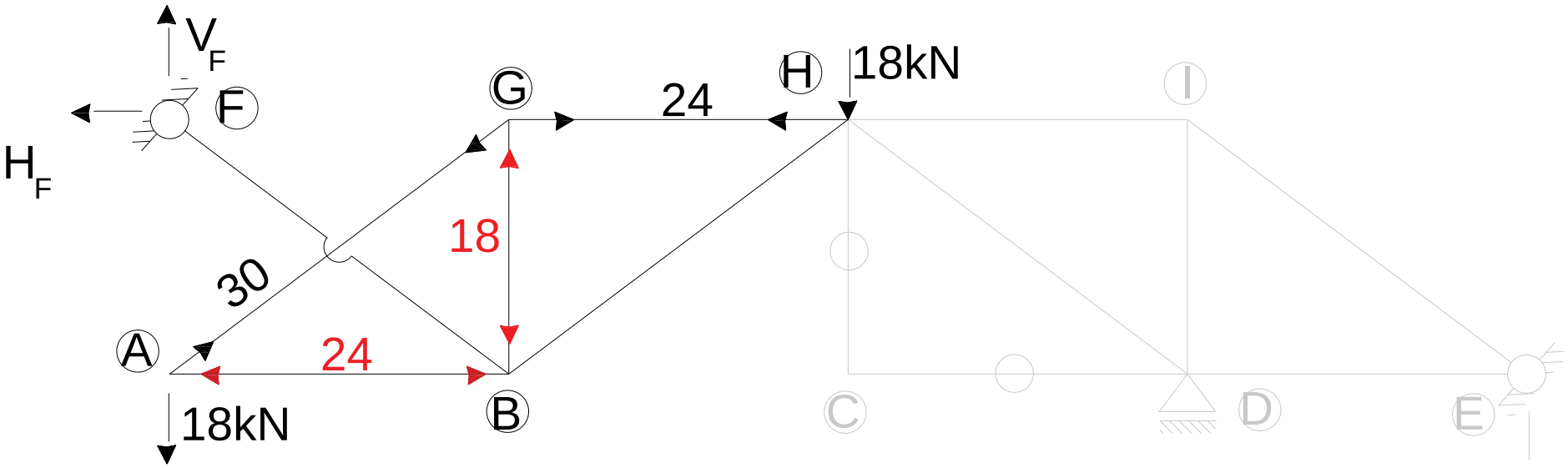
$$\downarrow$$

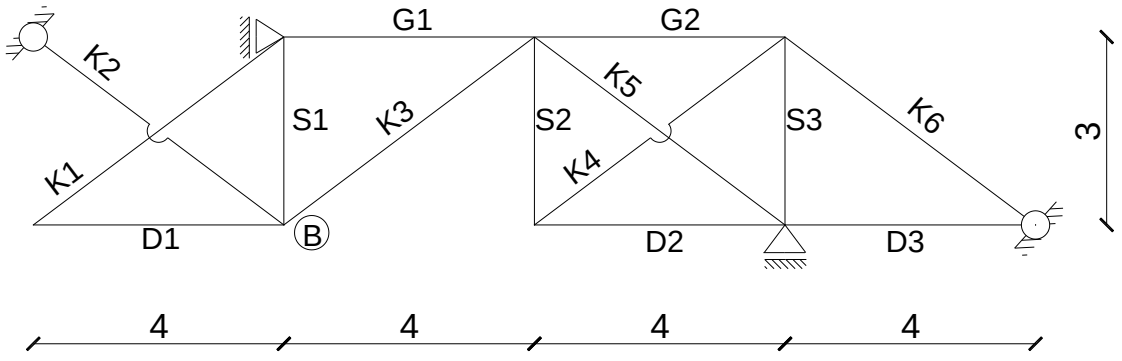
$$K1 = \frac{18}{\sin\alpha} = 18 \cdot \frac{5}{3} = 30\text{kN}$$

$$\sum R_X = D1 + K1 \cdot \cos\alpha = 0$$

$$D1 = -K1 \cdot \cos\alpha = -30 \cdot \frac{4}{5} = -24\text{kN}$$

Wykresy: obciążenie zewnętrzne – siły N0 [kN]





$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

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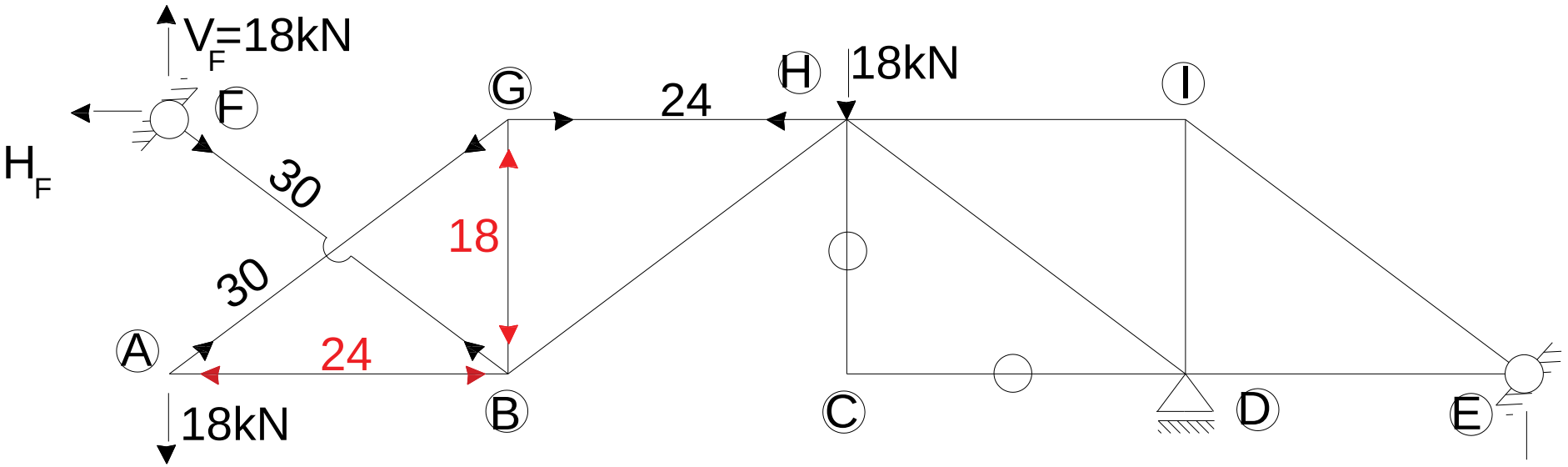
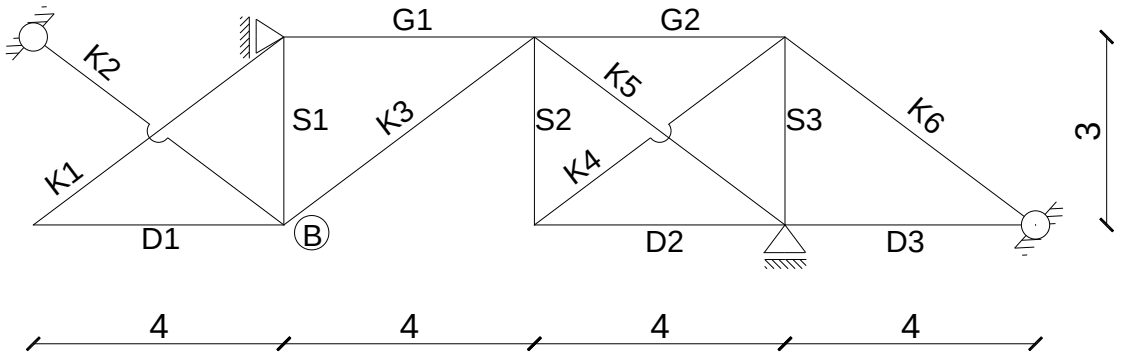
Wykresy: obciążenie zewnętrzne – siły N0 [kN]

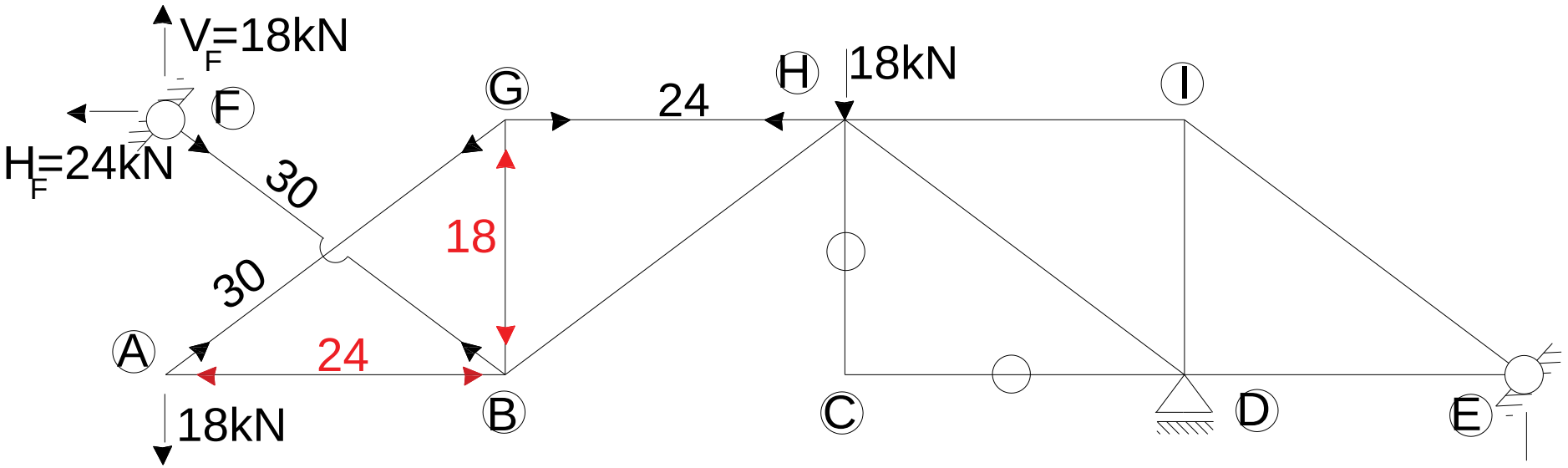


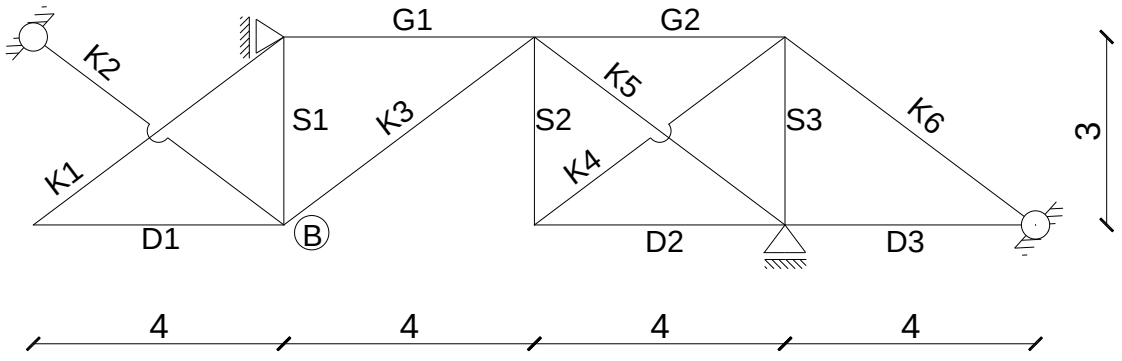


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

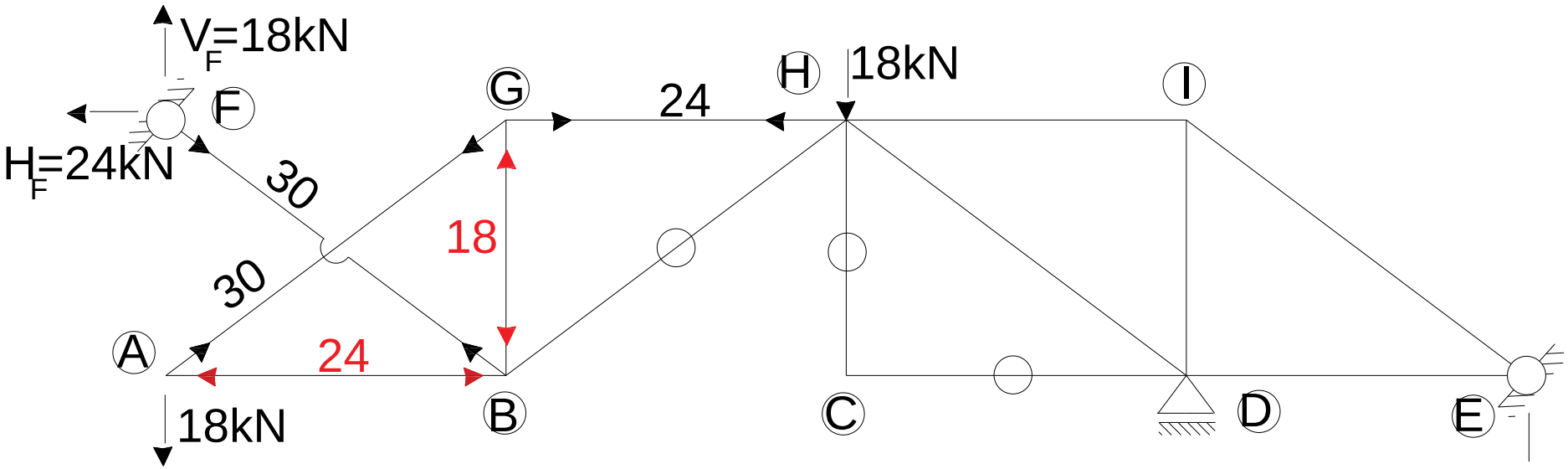


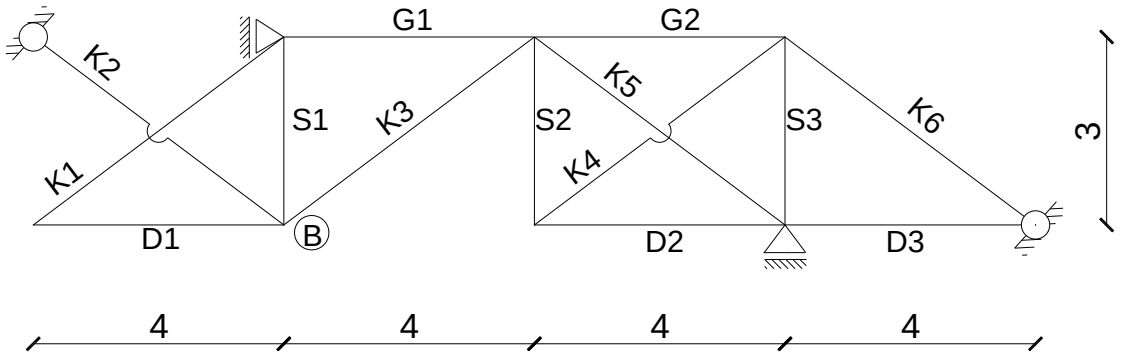


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

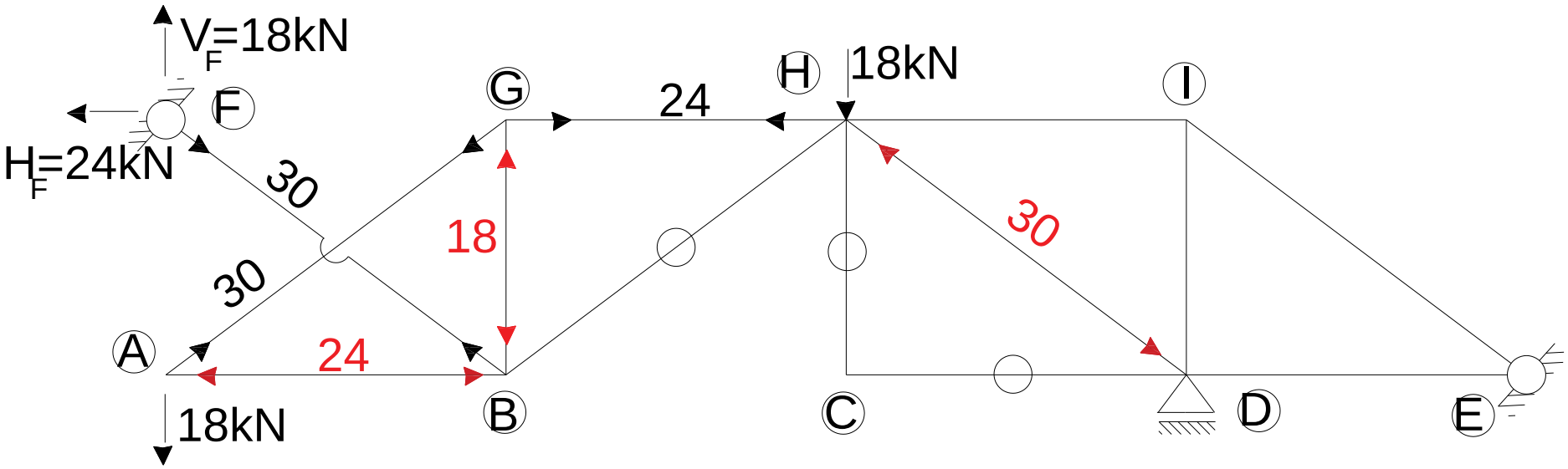


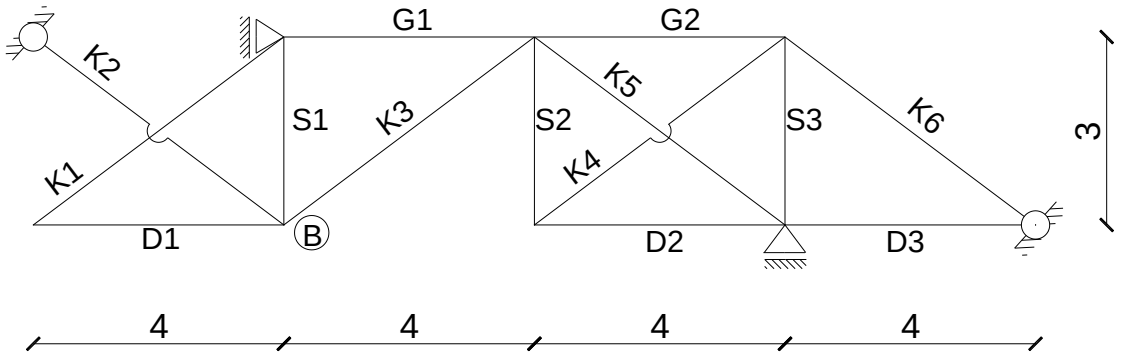


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

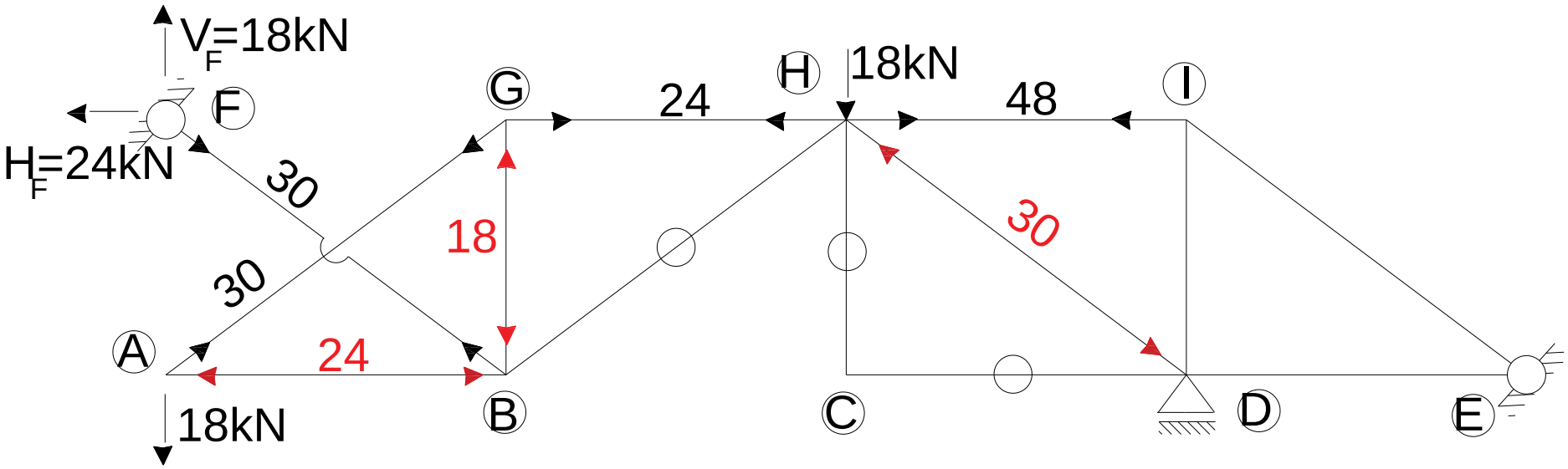


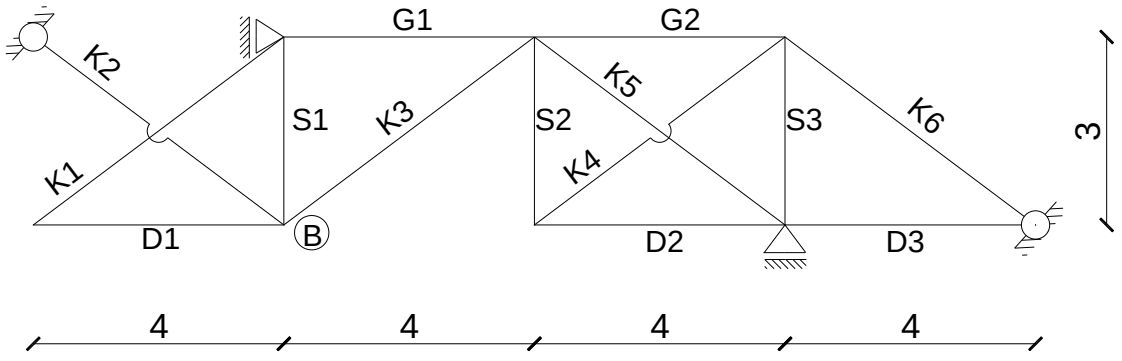


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

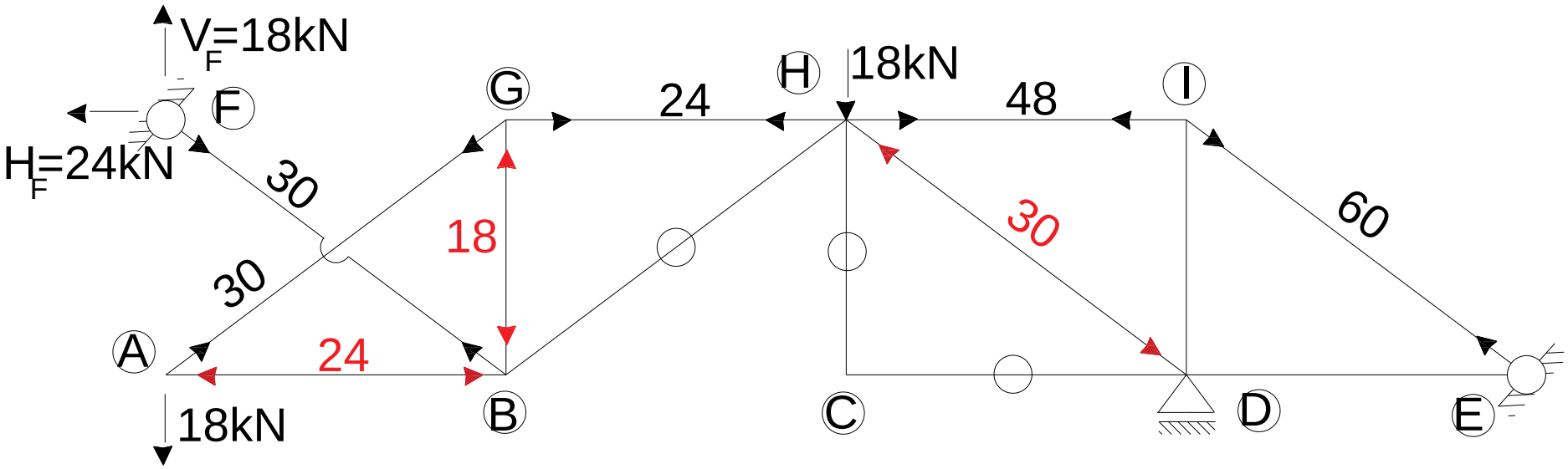


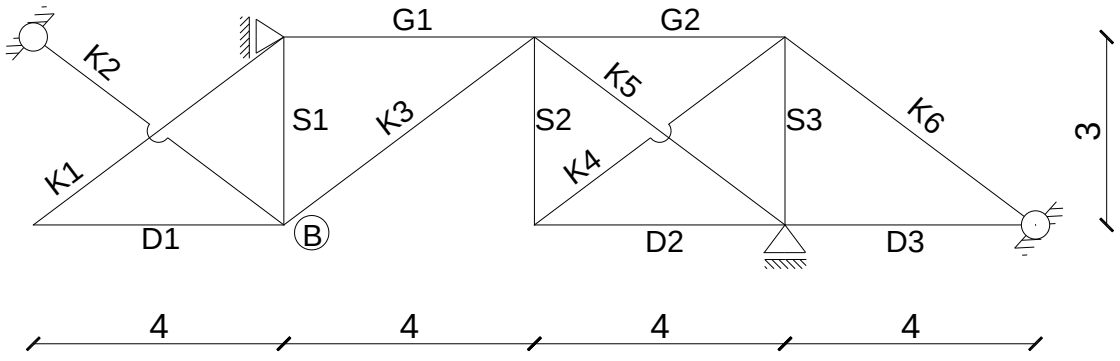


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

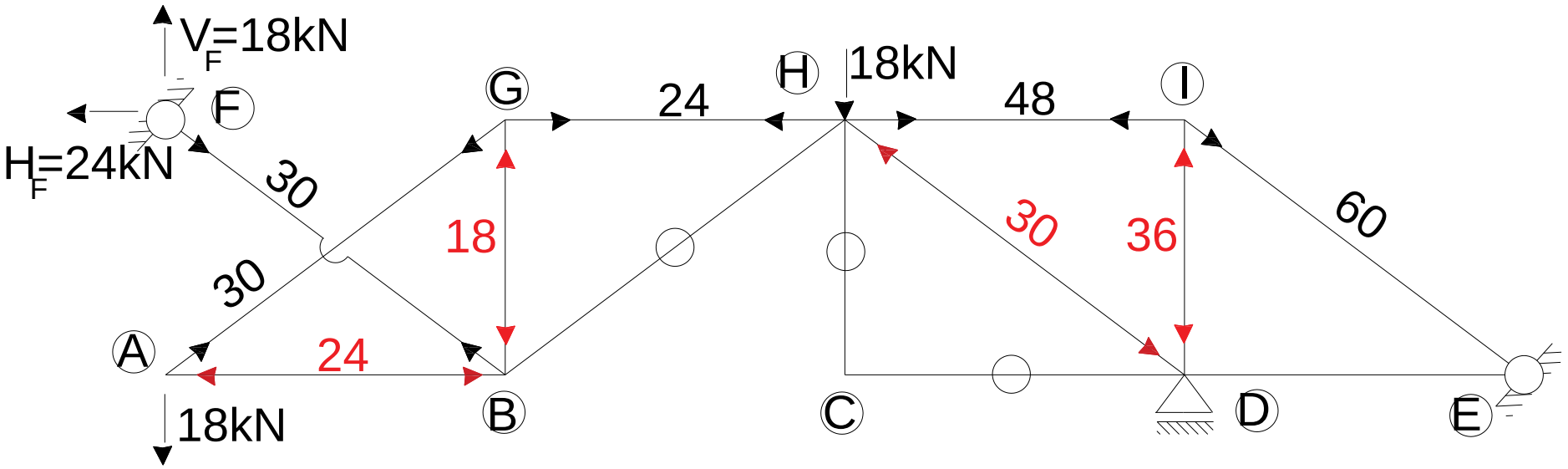


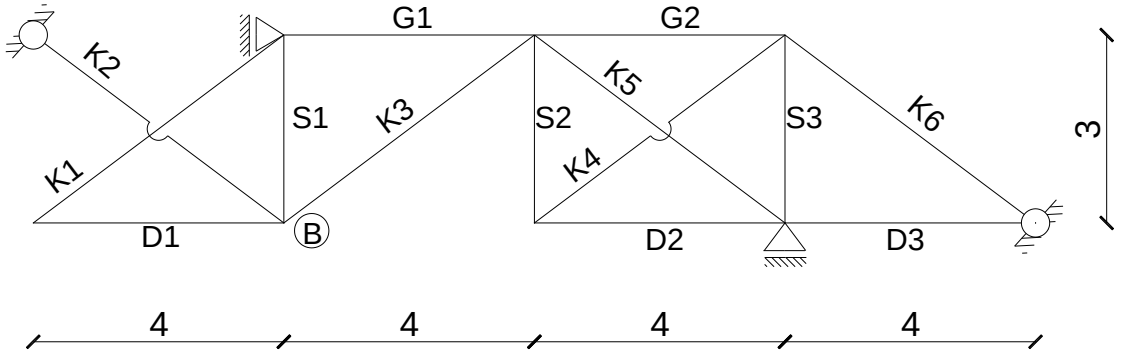


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

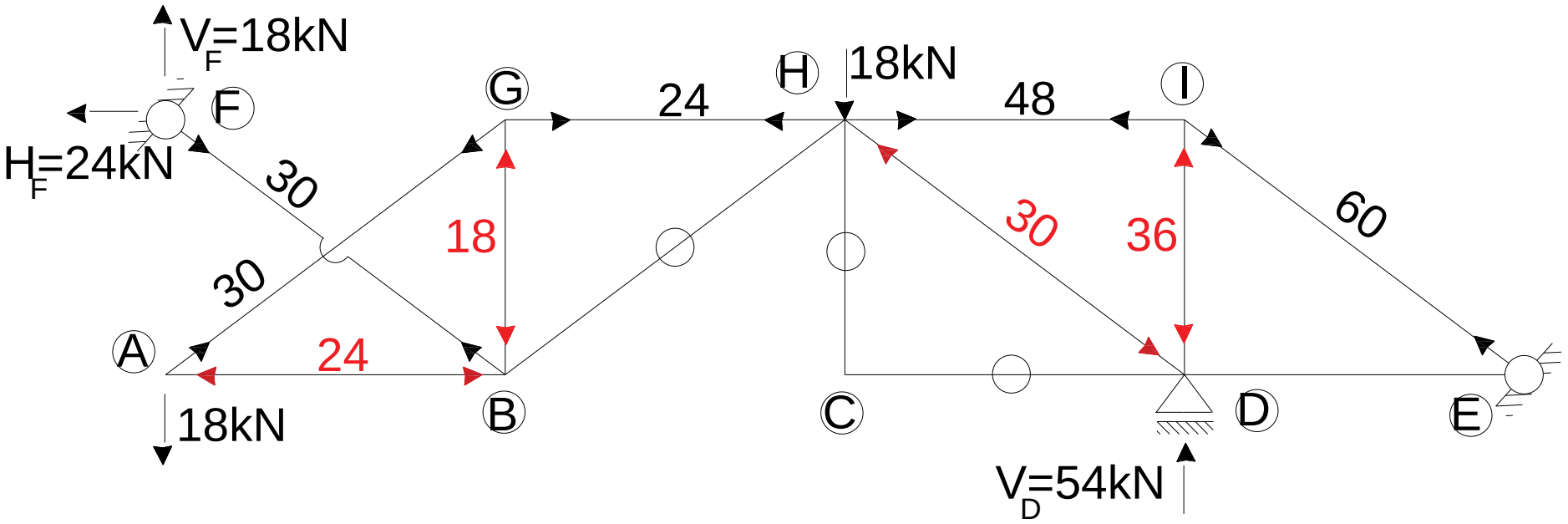


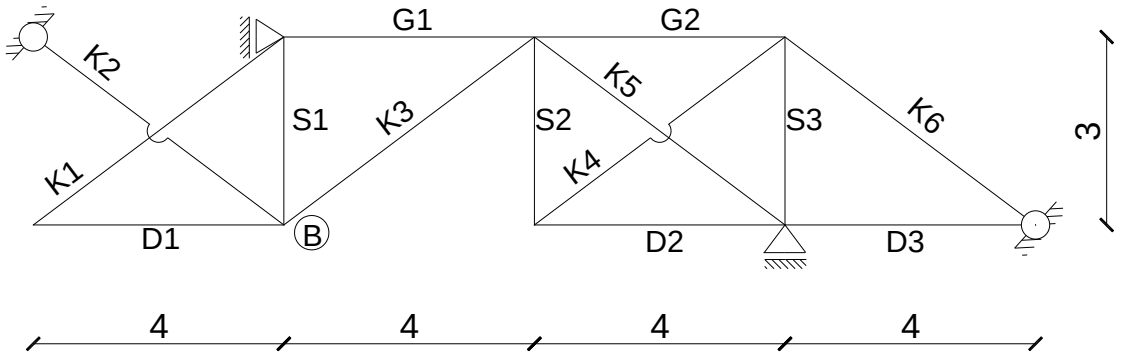


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

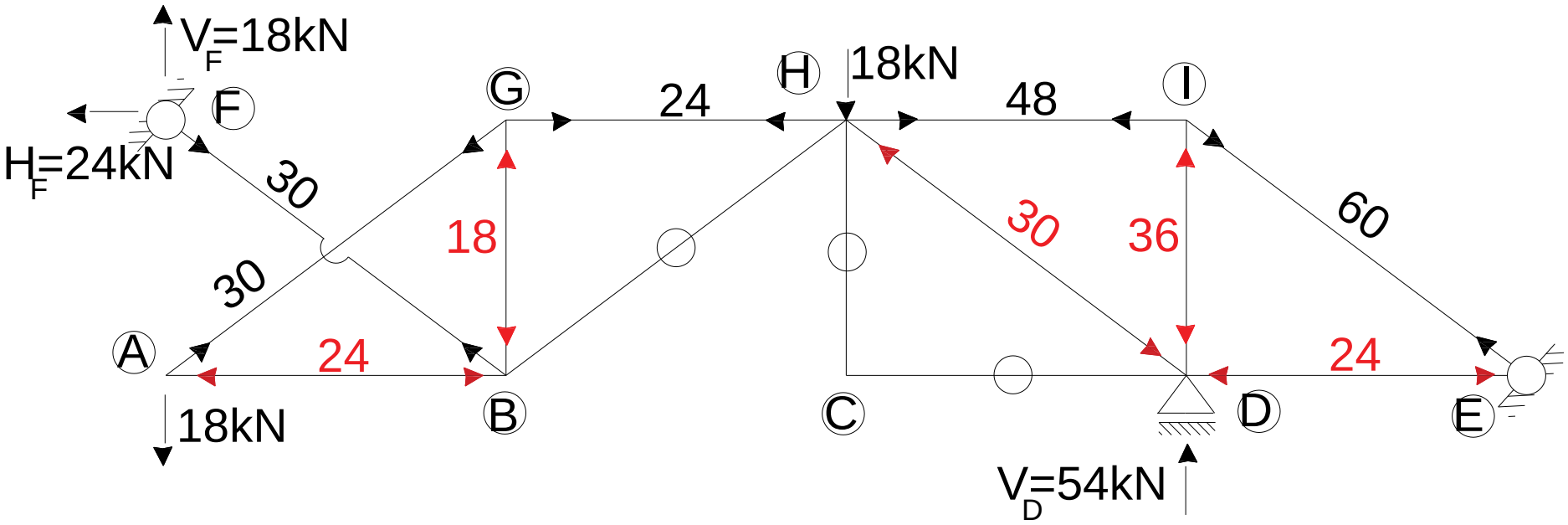


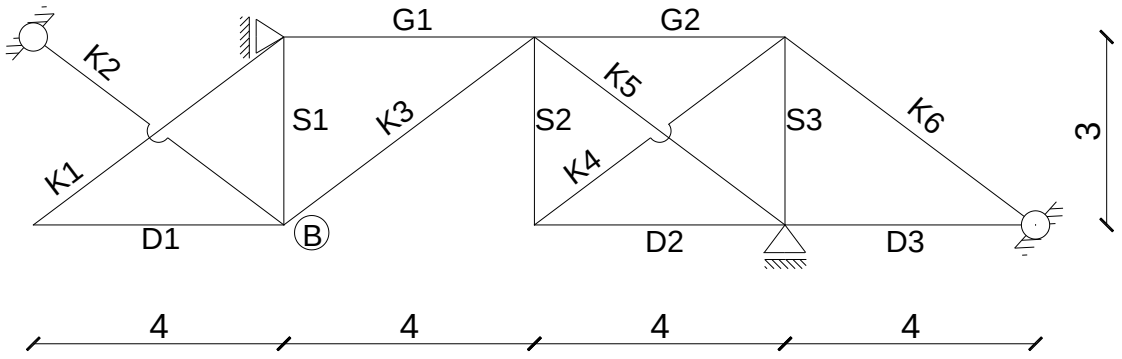


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

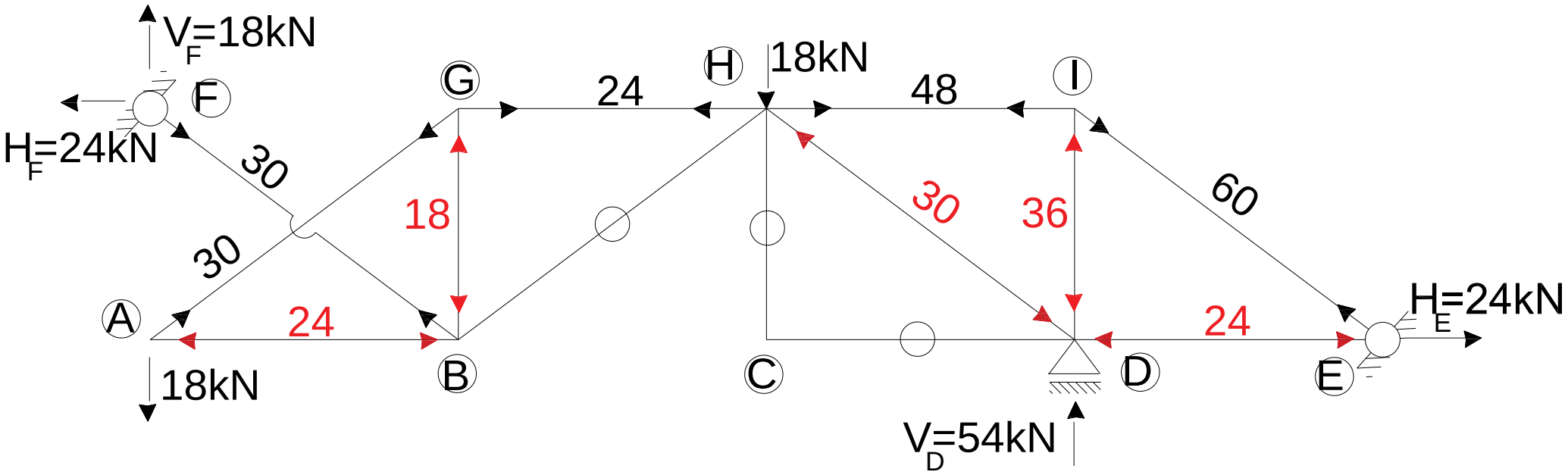


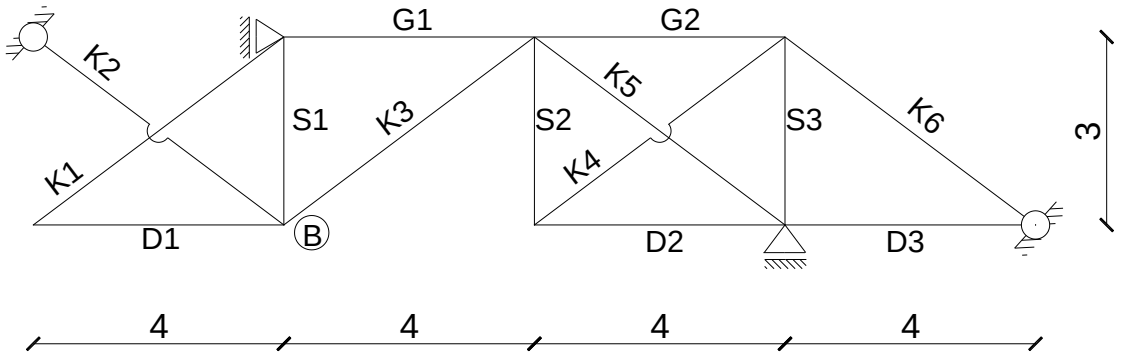


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

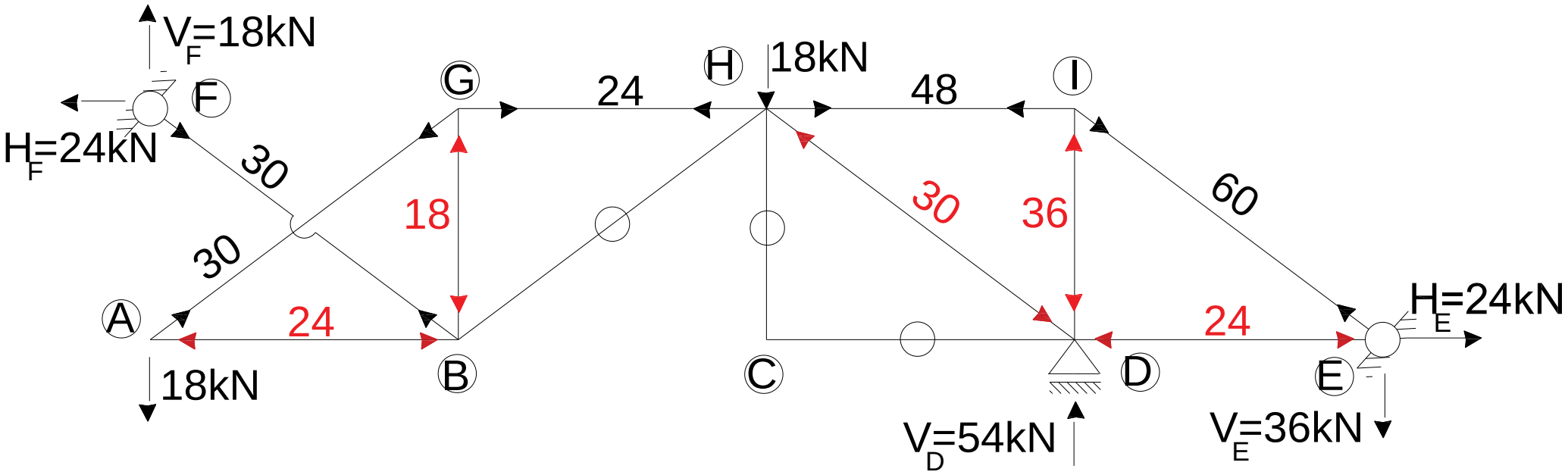


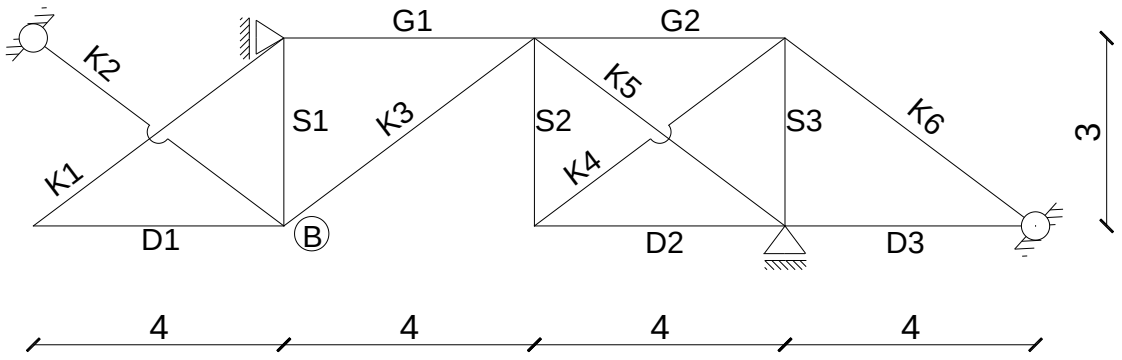


$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]

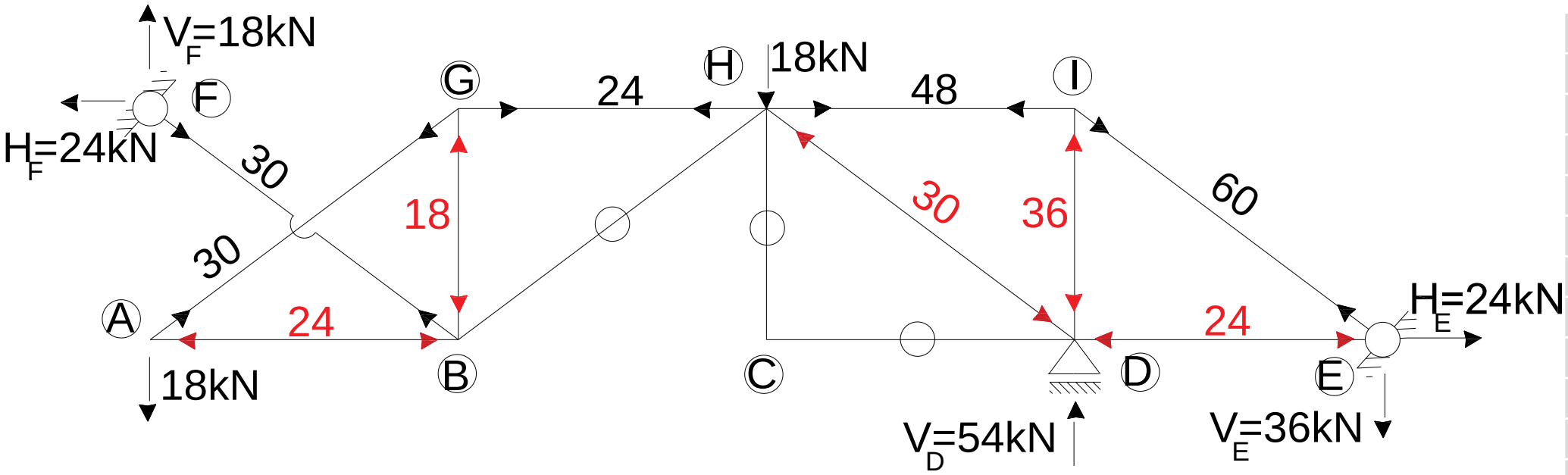




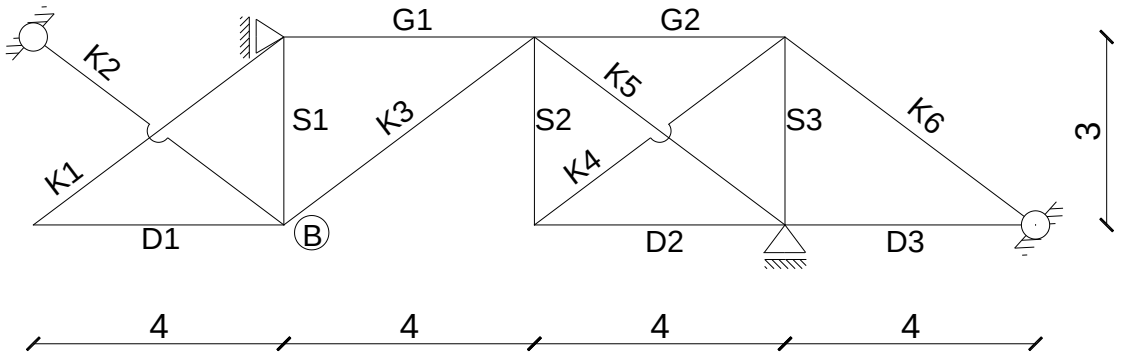
$$\sum M_H^L = -18 \cdot 8 + V_F \cdot 8 = 0 \rightarrow V_F = 18\text{kN}$$

dr inż. Hanna Weber

Wykresy: obciążenie zewnętrzne – siły N0 [kN]



Pręt	L/EA	N0
D1	4	-24
D2	4	0
D3	4	-24
G1	4	24
G2	4	48
S1	3	-18
S2	3	0
S3	3	-36
K1	2,5	30
K2	2,5	30
K3	2,5	0
K4	2,5	0
K5	2,5	-30
K6	2,5	60



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Obliczenia metody sił:

Pręt	L/EA	N1	N2	N0	N1*N1*L/EA	N1*N2*L/EA	N2*N2*L/EA	N1*N0*L/EA	N2*N0*L/EA	N1*X1	N2*X2	N	N1*N*L/EA	N2*N*L/EA
D1	4	0	0	-24	0	0	0	0	0	0,000	0,000	-24,000	0,000	0,000
D2	4	-0,8	0	0	2,56	0	0	0	0	-5,857	0,000	-5,857	18,743	0,000
D3	4	0	0	-24	0	0	0	0	0	0,000	0,000	-24,000	0,000	0,000
G1	4	0	-1	24	0	0	4	0	-96	0,000	-39,942	-15,942	0,000	63,766
G2	4	-0,8	-1	48	2,56	3,2	4	-153,6	-192	-5,857	-39,942	2,201	-7,044	-8,805
S1	3	0	0	-18	0	0	0	0	0	0,000	0,000	-18,000	0,000	0,000
S2	3	-0,6	0	0	1,08	0	0	0	0	-4,393	0,000	-4,393	7,907	0,000
S3	3	-0,6	0,75	-36	1,08	-1,35	1,6875	64,8	-81	-4,393	29,956	-10,437	18,786	-23,483
K1	2,5	0	0	30	0	0	0	0	0	0,000	0,000	30,000	0,000	0,000
K2	2,5	0	0	30	0	0	0	0	0	0,000	0,000	30,000	0,000	0,000
K3	2,5	0	0	0	0	0	0	0	0	0,000	0,000	0,000	0,000	0,000
K4	2,5	1	0	0	2,5	0	0	0	0	7,322	0,000	7,322	18,304	0,000
K5	2,5	1	0	-30	2,5	0	0	-75	0	7,322	0,000	-22,678	-56,696	0,000
K6	2,5	0	-1,25	60	0	0	3,90625	0	-187,5	0,000	-49,927	10,073	0,000	-31,478
					12,28	1,85	13,594	-163,8	-556,5				0,000	0,000
					δ11	δ12	δ22	δ10	δ20					

Układ równań metody sił:

$$\delta_{11}\cdot x1+\delta_{12}\cdot x2+\delta_{10}=0$$
$$\delta_{21}\cdot x1+\delta_{22}\cdot x2+\delta_{20}=0$$

$$\rightarrow$$

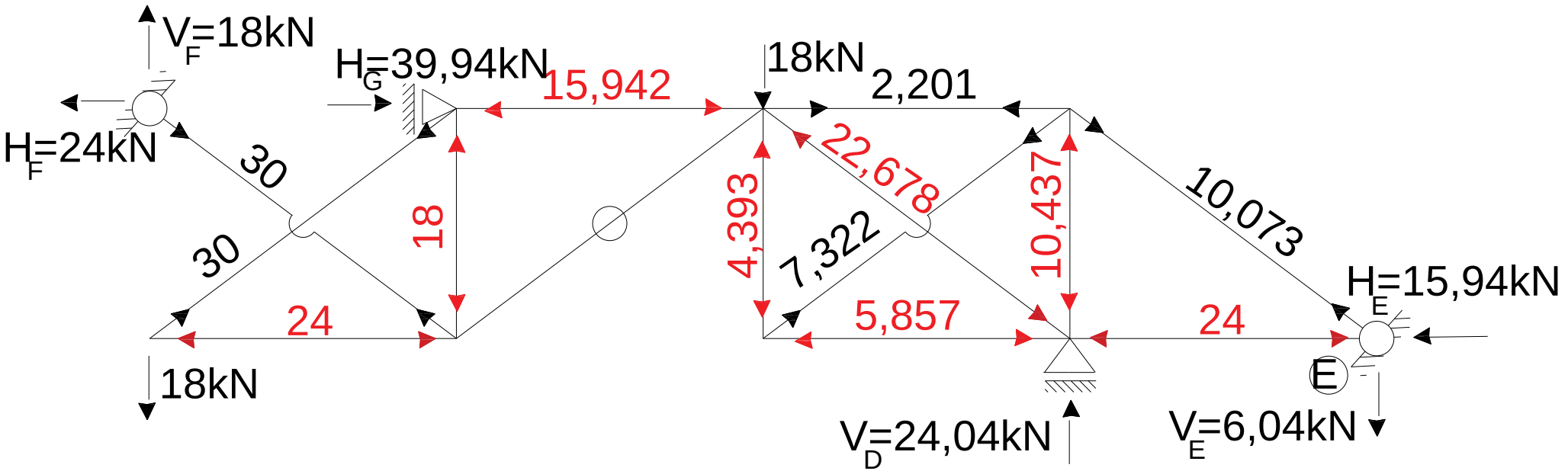
$$12{,}28\cdot x1+1{,}85\cdot x2-163{,}8=0$$
$$1{,}85\cdot x1+13{,}594\cdot x2-556{,}5=0$$

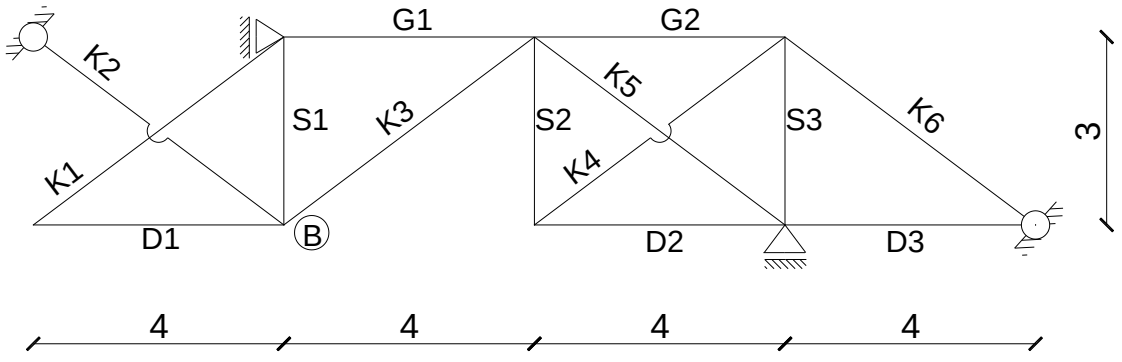
$$\rightarrow$$

$$x1=7{,}322\text{kN}$$
$$x2=39{,}942\text{kN}$$

dr inż. Hanna Weber

Końcowy wykres sił normalnych – N [kN]





Pręt	L/EA	N
D1	4	-24,000
D2	4	-5,857
D3	4	-24,000
G1	4	-15,942
G2	4	2,201
S1	3	-18,000
S2	3	-4,393
S3	3	-10,437
K1	2,5	30,000
K2	2,5	30,000
K3	2,5	0,000
K4	2,5	7,322
K5	2,5	-22,678
K6	2,5	10,073

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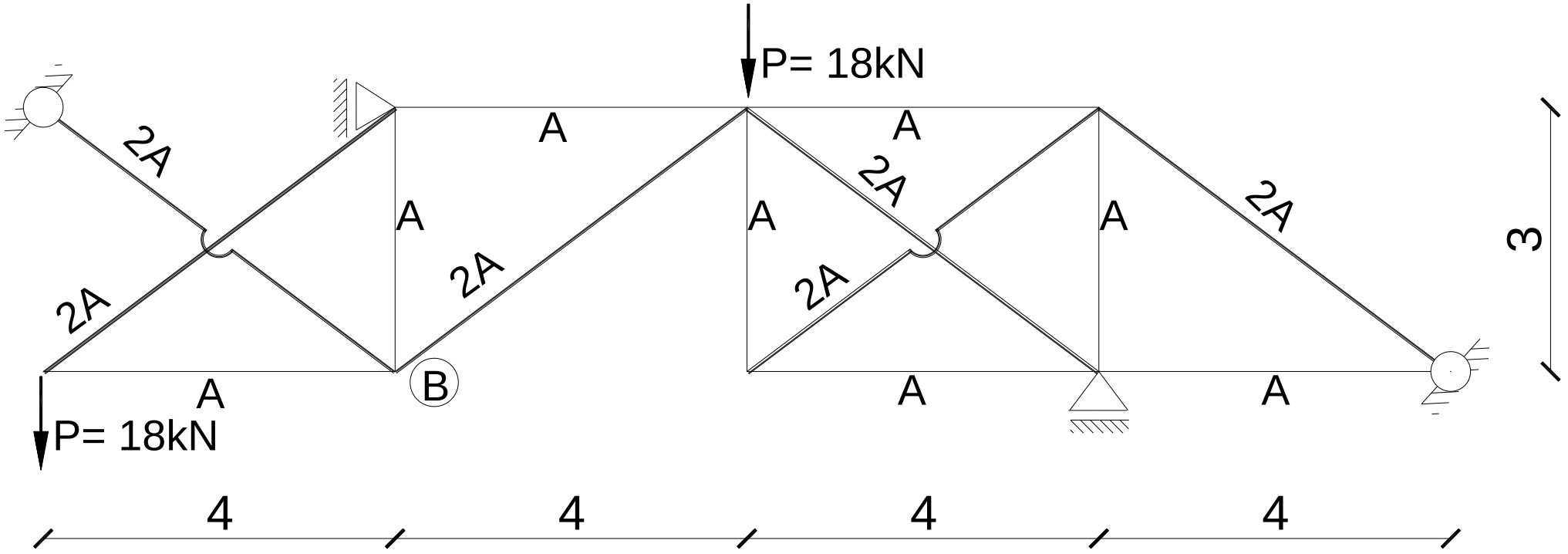
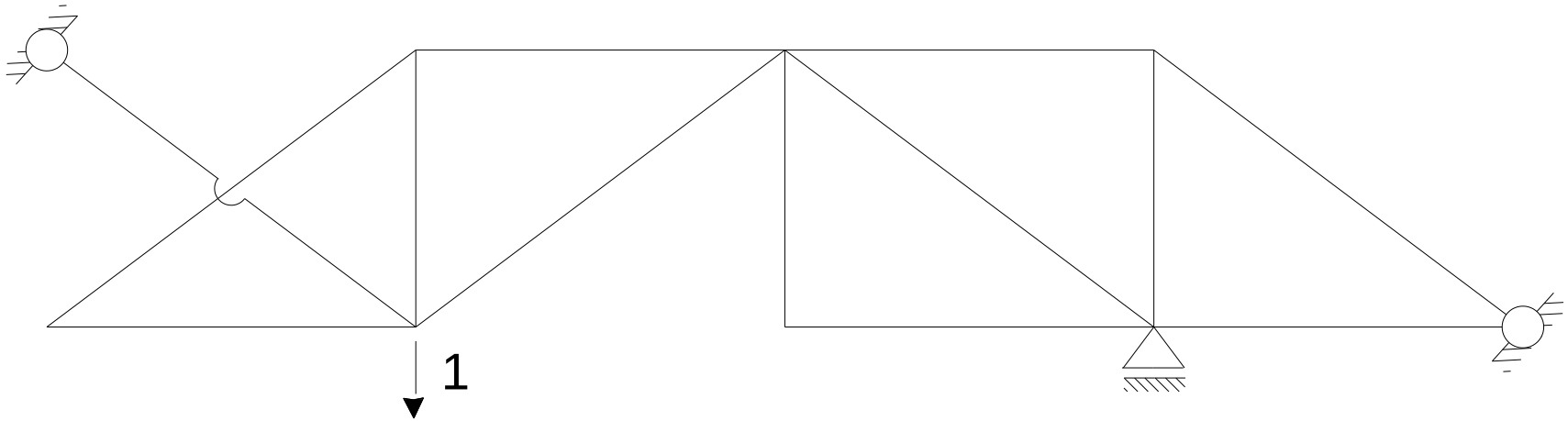
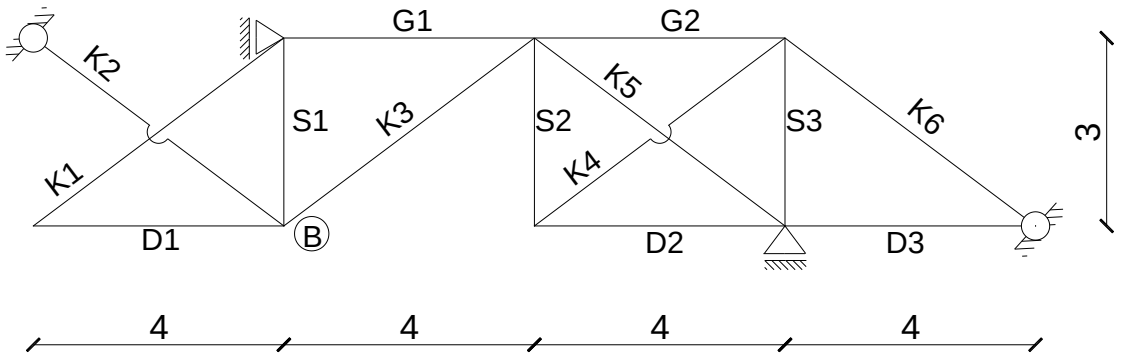
Zadanie 2 : Wykorzystując otrzymany wykres sił normalnych z zad.1. wyznacz przemieszczenie pionowe punktu B z twierdzenia redukcyjnego.



dr inż. Hanna Weber

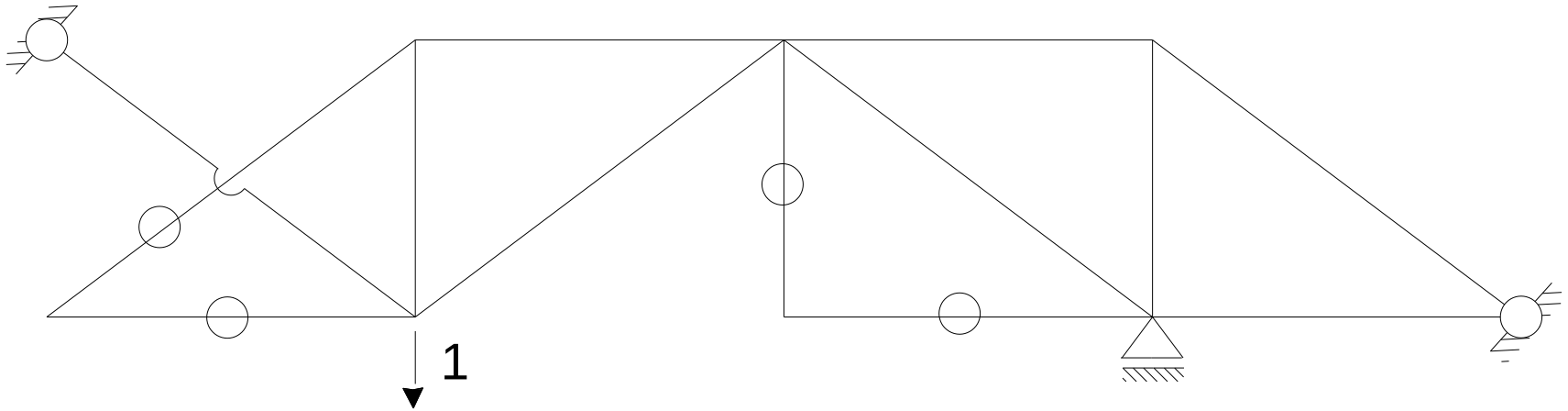
Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

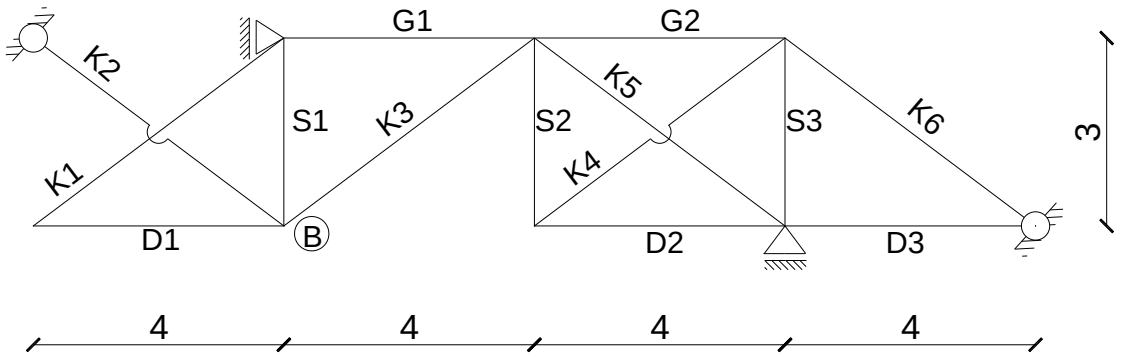




dr inż. Hanna Weber

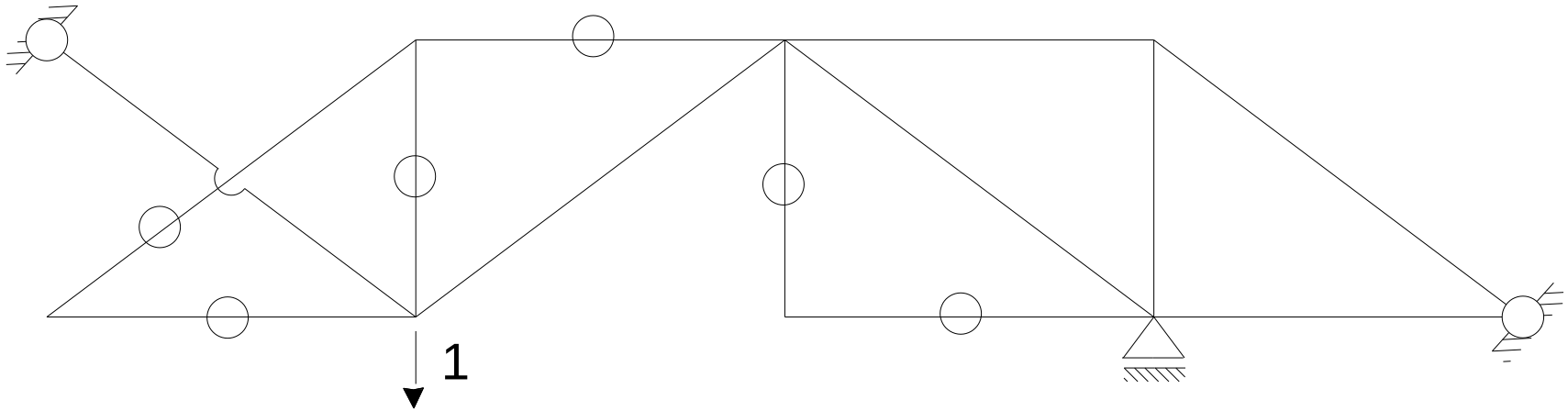
Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

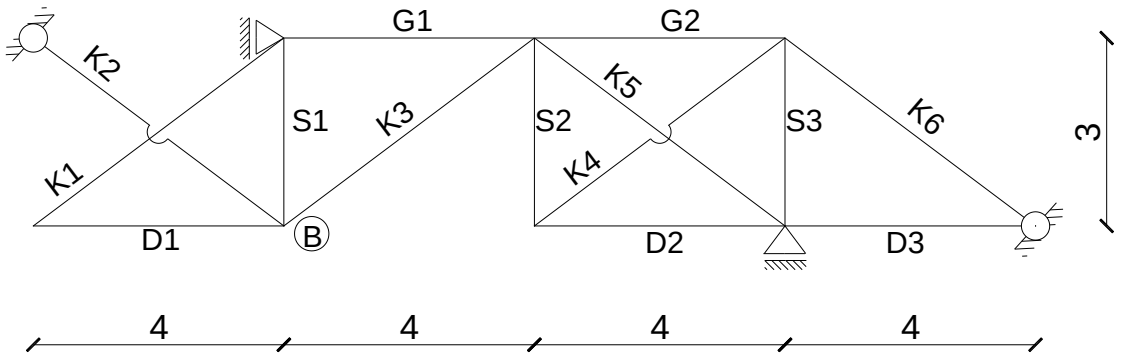




dr inż. Hanna Weber

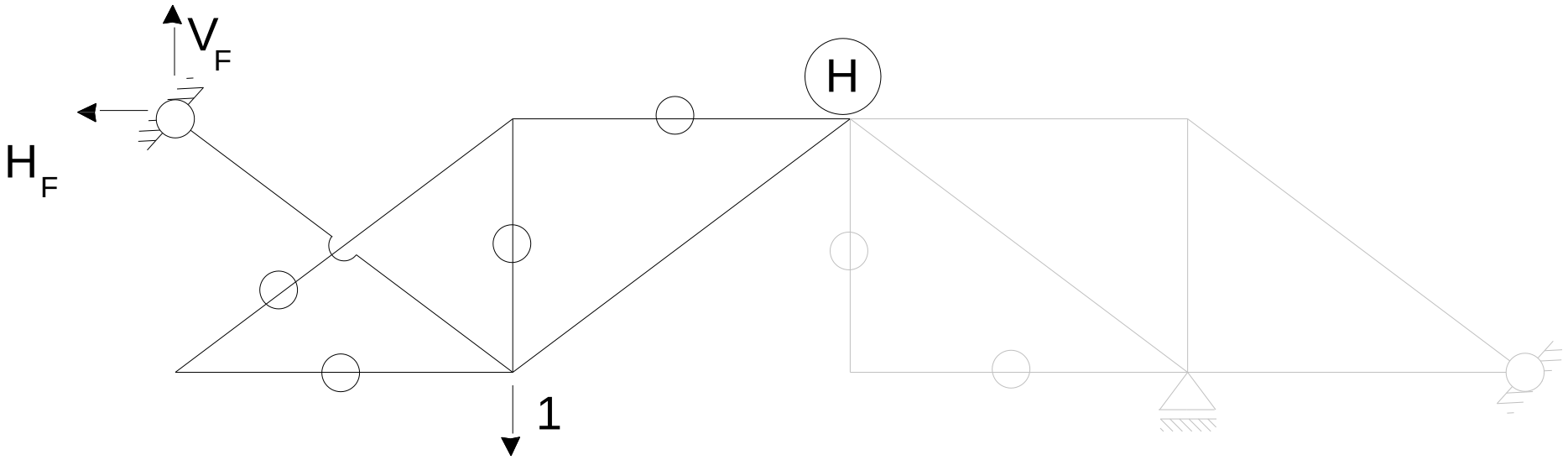
Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:



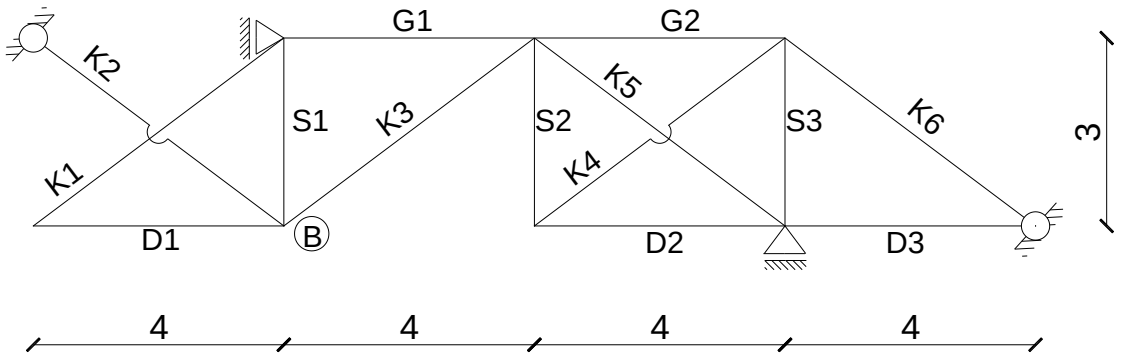


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

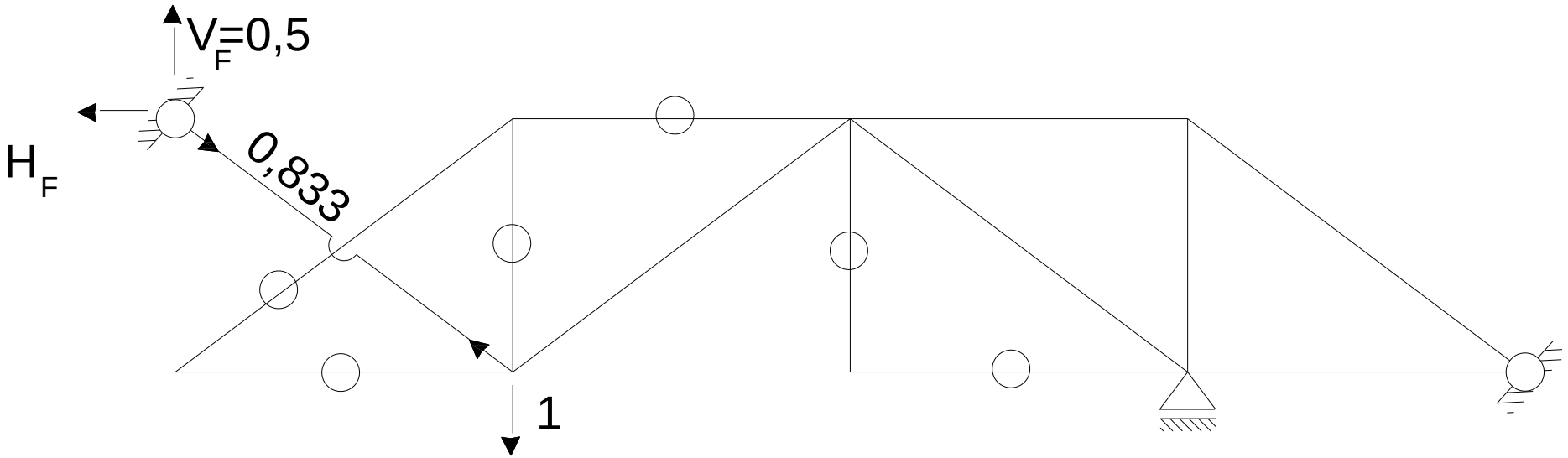


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

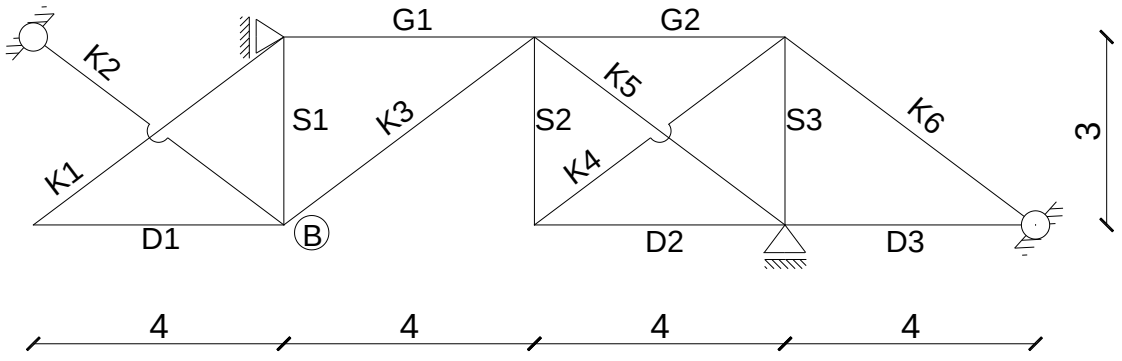


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

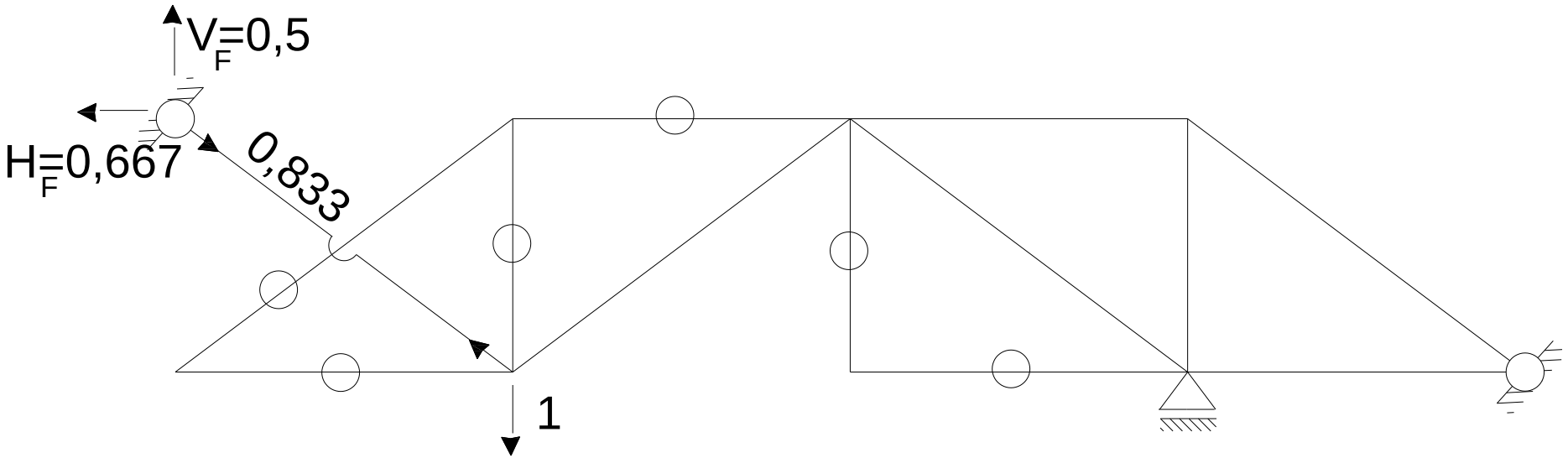


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

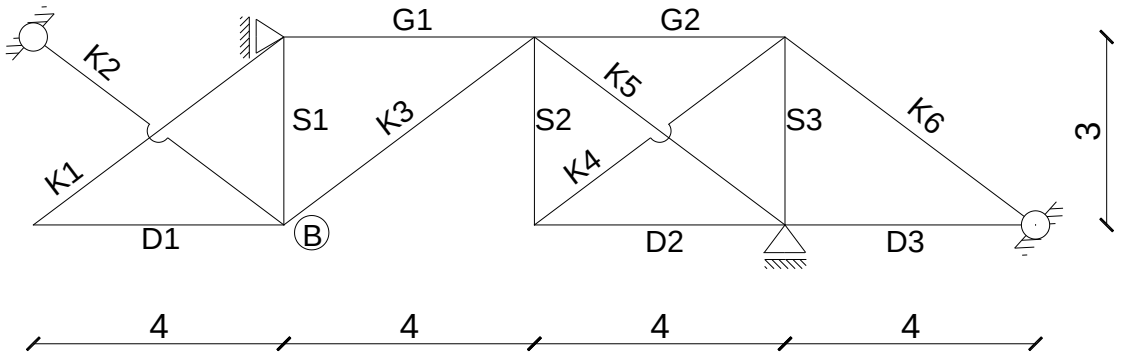


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

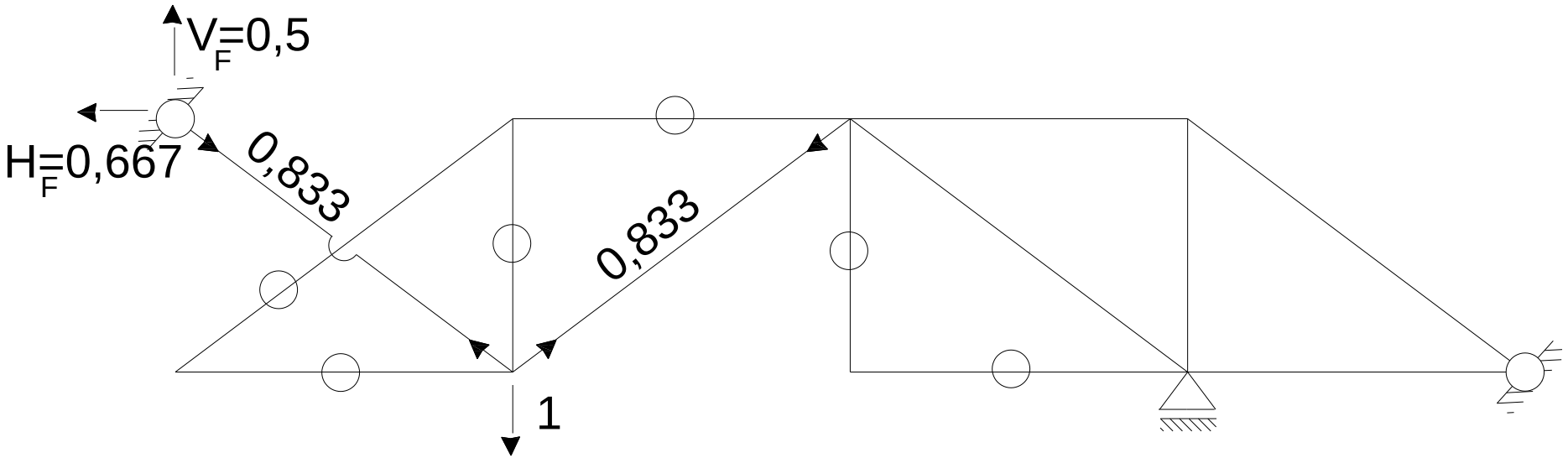


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

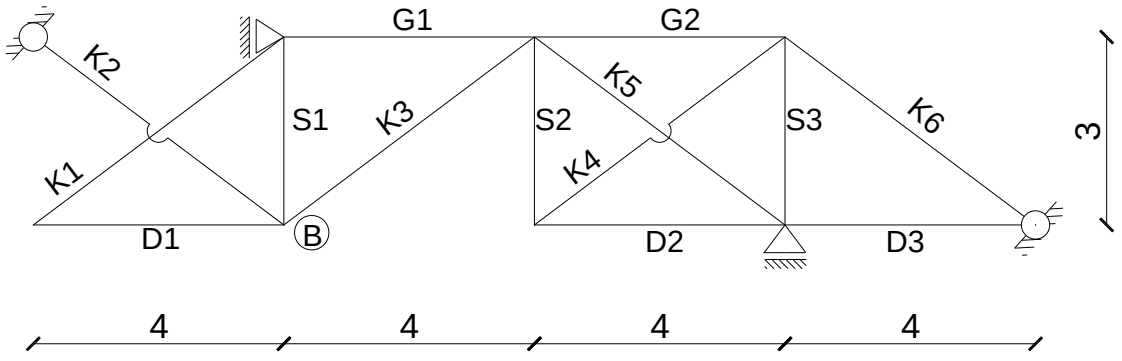


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

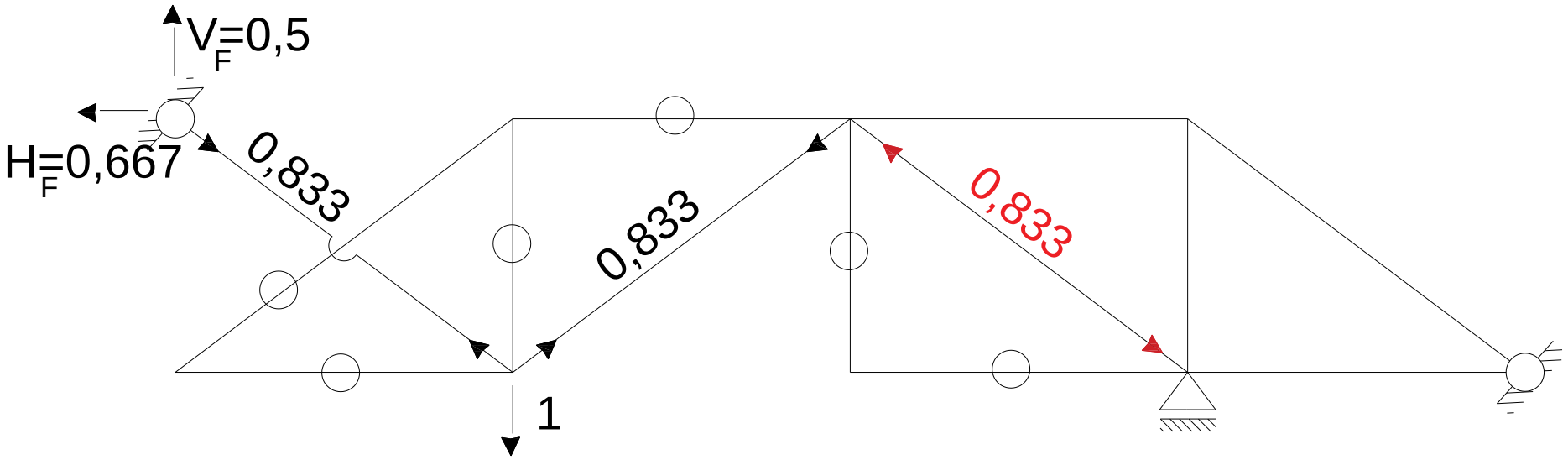


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

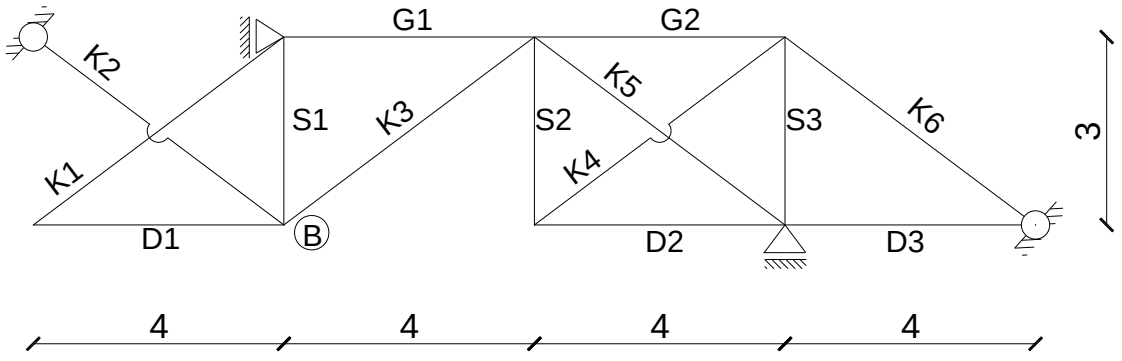


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

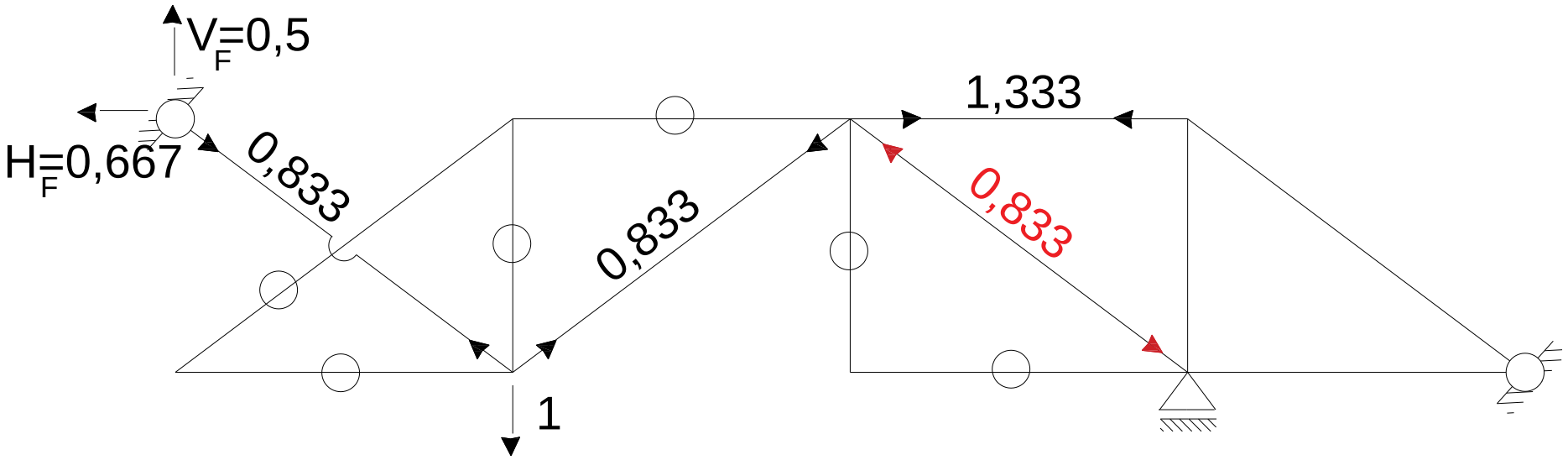


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

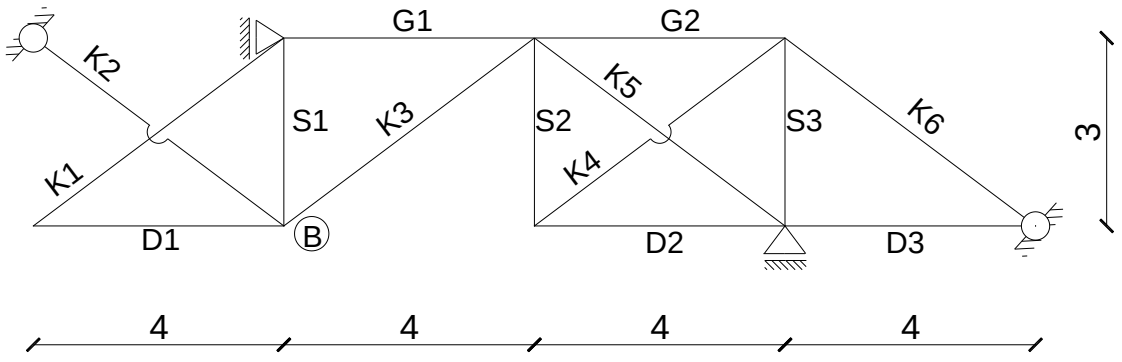


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

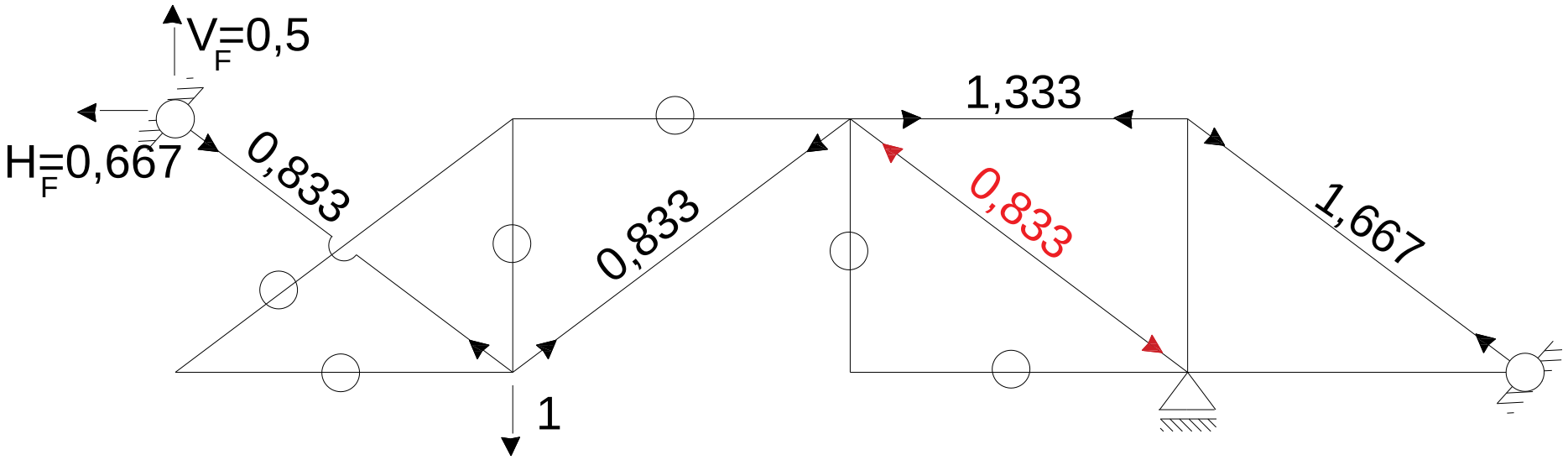


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

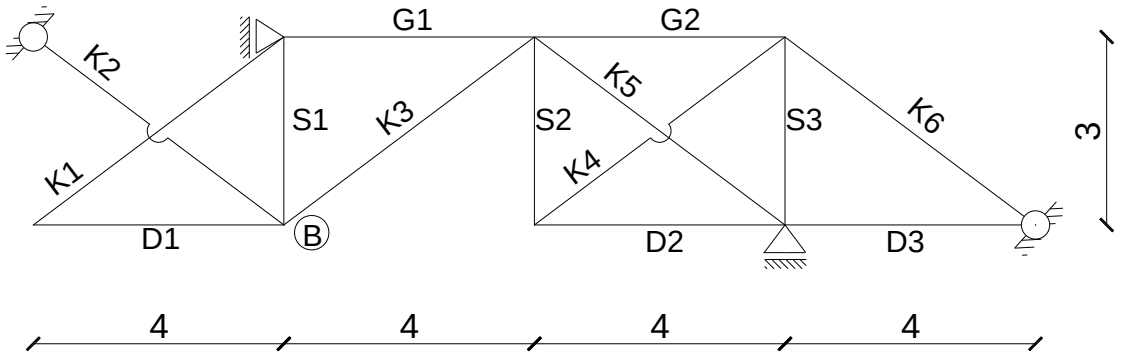


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

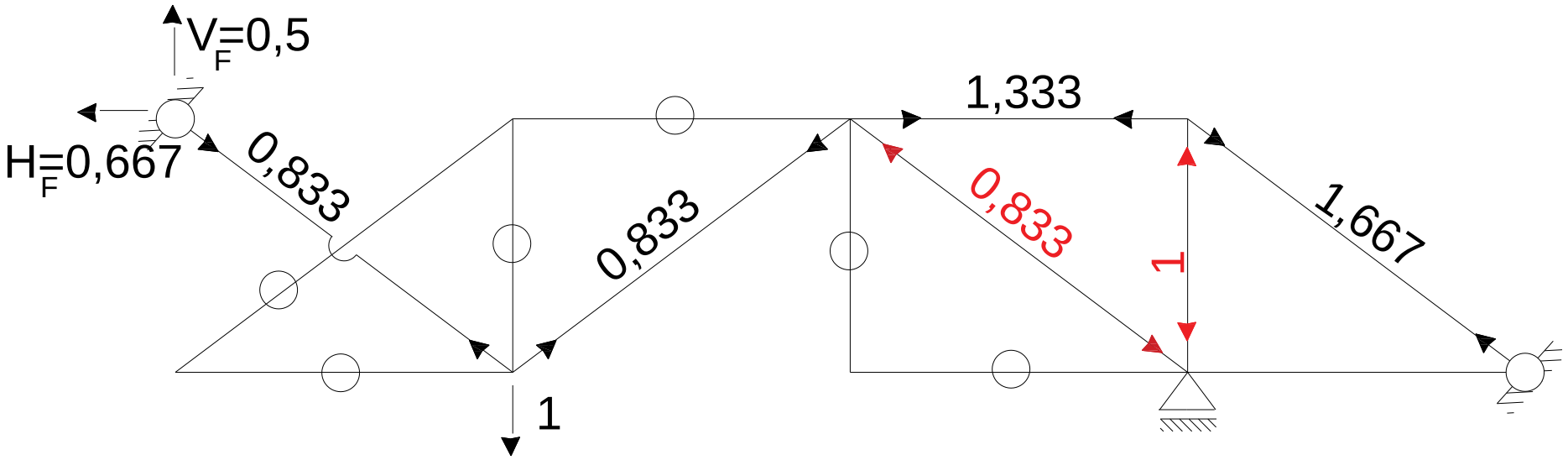


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$

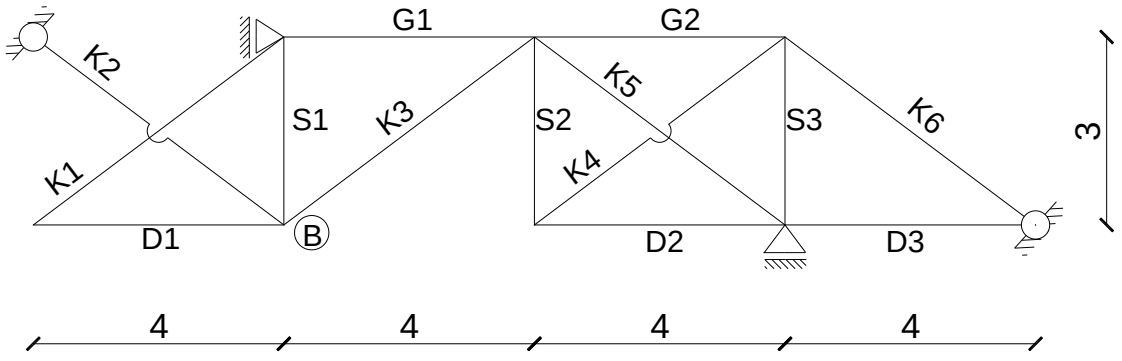


dr inż. Hanna Weber

Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:



$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$



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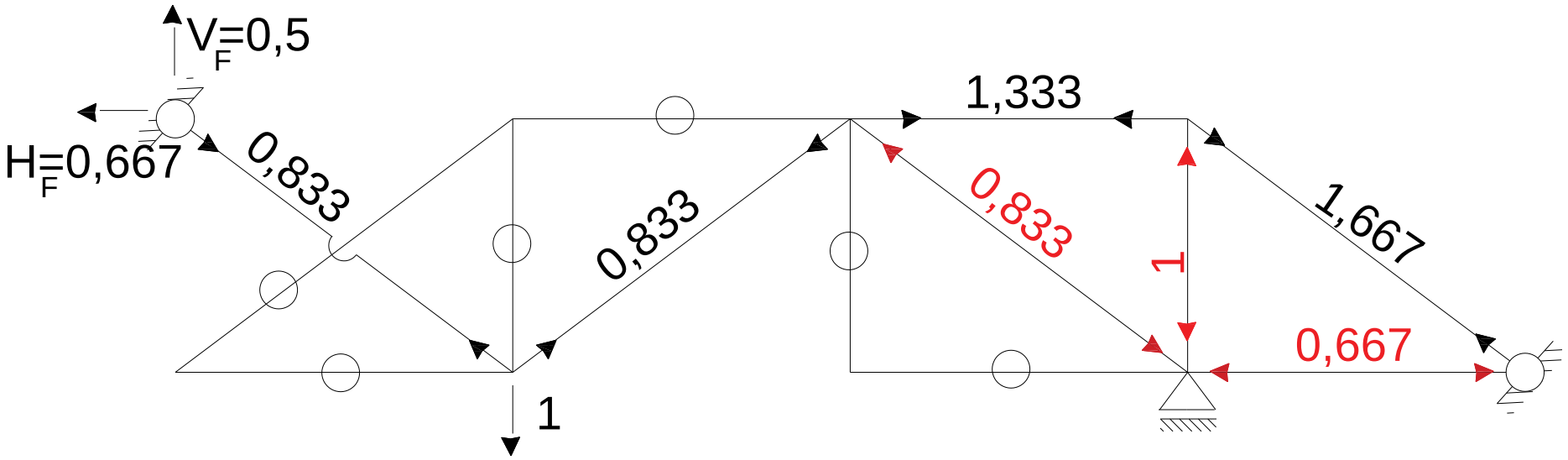
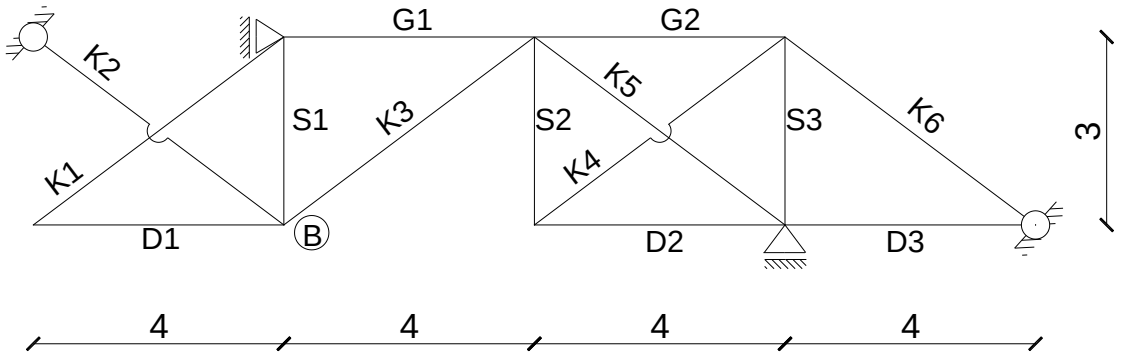


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$



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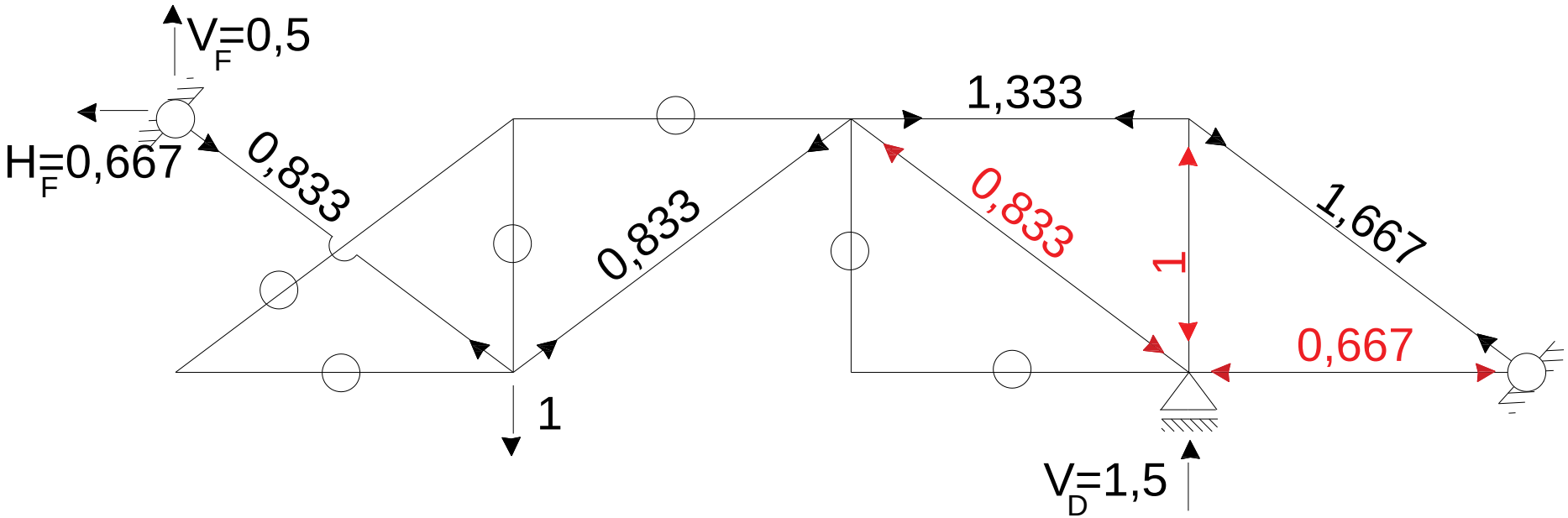
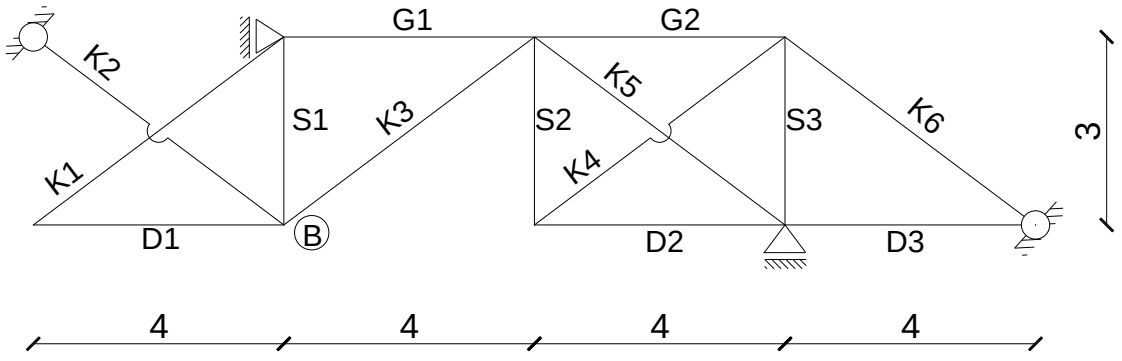


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$



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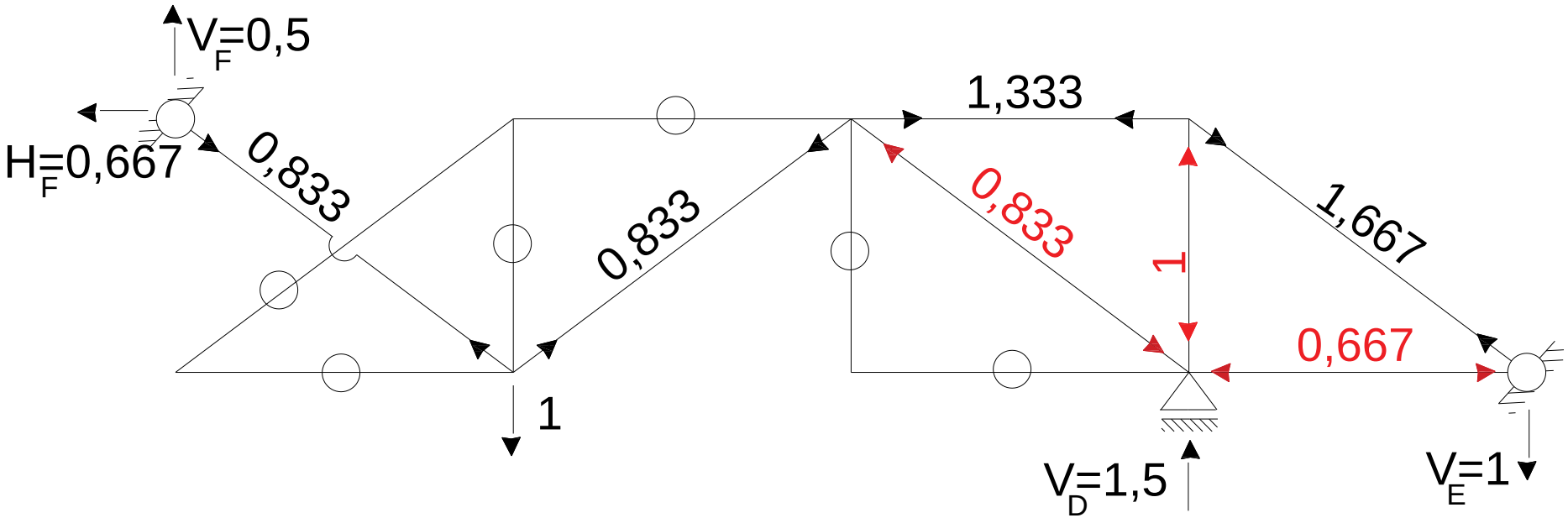
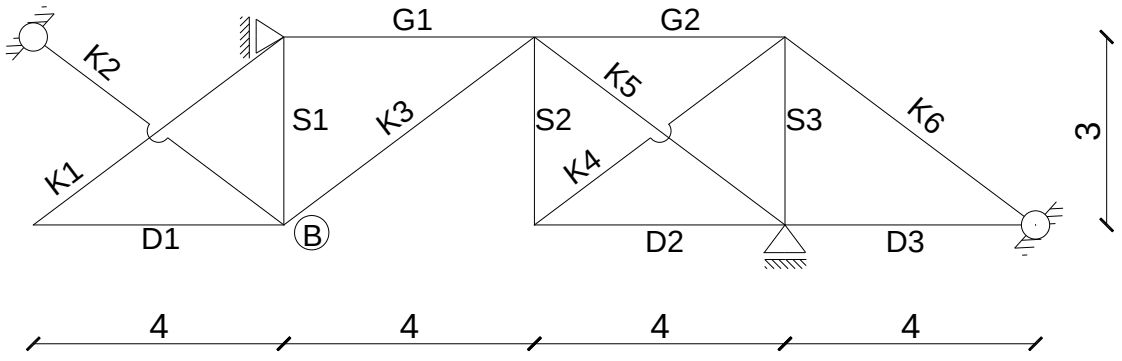


$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$



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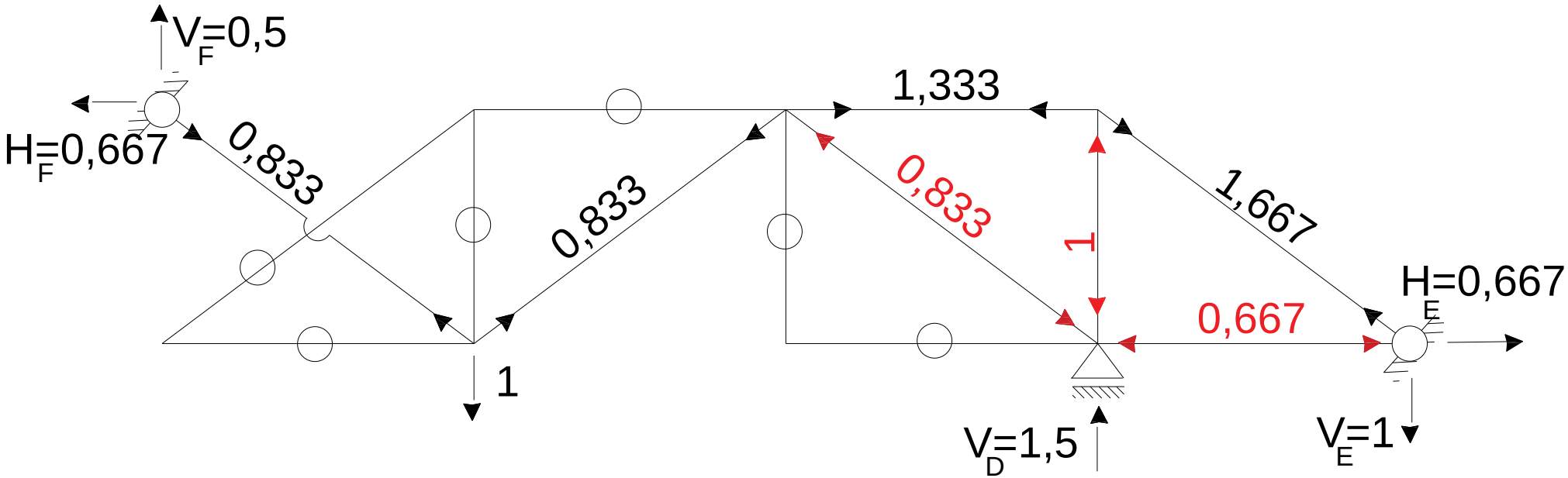
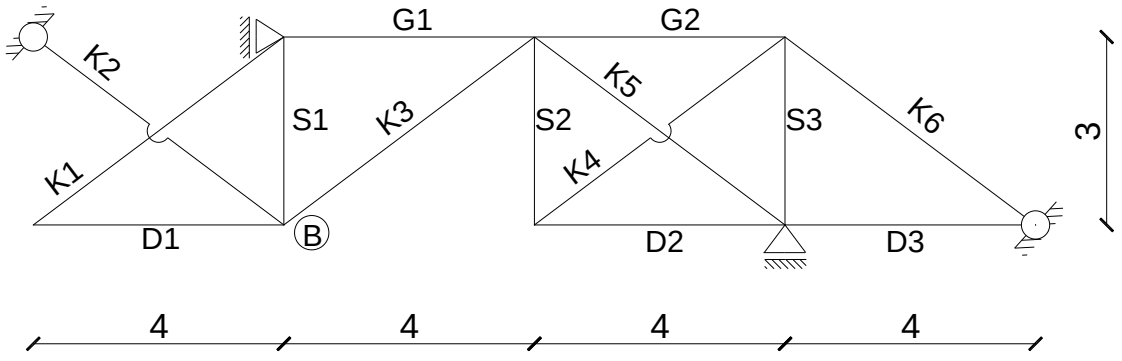
Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:



$$\sum M_H^L = -1 \cdot 4 - V_F \cdot 8 = 0 \rightarrow V_F = 0{,}5$$



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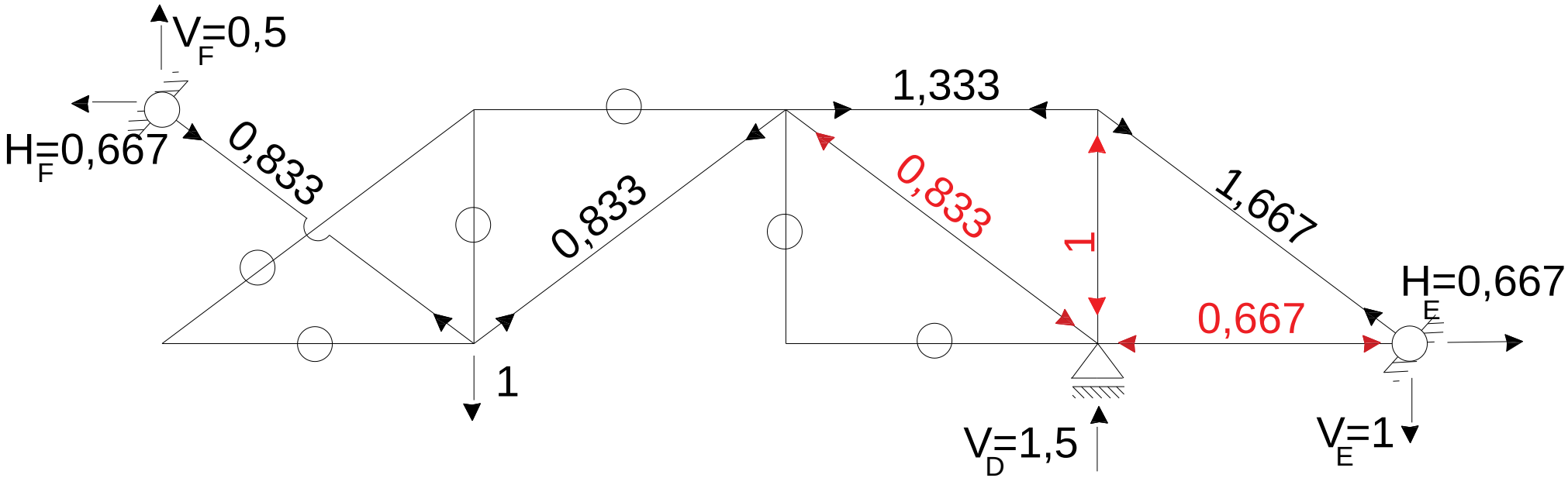
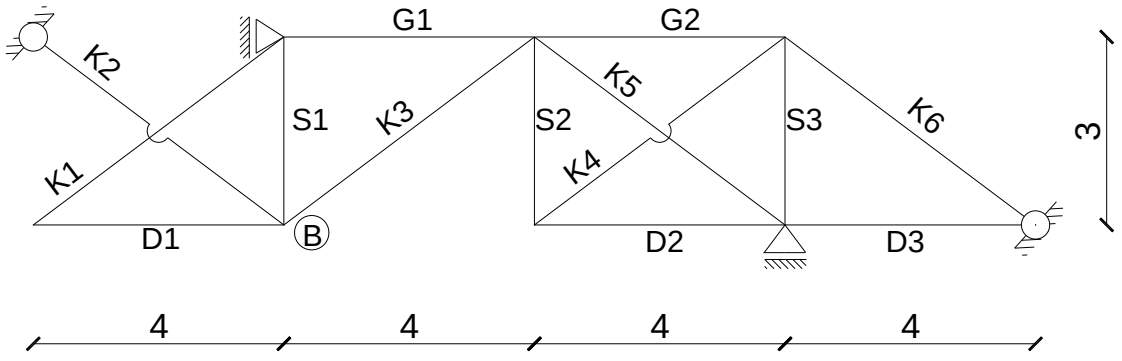
Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:



Pręt	L/EA	N
D1	4	0,000
D2	4	0,000
D3	4	-0,667
G1	4	0,000
G2	4	1,333
S1	3	0,000
S2	3	0,000
S3	3	-1,000
K1	2,5	0,000
K2	2,5	0,833
K3	2,5	0,833
K4	2,5	0,000
K5	2,5	-0,833
K6	2,5	1,667



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Obliczenie przemieszczenia pionowego punktu B z twierdzenia redukcyjnego:

Pręt	L/EA	$\bar{N}$	N	$\bar{N}$*N*L/EA
D1	4	0,000	-24,000	0,000
D2	4	0,000	-5,857	0,000
D3	4	-0,667	-24,000	64,000
G1	4	0,000	-15,942	0,000
G2	4	1,333	2,201	11,740
S1	3	0,000	-18,000	0,000
S2	3	0,000	-4,393	0,000
S3	3	-1,000	-10,437	31,310
K1	2,5	0,000	30,000	0,000
K2	2,5	0,833	30,000	62,500
K3	2,5	0,833	0,000	0,000
K4	2,5	0,000	7,322	0,000
K5	2,5	-0,833	-22,678	47,247
K6	2,5	1,667	10,073	41,971
				258,768

Przemieszenie pionowe punktu B wynosi $v_B = 258,768/EA$

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